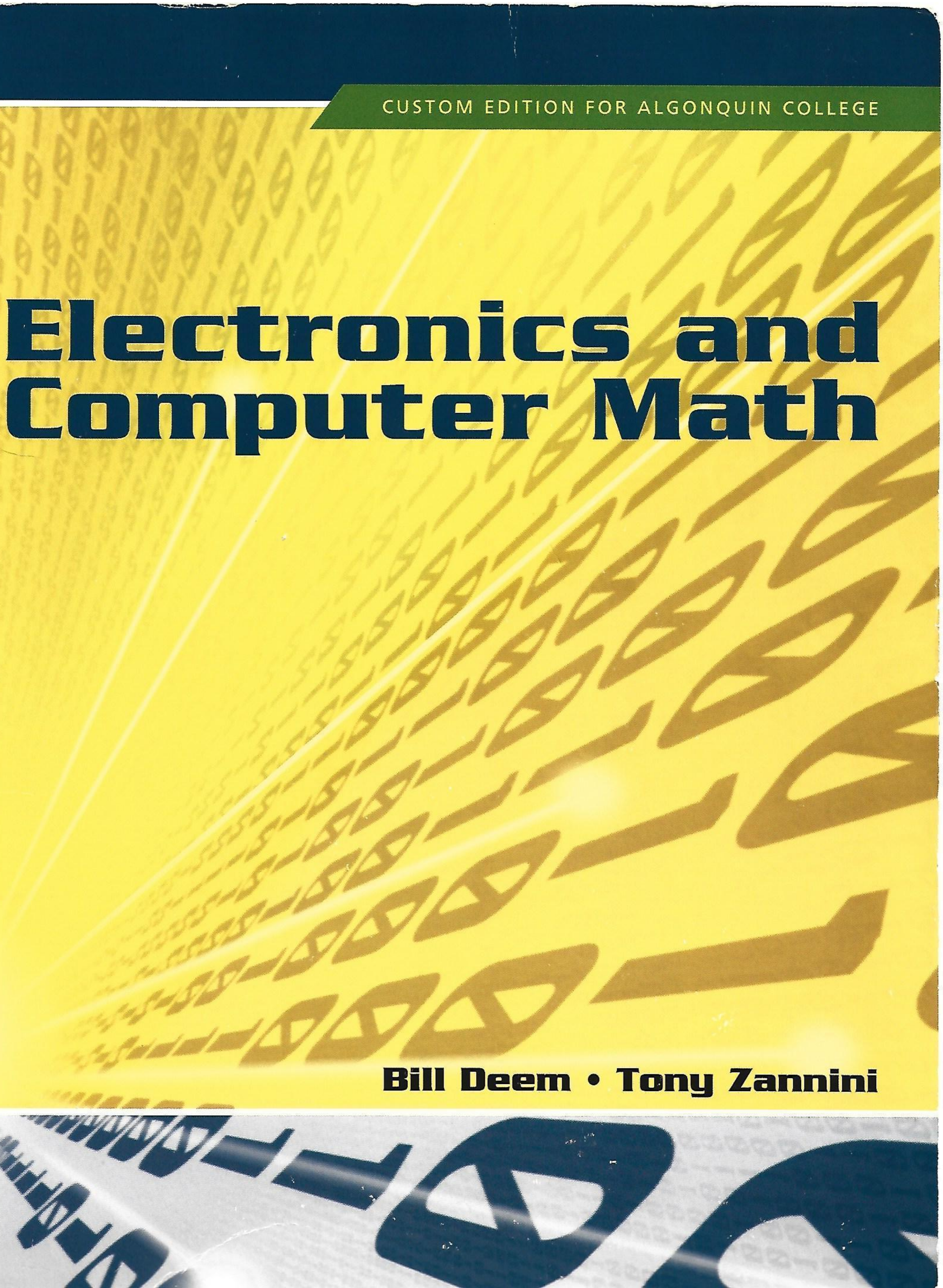


CUSTOM EDITION FOR ALGONQUIN COLLEGE

Electronics and Computer Math

Bill Deem • Tony Zannini

Table 3–1 Partial List of the International System of Units (SI).

SI PHYSICAL CHARACTERISTIC	NAME OF SI UNIT	SYMBOL
Capacitance (*C*)	farad	F
Admittance (*Y*)	siemens	S
Conductance (*G*)	siemens	S
Current (*I*, *i*)	ampere	A
Energy (*w*)	joule	J
Frequency (*f*)	hertz	Hz
Impedance (*Z*)	ohm	Ω
Inductance (*L*)	henry	H
Power (*P*)	watt	W
Reactance (*X*)	ohm	Ω
Resistance (*R*)	ohm	Ω
Susceptance (*B*)	siemens	S
Time (*t*)	second	s
Wavelength (λ)	meter	m
Electromotive force (*E*)	volt	V
Difference in potential (*V*)	volt	V

Table 3–2 Prefixes Used with Electrical Units Listed in Table 3–1.

PREFIX	SYMBOL	POWER OF TEN
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
kilo	k	10^{3}
Units	none	10^{0}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

EXPONENTS

1. $a^m = a_1 \times a_2 \times a_3 \times \cdots \times a_m$
2. $a^0 = 1$
3. $a^{-m} = \dfrac{1}{a^m}$
4. $(a\,b)^m = a^m b^m$
5. $(a \div b)^m = a^m \div b^m$
6. $a^m a^n = a^{m+n}$
7. $a^m \div a^n = a^{m-n}$
8. $(a^m)^n = a^{mn}$
9. $\sqrt{a} = a^{\frac{1}{2}}$
10. $\sqrt[m]{a} = a^{\frac{1}{m}}$
11. $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

LOGARITHMS

1. $\log_a x = y$ means $x = a^y$
2. Common log: $\log x = \log_{10} x$
3. Natural log: $\ln x = \log_e x$
4. $\log_a 1 = 0$
5. $\log_a a^x = x$
6. $\log_a x^z = z \times \log_a x$
7. $\log_a x^{-1} = -1 \times \log_a x$
8. $\log_a (x\,z) = \log_a x + \log_a z$
9. $\log_a (x \div z) = \log_a x - \log_a z$

ALGEBRAIC FACTORS AND PRODUCTS

$ax + ay + az = a(x + y + z)$

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$ and $a \neq 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

RIGHT TRIANGLE TRIGONOMETRY

$$\sin\theta = \frac{X}{Z}$$

$$\cos\theta = \frac{R}{Z}$$

$$\tan\theta = \frac{X}{R}$$

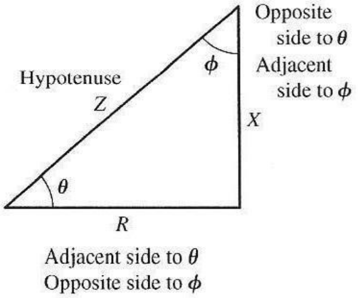


FIGURE 16–26

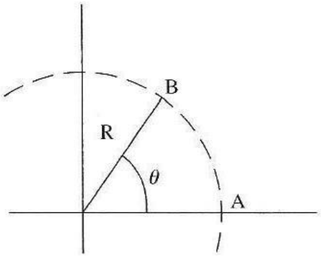
PYTHAGOREAN THEOREM

$$Z^2 = X^2 + R^2$$

RADIAN AND DEGREE EQUIVALENCIES

2π radians $= 360° =$ one cycle

$$\text{one radian} = \frac{360°}{2\pi} \approx 57.3°$$



When arc length AB = R then $\theta = 1$ radian

ANGULAR VELOCITY ω

$\omega = 360°\text{f}$ (in degrees)

$\omega = 2\pi\text{f}$ (in radians)

IMAGINARY NUMBER

Imaginary Number $= \sqrt{x}$ $\quad (x < 0)$

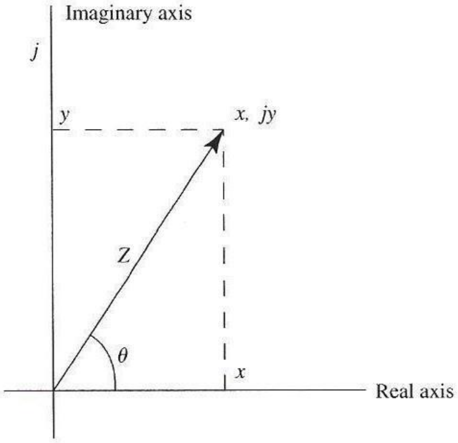
$j = \sqrt{-1}$

$j^2 = -1$

$j^3 = -\sqrt{-1} = -j$

$j^4 = 1$

COMPLEX NUMBER PLANE



COMPLEX NUMBERS

$x + jy$ (rectangular form)

$Z\angle\theta$ (polar form)

COMPLEX NUMBER OPERATIONS

$$(a + jb) \pm (c + jd) = (a \pm c) + j(b \pm d)$$

$$(a + jb)(c + jd) = (ac - bd) + j(ad + bc)$$

$$\frac{a + jb}{c + jd} = \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd} = \frac{ac + bd}{c^2 + d^2} + j\frac{bc - ad}{c^2 + d^2}$$

PEARSON ALWAYS LEARNING

Bill Deem • Tony Zannini

Electronics and Computer Math

Custom Edition for Algonquin College

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by Bill Deem and Tony Zannini

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BRIEF CONTENTS

SECTION 1 **Review of Arithmetic Fundamentals** 1

1 The Decimal Number System 2

2 Powers of Ten 26

3 Units and Prefixes 58

SECTION 6 **Math for Digital Electronics** 489

24 Computer Number Systems 490

25 Boolean Algebra 524

SECTION 7 **Introduction to Statistics in Electronics** 553

26 Introduction to Statistics 554

Appendix

C Answers to Selected Problems 594

CONTENTS

SECTION 1 **Review of Arithmetic Fundamentals** 1

1 The Decimal Number System 2

Math in Action 2
Introduction 2
1-1 Decimal Number System 3
1-2 Decimal Fractions 4
1-3 Whole Numbers and Fractions 8
1-4 Rounding Whole Numbers 9
1-5 Rounding Nonwhole Numbers 10
1-6 Significant Digits 11
1-7 Addition and Subtraction of Signed Numbers 13
1-8 Multiplication and Division of Signed Numbers 14
1-9 Mathematical Expressions and Terms 16
1-10 Parentheses ( ) and Other Signs of Grouping 17
Chapter 1 at a Glance 19
End of Chapter Problems 20

2 Powers of Ten 26

Math in Action 26
Introduction 26
2-1 Converting Numbers to Powers of Ten Form 27
2-2 Converting from Powers of Ten Form to Numbers 28
2-3 Converting Decimal Fractions to Powers of Ten Form 29
2-4 Converting Powers of Ten to Decimal Fractions 30
2-5 Multiplication in Powers of Ten Form 30
2-6 Division in Powers of Ten Form 31
2-7 Combined Multiplication and Division in Powers of Ten Form 33
2-8 Converting between Regular Numbers and Powers of Ten Notation 35
2-9 Expressing Numbers Greater Than One in Scientific Notation 36
2-10 Expressing Numbers Less Than One in Scientific Notation 37
2-11 Addition and Subtraction 39
2-12 Estimating 39
2-13 Problems with Complex Denominators 41
2-14 Reciprocals 42
2-15 Raising a Power of Ten to a Power 45
2-16 Square and Cube Roots in Power of Ten 46
2-17 Squaring Numbers 46
2-18 Square Roots of Numbers 47
Chapter 2 at a Glance 50
End of Chapter Problems 52

3 Units and Prefixes 58

Math in Action 58
Introduction 58
3-1 Units 59
3-2 Prefixes 62
3-3 Converting Numbers with Prefixes to Basic Units 64

3-4 Changing Prefixes 64
3-5 Applications 66
3-6 Systems of Measurement 69
Chapter 3 at a Glance 72
End of Chapter Problems 73

SECTION 6 Math for Digital Electronics 489

24 Computer Number Systems 490

Math in Action 490
Introduction 490
24-1 Binary Number System 490
24-2 Octal Number System 496
24-3 Hexadecimal Number System 498
24-4 Binary to Octal to Hexadecimal Conversions 500
24-5 Binary to Octal to Decimal to Hexadecimal Conversions 502
24-6 Decimal and Octal Addition 504
24-7 Adding Hexadecimal Numbers 506
24-8 Adding Binary Numbers 507
24-9 Decimal and Octal Subtraction 509
24-10 Subtracting Hexadecimal Numbers 511
24-11 Subtracting Binary Numbers 512
24-12 Complement Method of Subtraction 512
24-13 Sixteen's and Eight's Complement 516
Chapter 24 at a Glance 518
End of Chapter Problems 518

25 Boolean Algebra 524

Math in Action 524
Introduction 524
25-1 Logic Gates 525
25-2 Boolean Postulates and Theorems 533
25-3 De Morgan's Theorem and the Absorption Theorem 539
25-4 Application Problems 545
Chapter 25 at a Glance 547
End of Chapter Problems 548

SECTION 7 Introduction to Statistics in Electronics 553

26 Introduction to Statistics 554

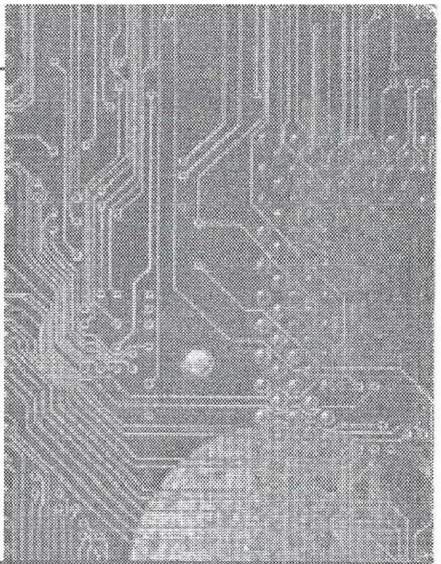
Math in Action 554
Introduction 554
26-1 Frequency Distribution 556
26-2 Measures of Central Tendency 561
26-3 Standard Deviation 565
26-4 The Normal Curve 568
Chapter 26 at a Glance 570
End of Chapter Problems 571

Appendix

C Answers to Selected Problems 594

SECTION ONE

Review of Arithmetic Fundamentals



1 Decimal Number System
2 Powers of Ten
3 Units and Prefixes

Topics in this section include a review of arithmetic fundamentals, scientific notation, electrical units, and terms related to electronics and computers. Many of the concepts in this section are usually first taught in grade or high school; consequently, detailed explanations are not always provided. It is important to master the concepts in this section because they form the foundation for the next section on algebra. Adult learners who have been out of school for several years or recent high school graduates who want to review their math skills need to practice solving these chapter problems until they can do them quickly with no errors.

Chapter 1 reviews the decimal number system, whole numbers, fractions, rounding, addition, subtraction, multiplication, division, and mathematical expressions and terms.

Chapter 2 covers powers of ten, scientific notation, reciprocals, and exponent operations. And a method of estimating calculations is presented.

Chapter 3 introduces engineering notation, the International System of Units, prefixes of electrical units, and conversions between the metric and English systems of measurements. Scientific units are introduced here so that students will become familiar with the terminology of typical electronics and computer problems. Students will learn the physical characteristics of units such as "volts" and "watts" in their Electronics courses.

"How many . . .?" is a common question in technology. Technologists use algebra (and sometimes calculus) to model electronic and computer systems so that mathematical techniques can be used to find solutions to problems. At some point in the problem solving process the topics covered in this section are used to calculate the answer to the question, "How many . . .?" Students who master the topics in this section will improve their problem solving skills and find the next section challenging and fun.

CHAPTER 1

The Decimal Number System

CHAPTER OBJECTIVES

In this chapter you will:

1. Develop an understanding of the decimal number system and the concept of place value.
2. Learn how to read whole numbers and decimal fractions.
3. Learn how to convert from decimal fractions to decimal numbers.
4. Learn the rules for rounding both whole and nonwhole numbers.
5. Learn how to work with signed numbers.
6. Learn how to add, subtract, multiply, and divide decimal numbers.
7. Learn how to recognize and simplify various mathematical expressions and terms.

MATH IN ACTION

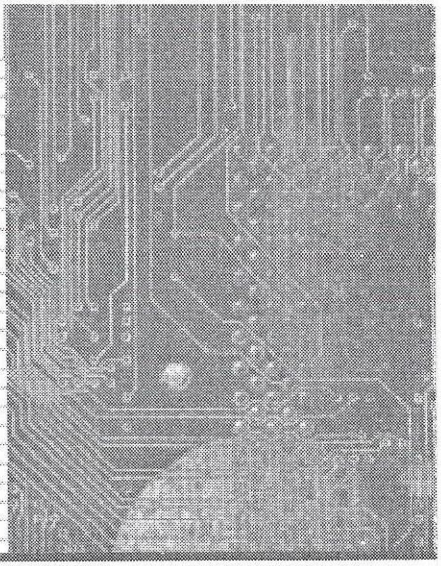
Mark needs to order supplies for the parts inventory in the computer repair and supply store where he works. His boss asks him to order 15 USB cables, 8 serial port cables, and 6 parallel port cables. Mark checks the cost of each item on the Internet and discovers there is a discounted price for orders of more than 10 of the same item. Mark's boss asks for a rough estimate of the cost and if it would be worth increasing the order to take advantage of the volume discounts. Mark rounds off the numbers to the nearest dollar and multiplies out the costs for each item.

Mark's boss decides to increase the number of serial port cables to 10, but doesn't make any changes for the other items. Now Mark must fill out a purchase requisition that indicates the unit cost, total cost per item, and total cost for all items. Since he now needs exact values, Mark multiplies and adds the complete decimal numbers. A shipping charge of $14.98 must also be added to the total.

Mark next checks on supplies of printer paper. He starts with the number of reams of paper in the store at the end of the previous day, and subtracts the number sold so far today and finds there are 36 reams left. He notes that there are 50 reams on order from the paper company, which should arrive in another six days. But will their current inventory last until then? Mark adds up the sales of paper over the last month and divides by the number of days the store was open. This tells him that, on average, the store sells 9 reams of paper per day. If the average sales holds for the next four days, they will run out of paper before the next shipment arrives. Mark puts in a rush order for another 50 reams of paper, to be delivered in two days.

INTRODUCTION

The decimal number system is the number system that we have always used. We all grew up using this system for all computations. Many of the concepts we will discuss in this chapter we have applied for years without really thinking about them. We are going to spend some time discussing the decimal numbering system in order to help us better understand working with numbers and to pave the way for a quicker understanding of other number systems. These other number systems, such as the *binary number system* and the *hexadecimal number system*, are important number systems used in the computer world and will be discussed in Chapter 24.



The chapter is primarily a review of basic arithmetic. We will review the process of adding, subtracting, multiplying, and dividing mathematical terms and expressions. In this chapter we will convert between decimal numbers and decimal fractions, and we will learn how to round numbers. Rounding numbers is a common practice in electronics because we seldom need accuracies beyond three decimal digits. The concepts learned or relearned in this chapter are used throughout the text.

1–1 DECIMAL NUMBER SYSTEM

Our number system is called the decimal number system. *Decimal* means ten. In the decimal number system there are ten symbols. These symbols are called *digits*. The ten digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Zero is the digit having the least value. In counting, when the count reaches 9, a limit has been reached because 9 is the digit of greatest value. An additional count produces a *carry*. This carry is equal to 10 and is said to occupy the tens position; the 0 occupies the ones position. This process continues each time 9 is reached in the ones position. When a 9 appears in the tens position, the next carry to the tens position produces a carry to the hundreds position. In Table 1–1 we show the names for places up to millions.

It is important to note that a 1 in the tens position possesses a value *(weight)* 10 times that of a 1 in the ones position. The same is true for each position of greater weight. Because of this relationship, the decimal numbering system is called a *place value system* and its base is 10.

DECIMAL NUMBER SYSTEM a number system that uses the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

CARRY In addition, if the sum of a column of digits exceeds 9, then the number transferred to the column to the left is called a carry and represents the number of tens that is to be transferred.

BASE the quantity of symbols used to represent numbers in a numerical system.

EXAMPLE 1–1

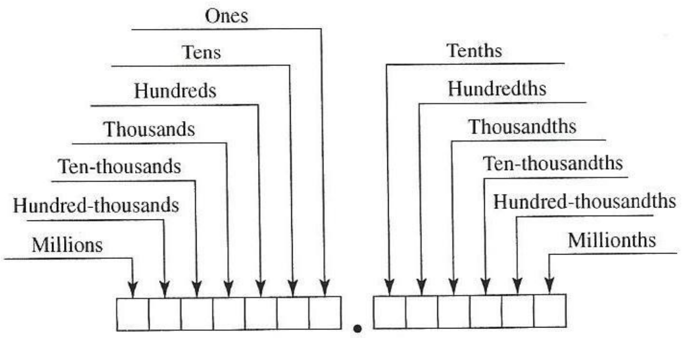
(a) In the number 24, what weight does 2 have?

SOLUTION 2 is in the tens position so its weight is (2 times 10), which equals 20.

(b) In the number 5,264, what weight does 2 have?

SOLUTION 2 is in the hundreds position so its weight is (2 times 100), which equals 200.

Table 1–1 Names for Places from Millions to Millionths.



(c) In the number 375, how many tens are in the tens position?

SOLUTION The tens position is the second from the right so there are 7 tens.

PRACTICE PROBLEMS 1–1[1]

1. In the number 893,462:
   (a) What weight does the 9 have?
   (b) What weight does the 4 have?
   (c) How many thousands are there?
2. In the number 7,603,418:
   (a) How many ten-thousands are there?
   (b) How many tens are there?
   (c) How many hundred-thousands are there?
   (d) How many ones are there?
3. The display readout on a piece of test equipment shows the number 823 and the display multiplier knob is set to ×1000. What value does the 3 represent?

1–1–1 Math Operator Symbols

The decimal number system not only includes the ten numerical symbols, it also includes several symbols that are called mathematical operators. Examples of these operator symbols are the addition sign (+) and the multiplication sign (×). These operator symbols are similar to punctuation marks in English, or any written language. Mathematics is the international language of numbers. It is a written and spoken language. To be an effective communicator in this language you must understand the meaning of the mathematical operator symbols. You can increase your math vocabulary, and become a better math communicator, by learning more of the math operators.

Math operator symbols usually tell us what to do, or what mathematical operations we should perform on numbers; for example, the addition sign in $6 + 2$ tells us to add six and two. Sometimes the symbols are used just to make our understanding of numbers easier; for example, we use a comma to separate thousands in the number 3,247. The following table describes some common math operators:

Table 1–2 Common Math Operators.

NAME	SYMBOL	EXAMPLE	USAGE
Decimal Point	.	3.7	The reference point for the place value of each numerical symbol
Addition Sign	+	$3 + 2$	Shows that 3 must be added to 2
Plus Sign	+	$+3$	Indicates a positive number and is usually omitted unless clarification is necessary
Subtraction Sign	−	$3 - 2$	Shows that 2 must be subtracted from 3
Negative Sign	−	-3	Indicates a negative number
Multiplication Sign	×	3×2	Shows that 3 must be multiplied by 2
Division Sign	÷ or /	$3 \div 2$ or $3/2$	Shows that 3 must be divided by 2
Equals Sign	=	$3 \times 2 = 6$	Shows that the quantity on the left is the same as the quantity on the right

FRACTION, PROPER a quantity of less than one that is expressed with a numerator and a denominator.

1–2 DECIMAL FRACTIONS

Let's review some names used in dealing with fractions. In the fraction $\frac{7}{10}$ the number above the line (7) is called the *numerator*. The number below the line (10) is called the *denominator*. The line itself is called the *vinculum*.

---

[1] At the end of each chapter there are additional problems labeled **End of Chapter Problems**. These problems are additional practice problems. For easy reference these End of Chapter Problems are numbered by section just like the Practice Problems. Thus, Practice Problems 1–1 and End of Chapter Problems 1–1 relate to the material covered in section 1–1. Practice Problems 1–4 relate to the material covered in section 1–4, and so on.

A decimal fraction is a fraction whose denominator is 10 or a multiple of 10 (100, 1000, 10,000, and so on). If the denominator is 10, the fraction is read one-tenth if the number is $\frac{1}{10}$, two-tenths if the number is $\frac{2}{10}$, and so on.

Here are some decimal fractions and how they are read:

$\frac{3}{100}$ 3 hundred*ths*

$\frac{1}{1000}$ 1 thousand*th*

$\frac{12}{10,000}$ 12 ten-thousand*ths*

Notice that we read the denominator just as we read a whole number except that we add *th* or *ths*. Similarly, if we write "4 thousandths," we know this is a fraction (the *ths* at the end of thousand tells us this); so we know the fraction is $\frac{4}{1000}$.

Table 1–1 gives the names of places from millions to millionths. The tenths position is the first position to the right of the decimal point. The hundredths position is the next position, and so on, to the millionths position, which is the sixth position to the right of the decimal point.

FRACTION, IMPROPER a quantity of one or more that is expressed with a numerator and a denominator.

EXAMPLE 1–2

Read the following fractions:

(a) $\frac{5}{10}$ (b) $\frac{27}{100}$ (c) $\frac{27}{1000}$ (d) $\frac{17}{10,000}$

SOLUTIONS

(a) 5 tenths (b) 27 hundredths (c) 27 thousandths
(d) 17 ten-thousandths

To express a written number as a decimal fraction, we determine the number in the denominator from Table 1–1. The numerator is the decimal number part. For example, if a number is written as 143 thousandths (one hundred forty-three thousandths), the "thousandths" tells us that the number in the denominator is 1000. The numerator is 143.

$$143 \text{ thousandths} = \frac{143}{1000}$$

The number 9 ten-thousandths is $\frac{9}{10,000}$. The "ten-thousandths" tells us that the denominator is 10,000. 9 is the value of the numerator.

EXAMPLE 1–3

Change the following numbers to fractions:

(a) 73 hundredths (b) two hundred eleven thousandths
(c) 33 hundred-thousandths

SOLUTIONS

(a) $\frac{73}{100}$ (b) $\frac{211}{1000}$ (c) $\frac{33}{100,000}$

These same numbers can be written as decimal numbers. In the number 143 thousandths, the "thousandths" tells us that there are three digits to the right of the decimal point as shown in

Table 1–1. Once we determine how many digits there must be, then we place the rightmost digit of the number, (3), in this position. The number would be written 0.143. (We placed a 0 to the left of the decimal point to remind us that there is no whole-number part, and to help us keep track of where the decimal point belongs.)

The number 6 ten-thousandths has four digits to the right of the decimal point. 6 would be placed in the ten-thousandths position and zeros would be placed in the thousandths, hundredths, and tenths positions signifying that there are no thousandths, no hundredths, and no tenths. The number would be written 0.0006.

EXAMPLE 1–4

Convert the following numbers to decimal numbers:

(a) 47 thousandths (b) 7 hundredths
(c) six hundred forty-nine ten-thousandths

SOLUTIONS

(a) 0.047 (b) 0.07 (c) 0.0649

To write a decimal number, determine the position of the rightmost digit (refer to Table 1–1). Then write the value of the number. Let's look at the number 0.003. The rightmost digit is 3 and occupies the thousandths position. Therefore, the number is 3 thousandths. In the number 0.0567, the 7 is in the ten-thousandths position so the number is 567 ten-thousandths.

EXAMPLE 1–5

Write the following decimal numbers:

(a) 0.432 (b) 0.0077 (c) 0.01047

SOLUTIONS

(a) 432 thousandths (b) 77 ten-thousandths
(c) 1047 hundred-thousandths

1-2-1 Converting from Decimal Fractions to Decimal Numbers

RULE 1-1 To convert decimal fractions to decimal numbers, determine the value of the denominator. Place the rightmost digit of the numerator in this position.

Usually quantities between zero and one are represented as decimal numbers or as decimal fractions. In this section we will convert from one to the other. First, let's convert from decimal fractions to decimal numbers. Consider the fraction $\frac{27}{100}$. To convert decimal fractions to decimal numbers, first determine the value of the denominator. In this fraction, the denominator is 100, making the fraction 27 hundredths. Place the rightmost digit of the numerator, 7, in the hundredths position. The 2 then must occupy the tenths position, so the answer is 0.27.

EXAMPLE 1–6

Change the decimal fraction $\frac{17}{100}$ to a decimal number.

SOLUTION The value of the denominator is 100. The rightmost digit in the numerator is 7; therefore, the 7 is placed in the hundredths position. This puts the 1 in the tenths position and the number is 0.17.

EXAMPLE 1-7

Change the decimal fraction $\frac{17}{1000}$ to a decimal number.

SOLUTION The denominator is 1000 so the 7 must appear in the thousandths position. That puts the 1 in the hundredths position. As with whole numbers, zeros must be used as place holders when necessary. Therefore, a zero goes in the tenths position. The decimal number is written 0.017 (17 thousandths).

Notice that in each case a zero was placed to the *left* of the decimal point. This is done to help us remember where the decimal point belongs. The zero also tells us that there is no whole-number part.

1-2-2 Converting from Decimal Numbers to Decimal Fractions

Now let's convert from decimal numbers to decimal fractions. Consider the number 0.039. Determine the position of the rightmost digit (9). This tells us the value of the denominator. The 9 is in the thousandths position, so the denominator is 1000. The numerator is the number with the decimal point and leftmost zeros removed. Stated numerically:

$$0.039 = \frac{39}{1000}$$

RULE 1-2 To convert decimal numbers to decimal fractions, determine the position of the rightmost digit. This tells us the value of the denominator. The numerator is the decimal number with the decimal point removed.

EXAMPLE 1-8

Convert 0.007 to a fraction.

SOLUTION The rightmost digit, 7, appears in the thousandths position so the answer is $\frac{7}{1000}$ (seven-thousandths).

EXAMPLE 1-9

Convert 0.0023 to a fraction.

SOLUTION The rightmost digit, 3, is in the ten-thousandths position so the answer is $\frac{23}{10{,}000}$ (twenty-three ten-thousandths).

PRACTICE PROBLEMS 1-2

1. Convert the following fractions to decimal numbers:
   (a) $\frac{4}{100}$ (b) $\frac{23}{1000}$ (c) $\frac{203}{100{,}000}$
2. Write the following numbers as decimal fractions:
   (a) 0.05 (b) 0.00073 (c) 0.00009
3. Write each of the following first as a decimal fraction and then as a decimal number:
   (a) 7 tenths (b) 37 hundredths
   (c) 7 ten-thousandths
   (d) 417 hundred-thousandths
   (e) 6 millionths
4. In the number 0.00246, in which place does the 2 appear? The 6?

5. Write the names of the following numbers:
   (a) 0.03 (b) 0.0005 (c) 0.00073

6. The display readout on a piece of test equipment shows the number 18 and the display multiplier knob is set to THOUSANDTHS. You must record the reading in a test report log as a decimal number. What number would you record?

7. You need to reset a machine in the shop. The machine must be set to remove 29 hundred-thousandths of an inch from a work piece and you must record the new setting in the machine log book as a fractional number. What number do you enter?

1–3 WHOLE NUMBERS AND FRACTIONS

Now let's look at numbers in which there is a whole-number part and a fractional part. For convenience, Table 1–3 (a repeat of Table 1–1) is shown below.

Consider the number 13.36. This would be read "thirteen *and* thirty-six hundredths." The word *and* is used in a number to define the position of the decimal point. We read the whole-number part, add the word *and*, and then read the fractional part. We could also write the number as a *mixed* number: $13\frac{36}{100}$. Mixed numbers indicate the addition of two numbers: a whole number and a fraction. $13.36 = 13\frac{36}{100} = 13 + \frac{36}{100}$. Typically, though, we wouldn't use the + sign but would simply write the number as a decimal number (13.36) or as a mixed number ($13\frac{36}{100}$).

KEY POINT Mixed numbers are numbers that are made up of a whole-number part and a fraction. When reading the number, the word and is used to connect the whole number and fractional parts.

Consider the number 432.243. This number is read "four hundred thirty-two *and* two hundred forty-three thousandths." Remember, the word *and* is used to connect the whole-number and fractional parts. As a mixed number we would write

$$432\frac{243}{1000}$$

EXAMPLE 1–10

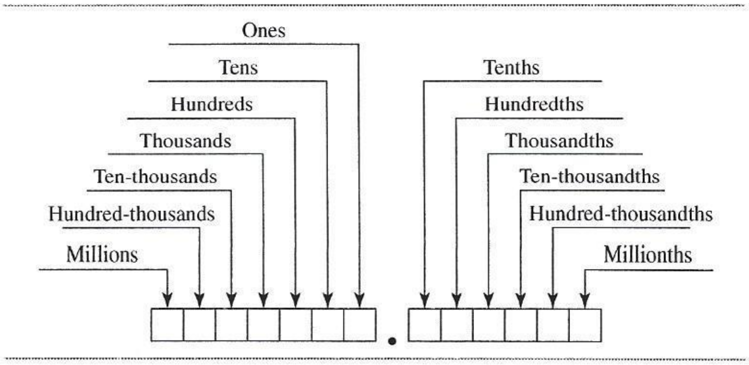
Convert the following decimal number to a mixed number.

57.045

SOLUTION Change the fractional part of the number (.045) to a decimal fraction, as we did in Examples 1–8 and 1–9. This yields $\frac{45}{1000}$. Then add the decimal fraction and the whole number to get the answer, which is $57\frac{45}{1000}$.

$$57.045 = 57\frac{45}{1000}$$

Table 1–3 Names for Places from Millions to Millionths.



EXAMPLE 1–11

Convert the following mixed number to a decimal number.

$$27\frac{7}{100}$$

SOLUTION Change the decimal fraction to a decimal number as we did in Examples 1–6 and 1–7. This yields 0.07. Then add the whole-number part and the fractional part to get the answer, which is 27.07.

$$27\frac{7}{100} = 27.07$$

EXAMPLE 1–12

Express the following number as a mixed number and then as a decimal number.

Three hundred two and seventy-three thousandths

SOLUTION The whole-number part is 302. The fractional part expressed as a decimal number is 0.073 (see Example 1–5), or expressed as a fraction is $\frac{73}{1000}$ (see Example 1–3). Combining the whole-number and fractional parts, we get $302\frac{73}{1000} = 302.073$.

PRACTICE PROBLEMS 1–3

1. Convert each of the following decimal numbers to mixed numbers:
   (a) 24.007 (b) 706.024 (c) 4.00017
   (d) 7.400 (e) 370.006 (f) 47.113

2. Convert each of the following mixed numbers to decimal numbers:
   (a) $1076\frac{7}{100}$ (b) $8\frac{23}{10{,}000}$ (c) $76\frac{14}{1000}$
   (d) $35\frac{270}{100{,}000}$ (e) $713\frac{7}{1000}$ (f) $8\frac{87}{10{,}000}$

3. Express the following numbers as decimal fractions and as mixed numbers:
   (a) Fifty-six and seventy-eight hundredths
   (b) Fifteen and thirty-five thousandths
   (c) One hundred six and eight ten-thousandths
   (d) Seven hundred four and two hundred fourteen hundred-thousandths
   (e) Four thousand seventy-five and fourteen ten-thousandths
   (f) Eighty and forty-three thousandths

4. You are instructed to take two current measurements with an ammeter and record the sum of the two measurements as a decimal number in a test log. The first measurement is 3 and the second measurement is 27 hundredths. What number would you record?

1–4 ROUNDING WHOLE NUMBERS

In problem solving it is common practice to simplify multidigit numbers. This simplification is done by replacing the rightmost nonzero digits with zeros. For example, the number 72,348 could be simplified to 72,350 to the nearest ten; 72,300 to the nearest hundred; 72,000 to the nearest thousand; or 70,000 to the nearest ten-thousand. This simplification is called *rounding*.

Rounding numbers is done to make problem solving easier. Some accuracy is lost as a result of the rounding. The more rounding we do, the less accurate the number. In electronics we work

RULE 1–3 When rounding a whole number to a specific place value, round up if the digit to the immediate right of the place value is a 5, 6, 7, 8, or 9; to round up, set all the digits to the right to zero and add one to the specific place value. Round down if the digit to the immediate right of the specific place value is a 0, 1, 2, 3, or 4; to round down, set all the digits to the right to zero.

with components that typically have 5% or 10% tolerances. We normally make measurements with instruments whose tolerances are within the 5% or 10% range (or worse). For these reasons, great accuracies in problem solving are not necessary.

Let's look at the number 72,348 again. When we rounded to the nearest ten, we rounded to 72,350. We had the choice of rounding *down* to 72,340 or rounding *up* to 72,350. Since 48 is closer to 50 than to 40, we rounded *up* to 72,350. When we rounded to the nearest hundred, we rounded to 72,300. We had the choice of rounding *down* to 72,300 or rounding *up* to 72,400. Since 348 is closer to 300 than to 400, we rounded *down* to 72,300. The same logic was used in rounding to the nearest thousand. Since 2348 is closer to 2000 than to 3000, we rounded to 72,000. Finally, 72,000 was rounded to 70,000 because 72,000 is closer to 70,000 than to 80,000.

Although the digit 5 could indicate either up or down, it is typically rounded *up*.

EXAMPLE 1–13

Round the number 2735 to the nearest ten, hundred, and thousand.

SOLUTION To round to the nearest ten, we examine the digit in the ones position. Since the digit is 5, we round *up* to 2740 by replacing 5 with 0 and adding 1 to the next significant digit. To round to the nearest hundred, we note that the digit in the tens position is 3; so we round *down* to 2700 by replacing 3 with 0. (The digit in the ones position is also changed to 0 since we are rounding to the nearest hundred.) Since 7 is the digit in the hundreds position, we round *up* to the nearest thousand and the answer is 3000.

PRACTICE PROBLEMS 1–4

Round the following numbers to the nearest ten, hundred, and thousand:

1. 2714
2. 6526
3. 43,258
4. 76,547
5. 82,803
6. 26,764
7. 78,226
8. 18,999
9. You are asked to sort resistors by measuring them with an ohmmeter, rounding the measured value to the nearest hundred, and then placing them in bins marked in increments of 100 ohms. The first resistor measures 8348 ohms. Into what bin would you place this resistor?

1–5 ROUNDING NONWHOLE NUMBERS

Nonwhole numbers are rounded by using the same rules as for whole numbers. As with whole numbers, we work from the rightmost digit to the left. Consider the number 12.736. We could round to the nearest hundredth, tenth, one, or ten. To round to the nearest hundredth, we would note the digit in the thousandths position. Since the digit is 6, we change the digit to 0 and add 1 to the next digit, making the number 12.74. We don't write the number as 12.740. The zero is not accurate since we rounded to the nearest hundredth. Therefore, we just drop the zero. To round to the nearest tenth, we note the digit in the hundredths position. Since the number is 3, we simply drop it and the answer is 12.7. To round to the nearest one, we note the digit in the tenths position. Because it is a 7, a number that is 5 or greater, we drop it and add 1 to the number in the ones position. The answer is 13. To convert to the nearest ten, we change the 2 to a 0. The answer is 10. Remember that in rounding whole numbers the zero is a place holder and cannot be dropped. In a fractional part, if a zero exists in the rightmost position as a result of rounding, the zero is dropped because it is not accurate.

PRACTICE PROBLEMS 1–5

Round the following numbers to the nearest hundredth, tenth, one, and ten:

1. 73.647
2. 26.401
3. 17.0419
4. 30.6908
5. 50.4736
6. 48.047
7. 33.781
8. 68.147
9. Your lead technician asks you to measure the voltage output of a power supply and record its value rounded to the nearest tenth volt. The voltmeter indicates that the output voltage is 28.26 volts. What value would you record?
10. In working on your taxes, your instructions are to round all numbers to the nearest ten. The numbers are 1017.4, 6269.7, and 8764.83. What are the rounded numbers?
11. Round the number 47.062 to the nearest tenth and the nearest ten.

1–6 SIGNIFICANT DIGITS

KEY POINT
Significant digits, also called significant figures, are the digits in a number (not counting leading zeros) that are known to be accurate.

The term "significant digits" is used in conjunction with the process of rounding numbers and with scientific notation, which will be covered in the next chapter. When we say we are going to round a number to "two significant digits," we mean the same as rounding to two places.

Leading zeros are the zeros to the left of the leftmost non-zero digit. When writing whole numbers, leading zeros are meaningless and are usually not written. For example, we would not write the number 52 as 052 because the leading zero adds no meaning. When writing a number with a value between zero and one such as 0.007, the leading zeros are important as placeholders, but are not customarily counted as significant digits. For example: 0.04 has one significant digit, which is four. The zero to the left of the decimal point tells us that there is no whole number part. The second zero is important because it states that there are no tenths in the number. But neither of these two zeros are considered significant digits.

Trailing zeros (the rightmost zeros) are considered significant digits when to the right of the decimal point, because their presence indicates a known accuracy. For example: the number 52.00 tells us that there are zero tenths and zero hundredths associated with this number. Their presence indicates their accuracy, therefore 52.00 has four significant digits. The absence of any digit in the thousandths position indicates an uncertainty as to the quantity of thousandths in the measurement.

There is an ambiguous case; when a whole number has trailing zeros we cannot be sure of their accuracy. For example: 5,000 has 4 significant digits according to the above definition but it may be a number that has been rounded to the nearest thousand. If it has been rounded then the trailing zeros are not accurate.

The ambiguity problem is eliminated by the use of scientific notation covered in the next chapter. For all problems in this text, we will assume that trailing zeros in whole numbers are accurate and therefore they should be considered significant digits.

EXAMPLE 1–14

How many significant digits are in the following numbers?

1. 593   2. 8000   3. 27,061   4. 0905

SOLUTIONS

1. 3   2. 4   3. 5   4. 3

Significant digits are independent of the decimal point. The following numbers each have four significant digits:

5432   543.2   54.32   5.432   0.5432   0.05432   0.005432

Let's examine the number 687. The digit 7 is in the ones position so its value is 7. The 8 is in the tens position so its value is 80 (10 × 8). The 6 is in the hundreds position so its value is 600 (100 × 6). In this number the 7 has the least value because it is in the ones position. A digit in the rightmost position is referred to as the *least significant digit* (LSD).

The digit with greatest value will always be the leftmost nonzero number. In this number the digit 6 has the greatest value or weight. A nonzero digit in the leftmost position is called the *most significant digit* (MSD).

KEY POINT
The most significant digit (MSD) is the leftmost nonzero digit. The least significant digit (LSD) is the rightmost digit.

EXAMPLE 1–15

In the following numbers, (a) which digit is the most significant digit? (b) Which digit is the least significant digit? (c) List each digit and its value.

1. 79,630   2. 485,607

SOLUTION

1. (a) 7 is the most significant digit (MSD). (b) 0 is the least significant digit (LSD). (c) 7 occupies the ten-thousands position so it has a value of 70,000 (7 × 10,000). 9 occupies the thousands position so it has a value of 9000 (9 × 1000). 6 is in the hundreds position, making its value 600 (6 × 100). 3 is in the tens position so its value is 30 (3 × 10), and 0 is in the ones position and its value is 0 (0 × 1).
2. (a) 4 is the most significant digit (MSD). (b) 7 is the least significant digit (LSD). (c) 4 occupies the hundred-thousands position, making its value 400,000 (4 × 100,000). 8 occupies the ten-thousands position so its value is 80,000 (8 × 10,000). 5 is in the thousands position, giving it a value of 5000 (5 × 1000). 6 is in the hundreds position so its value is 600 (6 × 100). 0 is in the tens position so its value is 0 (0 × 10), and 7 is in the ones position, making its value 7 (7 × 1). See Table 1–1.

PRACTICE PROBLEMS 1–6

1. How many significant digits are in the following numbers?
   (a) 489.0 (b) 0.489
   (c) 40,089 (d) 4809.0
   (e) 0.00489
2. In the number 4762:
   (a) Which digit is the most significant digit (MSD)?
   (b) Which digit is the least significant digit (LSD)?
   (c) Which digit occupies the hundreds position?
3. You are testing the output of a pseudo-random number generator. You must record just the most significant digit and the least significant digit. The first output is 7894563. What numbers do you record?

SELF-TEST 1–1 THROUGH 1–6

1. In the number 20,378:
   (a) Which digit is the MSD?
   (b) Which digit is the LSD?
   (c) Which digit occupies the thousands position?
   (d) Which digit occupies the tens position?
   (e) What weight does the digit 3 have?
   (f) How many ones are there?
2. Convert the following fractions to decimal numbers:
   (a) $\frac{41}{100}$ (b) $\frac{9}{1000}$ (c) $\frac{1783}{100{,}000}$

3. Write each of the following first as a decimal fraction and then as a decimal number:
   (a) Three-tenths
   (b) Eighty-five hundredths
   (c) Eighteen ten-thousandths
4. Write the names of the following numbers:
   (a) 0.33 (b) 0.004
5. Convert each of the following decimal numbers to mixed numbers:
   (a) 7.46 (b) 18.006
6. Convert each of the following mixed numbers to decimal numbers:
   (a) $76\frac{14}{100}$ (b) $6\frac{23}{1000}$
7. Express the following numbers as decimal fractions and as mixed numbers:
   (a) Three and seven-hundredths
   (b) Twenty-eight and sixty-three thousandths
8. Round to the nearest ten, hundred, and thousand:
   (a) 4765 (b) 9705
   (c) 15,789 (d) 37,046
9. Round to the nearest tenth, hundredth, and thousandth:
   (a) 0.0746 (b) 0.4605
   (c) 0.4056 (d) 0.3748
10. Round to the nearest ten, one, tenth, and hundredth:
    (a) 17.486 (b) 23.462
    (c) 36.547 (d) 20.706

1–7 ADDITION AND SUBTRACTION OF SIGNED NUMBERS

1–7–1 Signed Numbers Defined

In mathematics, numbers may be either positive (+) or negative (−). The plus sign is not usually written. If we saw the numbers 6, -4, 15, and -9 written, we would recognize that the numbers 6 and 15 are positive and the numbers 4 and 9 are negative.

In this section we introduce the symbols $<$ and $>$. The symbol $<$ means *less than* and the symbol $>$ means *greater than.*

In Figure 1–1 we have drawn a straight line that starts at zero and extends both right and left. Notice that numbers starting at zero and extending to the right are positive. Numbers starting at zero and extending to the left are negative. In comparing signed numbers, we see that the rightmost number is the greater and the leftmost number is the lesser. For example, compare 4 and -3. We could say that 4 is greater than -3 (expressed mathematically $4 > -3$), or we could say that -3 is less than 4 (expressed mathematically $-3 < 4$).

Consider the numbers -7, -2, and 6. We could say that $-7 < -2 < 6$. Or we could say that $6 > -2 > -7$. Notice that in comparing -2 and -7 we said that $-2 > -7$. Remember, on the number scale -2 is to the right of -7; therefore, it is greater. -2 is more positive than -7. -7 is more negative than -2. If we compare 3 and 11, we would say that $3 < 11$. We could also say that 3 is more negative than 11.

NEGATIVE NUMBER a number less than zero; in practical applications negative numbers denote a direction opposite that of positive numbers.

The *absolute* value of a number is that number without regard to sign. The absolute value of $+5$ and -5 is 5 and is symbolized as $|5|$. This tells us that no matter whether 5 is − or +, the distance from zero is 5.

ABSOLUTE VALUE a number without regard to its sign.

1–7–2 Addition and Subtraction

In addition to using the symbols + and − to denote the sign of a number, we also use these symbols to indicate the operations of addition and subtraction.

Anyone who has trouble visualizing adding signed numbers should refer to Figure 1–1. Adding two or more positive numbers does not present any problem with signs. $2 + 3 + 5 = 10$.

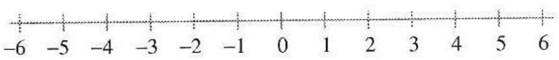


FIGURE 1–1 Scale showing positive and negative numbers.

Let's add two negative numbers: $(-2) + (-3)$. -2 and -3 are enclosed in parentheses to indicate that the $-$ is the sign of the number and not a sign of subtraction.

RULE 1–4 To add numbers with like signs, add the numbers and affix the sign to the answer.

To add numbers with unlike signs, subtract the smaller from the larger and affix the sign of the larger to the answer.

$$(-2) + (-3) = -5$$

$$(-6) + 7 + (-9) = -8$$

We could simplify the problem before finding the sum by adding the negative numbers together first:

$$-6 + 7 + (-9) = -6 - 9 + 7 = -15 + 7 = -8$$

In each case we move left on the number scale if the number is $-$ and we move right if the number is $+$.

RULE 1–5 To subtract, change the sign of the subtrahend and then add, following the rule for addition.

In subtraction we call the first number the *minuend*. The second number is the *subtrahend*. The answer is the *difference*.

$$\begin{array}{rl} 6 & \text{minuend} \\ \underline{-3} & \text{subtrahend} \\ 3 & \text{difference} \end{array}$$

$$6 - (+3) = 6 + (-3) = 3$$

$$\begin{array}{r} 6 \\ \underline{-3} \\ 3 \end{array} = \begin{array}{r} 6 \\ \underline{+\ -3} \\ 3 \end{array}$$

When more than two numbers are subtracted, the same rule applies. Whenever subtracting is indicated, we change the sign of the following numbers and add. $17 - (+6) - (+3) = 17 + (-6) + (-3) = 17 + (-9) = 8$. When addition and subtraction are both indicated, we follow the rules for both.

$$25 - (+6) + 4 = 25 + (-6) + 4 = 23$$

$$-7 + 16 - (-4) = -7 + 16 + 4 = 13$$

PRACTICE PROBLEMS 1–7

Perform the indicated operations:

1. $6 + (-4)$
2. $-14 + (-6)$
3. $16 - (+4)$
4. $9 - (-3)$
5. $-10 - (-6)$
6. $-15 - (+4)$
7. $25 + (-3) - (+4)$
8. $4 - (-6) + (-15)$
9. $-6 - (-3) - (-10)$
10. $40 + (-30) - (-6)$
11. Add the following numbers: 6, -14, -3, and 8.
12. When troubleshooting a circuit you calculate the following node currents: 5 amps, -3 amps, -4 amps, and 2 amps. What is the sum of the calculated node currents?

1–8 MULTIPLICATION AND DIVISION OF SIGNED NUMBERS

Both × and · are symbols used to indicate multiplication. The symbols ÷ and / are used to indicate division. Each part of a multiplication or division has a name as shown in Figure 1–2 on the following page.

6	×	7	=	42
Multiplicand		Multiplier		Product
14	÷	7	=	2
Dividend		Divisor		Quotient

FIGURE 1–2 Parts of multiplication and division identified.

When the multiplicand and multiplier are whole numbers they are also called the factors of the product.

Signed numbers are multiplied and divided by using the same methods learned in arithmetic. The rules are:

> **RULE 1–6** When multiplying or dividing signed numbers, if all numbers are positive, the answer is positive. If there is an even number of negative signs, the answer is positive. If there is an odd number of negative signs, the answer is negative.

The general equations are:

$$\begin{aligned}(+)\cdot(+) &= +\\ (+)\cdot(-) &= -\\ (-)\cdot(-) &= +\end{aligned}$$

Examples of Rule 1–6

$$\begin{aligned}
9 \times 8 &= 72 & (+)\cdot(+) &= +\\
7 \times (-6) &= -42 & (+)\cdot(-) &= -\\
-8 \times (-6) &= 48 & (-)\cdot(-) &= +\\
16 \div 4 &= 4 & (+)\div(+) &= +\\
12 \div (-4) &= -3 & (+)\div(-) &= -\\
-20 \div 5 &= -4 & (-)\div(+) &= -\\
-35 \div -7 &= 5 & (-)\div(-) &= +
\end{aligned}$$

PRACTICE PROBLEMS 1–8

Perform the indicated operations:

1. $17 \times (-5)$
2. $-14 \times (-16)$
3. -9×12
4. -12×14
5. $-14 \div 2$
6. $-42 \div 6$
7. $172 \div (-4)$
8. $98 \div (-14)$
9. $-121 \div (-11)$
10. $-144 \div (-12)$
11. What is the product of 73 and 6?
12. What is the quotient when 650 is divided by 25?

13. Find the product of 24 and 9 to the nearest ten.

14. When troubleshooting an electronics circuit, you must calculate the voltage (E) by using the equation $E = IR$. You measure the current (I) to be -3 amps. The resistance (R) is 8 ohms. What is the calculated voltage?

1–9 MATHEMATICAL EXPRESSIONS AND TERMS

Many problem-solving situations in electronics involve all four operations: addition, subtraction, multiplication, and division. Therefore, it is important that we understand the order in which these operations are performed.

Before we talk about these problem-solving situations, we must define some new words. Here and in future chapters we are going to be talking about mathematical *terms* and *expressions*.

TERM (MATHEMATICAL) a quantity preceded by a plus or minus sign.

EXPRESSION (MATHEMATICAL) a quantity denoted by one or more terms.

- A mathematical *term* is a quantity preceded by a + or − sign.
- A mathematical *expression* is a quantity denoted by one or more terms.

Thus, $4 + 7$ is an expression containing two terms. This expression can be reduced to *one* term (11).

$4 + (7 \times 3)$ is a two-term expression. 4 is one term and 7×3 is the other. We have used parentheses to help you identify the term. Remember, *terms* are separated by + and − signs. Signs of multiplication and division are signs within a term. In problem solving, all operations within a term are performed first and then the terms are combined. The rule is:

RULE 1–7 Unless directed by other mathematical operators, such as parentheses, a mathematical expression can be simplified by performing the multiplications and divisions first and then the additions and subtractions.

Here are some examples:

$$(7 \times 6) + (4 \times 2) = 42 + 8 = 50$$

$$(-14 \div 7) - (-6 \times 3) = -2 - (-18) = -2 + 18 = 16$$

$$4 + (-3) - (-4 \times 6) = 4 - 3 - (-24) = 4 - 3 + 24 = 25$$

Notice in each example that the operations within the terms were performed *first*, and then the terms were combined.

My colleague, Barbara Snyder, uses the phrase "**M**y **D**ear **A**unt **S**ally" to help her students remember which function is performed first—**M**ultiply, **D**ivide, **A**dd, and **S**ubtract.

PRACTICE PROBLEMS 1–9

Simplify the following expressions to one term:

1. $8 + 6 \times 3$
2. $-3 + 12 \div (-2)$
3. $6 \times 3 - 2 \times 4$
4. $16 \div (-2) - (-4) + 6 \times (-6)$
5. $(-6) \times (-4) + 7 \times (-3)$
6. $6 \times (-4) \div 2 - 4 + (-6)$
7. $15 \times 3 \div 3 + 4$
8. $8 \times 4 - (-6) \times (-4)$
9. When troubleshooting a circuit you must calculate the voltage by using the equation $E = I_1 \times R_1 + I_2 \times R_2$. You measure the currents and find that $I_1 = 3$ amps and $I_2 = 7$ amps. You know that $R_1 = 8$ ohms and $R_2 = 4$ ohms. What is the calculated voltage E?

1-10 PARENTHESES ( ) AND OTHER SIGNS OF GROUPING

Parentheses are mathematical symbols that can be used in a mathematical expression to direct the sequence of performing calculations. For example, when an expression is written as $3 \times 4 - 5$ we should first multiply 3 times 4, which is 12, and then subtract 5 to get 7. $3 \times 4 - 5 = 12 - 5 = 7$

If the expression was written as $3 \times (4 - 5)$ we should first perform the mathematical operation inside the parentheses. The sequence of operations, as directed by the parentheses, is:

$$3 \times (4 - 5)$$

$$3 \times (-1)$$

$$-3$$

In complex technical problems, we often have terms within terms. For example, in the expression $3 + 9(5 + 2)$, the first term is the 3. The second term is $9(5 + 2)$ but it contains two other terms, $5 + 2$, which must be evaluated before being multiplied by 9. This example can be further complicated by dividing it by 5, which we could write as $(3 + 9(5 + 2))/5$. The order of operation, as directed by the parentheses, is:

$(3 + 9(5 + 2))/5$		
$(3 + 9(7))$	/5	Perform the operation within the innermost parentheses $(5 + 2)$.
$(3 + 63)$	/5	Remove the innermost parentheses by performing the indicated multiplication.
(66)	/5	Perform the operation within the parentheses $(3 + 63)$.
13.2		Divide 66 by 5.

When we have terms within terms, we refer to them as *nested* terms. The order of operation is to remove the innermost parentheses by performing its operation first, like the $(5 + 2)$ above. The next operation is to remove the innermost parentheses which enclose the 7. We would continue to remove parentheses until the outermost parentheses are gone. Then we would use "My Dear Aunt Sally" to remove the remaining operators.

RULE 1-8 When there are nested terms in a mathematical expression, simplify (evaluate) the innermost term first and then work toward the outermost term.

When we have nested terms, we can use brackets [ ] and braces { } along with parentheses ( ) to identify the beginning and end of terms. If a division is involved, like the example above, then the *vinculum* (the horizontal bar that separates the numerator from the denominator) can be used to dictate the order of operations. For example, if the above problem is written as:

$$\frac{3 + 9(5 + 2)}{5}$$

then the order of operations is the same as before. The vinculum tells us that the numerator must be reduced to a single term before the division can be performed. The vinculum is doing the same job as the outermost parentheses in $(3 + 9(5 + 2))$.

Parentheses ( ), brackets [ ], braces { }, and the vinculum are often called *signs of grouping* because they are used to group terms together to show the order in which mathematical operations should be performed. For example, in the expression $(2 + 4 + 6 + 8) \times (3 + 5 + 7 + 9)$ the two groups of terms in the parentheses must be evaluated before the multiplication can be performed.

If all four signs of grouping are used, for example, in the problem $\{(3+9[5+2])/5\}+9$, we still work from the innermost group toward the outermost group. Using different signs of grouping often makes it easier to see the order of operations.

EXAMPLE 1–16

Perform the indicated operations:

(a) $7(6 + 3)$ (b) $4 - (-6 + 3) \times 2$ (c) $3[4(3 + 2) - 4]$
(d) $-3 - [2 + 4 \times (-6)]$

SOLUTIONS

(a) $7(6 + 3) = 7 \times 9 = 63$
(b) $4 - (-6 + 3) \times 2 = 4 - (-3) \times 2 = 4 - (-6) = 4 + 6 = 10$
(c) $3[4(3 + 2) - 4] = 3(4 \times 5 - 4) = 3(20 - 4) = 3(16) = 48$
(d) $-3 - [2 + 4 \times (-6)] = -3 - [2 + (-24)] = -3 - (-22) = -3 + 22 = 19$

Notice that when we simplified (c) to $3(4 \times 5 - 4)$, we then applied Rule 1–7 and performed the multiplication, $(4 \times 5 = 20)$, first. The problem then simplified to $3(20 - 4)$. Then $3(20 - 4) = 3(16) = 48$. Had we *subtracted* first $(5 - 4 = 1)$, we would have had $3(4 \times 1) = 3(4) = 12$—which, of course, is a wrong answer.

PRACTICE PROBLEMS 1–10

Perform the indicated operations:

1. $-6 - (-3) + 3(3 - 4)$
2. $6(-3) - [6(-3)]$
3. $14 - (-4 + 18 \div 3)$
4. $(4 + 3)(6 - 3)$
5. $4 - [6(7 - 3)]$
6. $6[18 \div (-6 + 3)] + 7 \times 4$
7. $10 - (6 + 3) \times 4$
8. $18 - 3[(3 + 4)(8 - 12)]$
9. $4[7 - (-3) + (2 - 6)(3 - 7)]$
10. $4 - 3\{-2[6(2 + 3)] - 6\}$
11. When troubleshooting a circuit you must calculate the voltage by using the equation $E = 18[(5 + 7)/24] + 22[(8 + 13)/42]$. What is the calculated voltage E?

SELF-TEST 1–7 THROUGH 1–10

Perform the indicated operations:

1. $14 - (+4) + (-5)$
2. $-16 - (-4) - (+6)$
3. -8×6
4. $-9 \times (-4)$
5. $4 \times (-3)$
6. $51 \div (-3)$
7. $-64 \div (-4)$
8. $-36 \div 4$
9. $5 \times (-3) + (-6) \times 7$
10. $-9 \div 3 + 4 \times (-6)$
11. $16 - (-6) + 27 \div (-3)$
12. $4 - (-6) + 4(5 - 2)$
13. $7 \times (-2) - [4(-6)]$
14. $7[15 \div (-3 + 6)] + 8 \div 4$
15. $6 - 5\{-4[8(4 + 2)] - 4\}$

CHAPTER 1 AT A GLANCE

PAGE	KEY POINT OR RULE	EXAMPLE
3	*Decimal Number System:* A number system that uses the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
3	*Carry:* In addition, if the sum of a column of digits exceeds 9 then the number transferred to the column to the left is called a carry and represents the number of tens that is to be transferred.	$\begin{array}{r} 29 \\ 28 \\ 17 \\ \underline{39} \\ 113 \end{array}$ The sum of digits in the first column is 33 so a carry of 3 is added to the next column.
3	*Base:* The quantity of symbols used to represent numbers in a numerical system.	In the decimal number system there are ten symbols, so the base is ten.
4	*Fraction, Proper:* A quantity of less than one that is expressed with a numerator and a denominator.	$\frac{1}{2}$
5	*Fraction, Improper:* A quantity of one or more that is expressed with a numerator and a denominator.	$\frac{3}{2}$ is greater than one $\frac{2}{2}$ is equal to one
6	*Rule 1–1:* To convert decimal fractions to decimal numbers, determine the value of the denominator. Place the rightmost digit of the numerator in this position.	$\frac{37}{1000} = 0.037$ The digit 7 goes in the thousandths position, which puts the 3 in the hundredths position. Zero is a place holder and goes in the tenths position.
7	*Rule 1–2:* To convert decimal numbers to decimal fractions, determine the position of the rightmost digit. This tells us the value of the denominator. The numerator is the decimal number with the decimal point removed.	$0.0019 = \frac{19}{10{,}000}$ The 9 is in the ten-thousandths position, so the denominator is 10,000 and the numerator is 19.
8	*Key Point:* Mixed numbers are numbers that are made up of a whole-number part and a fraction. When reading the number, the word *and* is used to connect the whole-number and fractional parts.	$6\frac{3}{10}$ 6 and 3 tenths $40\frac{11}{100}$ 40 and 11 hundredths
10	*Rule 1–3:* When rounding a whole number to a specific place value, round up if the digit to the immediate right of the place value is a 5, 6, 7, 8, or 9; to round up, set all the digits to the right to zero and add one to the specific place value. Round down if the digit to the immediate right of the specific place value is a 0, 1, 2, 3, or 4; to round down set all the digits to the right to zero.	4747 rounded to the nearest ten is 4750. To the nearest hundred, it is 4700, and to the nearest 1000, it is 5000.

10	Apply Rule 1–3 to nonwhole numbers.	4.074 rounded to the nearest hundredth is 4.07. To the nearest tenth, it is 4.1, and to the nearest one, it is 4.
11	*Key Point:* Significant digits are the digits in a number (not counting leading zeros) that are known to be accurate.	0.04 has one significant digit; 52.00 has four significant digits.
12	*Key Point:* The most significant digit (MSD) is the leftmost nonzero digit. The least significant digit (LSD) is the rightmost digit.	In the number 2740, the MSD is 2 and the LSD is 0.
13	*Negative Number:* A number less than zero; in practical applications negative numbers denote a direction opposite that of positive numbers.	When your checking account balance is −$100.00 it means that more money has gone out of your account than has gone into it.
13	*Absolute Value:* A number without regard to its sign.	$\lvert -8 \rvert = \lvert +8 \rvert = 8$
14	*Rule 1–4:* To add numbers with like signs, add the numbers and affix the sign to the answer. To add numbers with unlike signs, subtract the smaller from the larger and affix the sign of the larger to the answer.	$-4 + (-3) = -7$ $-4 + 8 = 4$
14	*Rule 1–5:* To subtract, change the sign of the subtrahend and then add, following the rule for addition.	$-4 - (-3) = -4 + 3 = -1$
15	*Rule 1–6:* When multiplying or dividing signed numbers, if all numbers are positive, the answer is positive. If there is an even number of negative signs, the answer is positive. If there is an odd number of negative signs, the answer is negative.	$(+) \cdot (+) = +$ $(-) \cdot (-) = +$ $(+) \cdot (-) = -$
16	*Term (mathematical):* A quantity preceded by a plus or minus sign.	−53
16	*Expression (mathematical):* A quantity denoted by one or more terms.	$4 + 7$
16	*Rule 1–7:* Unless directed by other mathematical operators, such as parentheses, a mathematical expression can be simplified by performing the multiplications and divisions first and then the additions and subtractions.	$(4 \times 3) - (9 \div 3) = 12 - 3 = 9$
17	*Rule 1–8:* When there are nested terms in a mathematical expression, simplify (evaluate) the innermost term first and then work toward the outermost term.	

END OF CHAPTER PROBLEMS 1–1

When working *End of Chapter Problems*, if you need to go back to the section where the material was presented, just look at the problem set number. For example, if you are working problems in *End of Chapter Problems 1–3* and you need help, go back to section 1–3 and review the material and examples presented there.

1. In the number 8042:
    - (a) What digit occupies the hundreds position?
    - (b) What digit occupies the ones position?
    - (c) What weight does the 8 have?
    - (d) How many tens are there?
3. In the number 67,403:
    - (a) What digit occupies the thousands position?
    - (b) What digit occupies the tens position?
    - (c) What weight does the 7 have?
    - (d) How many hundreds are there?
5. In the number 894,307:
    - (a) What digit occupies the hundreds position?
    - (b) What digit occupies the ten-thousands position?
    - (c) What weight does the 8 have?
    - (d) How many thousands are there?
7. The display readout on a piece of test equipment shows the number 497 and the display multiplier knob is set to × 100. What value does the 9 represent?

2. In the number 2074:
    - (a) What digit occupies the hundreds position?
    - (b) What digit occupies the ones position?
    - (c) What weight does the 4 have?
    - (d) How many tens are there?
4. In the number 75,403:
    - (a) What digit occupies the thousands position?
    - (b) What digit occupies the tens position?
    - (c) What weight does the 5 have?
    - (d) How many hundreds are there?
6. In the number 190,348:
    - (a) What digit occupies the hundreds position?
    - (b) What digit occupies the ten-thousands position?
    - (c) What weight does the 4 have?
    - (d) How many thousands are there?
8. The display readout on a piece of test equipment shows the number 2561 and the display multiplier knob is set to × 100. What value does the 5 represent?

END OF CHAPTER PROBLEMS 1–2

1. Convert the following fractions to decimal numbers:
    - (a) $\frac{3}{10}$ (b) $\frac{16}{1000}$ (c) $\frac{278}{100{,}000}$
    - (d) $\frac{1763}{10{,}000}$ (e) $\frac{435}{1000}$ (f) $\frac{2060}{10{,}000}$
3. Write the following numbers as decimal fractions:
    - (a) 0.007 (b) 0.0432 (c) 0.174
    - (d) 0.000065 (e) 0.00016
    - (f) 0.01234
5. Write each of the following first as a decimal fraction and then as a decimal number:
    - (a) 17 thousandths
    - (b) 4 hundredths
    - (c) 460 ten-thousandths
    - (d) 27 millionths
    - (e) 1780 hundred-thousandths
    - (f) 65 thousandths
7. In the number 0.001642, in which place does the
    - (a) 1 appear? (b) 2 appear?
    - (c) 6 appear?
9. In the number 0.508743, in which place does the
    - (a) 7 appear? (b) 3 appear?
    - (c) 0 appear?
11. Write the names of the following:
    - (a) 0.006 (b) 0.147 (c) 0.00092
13. The display readout on a piece of test equipment shows the number 27 and the display multiplier knob is set to HUNDREDTHS. You must record the reading in a test report log as a decimal number. What number would you record?
15. You need to reset a machine in the shop. The machine must be set to remove 43 thousandths of an inch from a work piece and you must record the new setting in the machine log book as a fractional number. What number do you enter?

2. Convert the following fractions to decimal numbers:
    - (a) $\frac{36}{1000}$ (b) $\frac{9}{10{,}000}$ (c) $\frac{289}{1000}$
    - (d) $\frac{83}{100{,}000}$ (e) $\frac{27}{100}$ (f) $\frac{980}{10{,}000}$
4. Write the following numbers as decimal fractions:
    - (a) 0.00273 (b) 0.0906
    - (c) 0.00005 (d) 0.067
    - (e) 0.417 (f) 0.000813
6. Write each of the following first as a decimal fraction and then as a decimal number:
    - (a) 6 tenths
    - (b) 83 hundred-thousandths
    - (c) 48 thousandths
    - (d) 230 millionths
    - (e) 73 hundredths
    - (f) 895 ten-thousandths
8. In the number 0.000867, in which place does the
    - (a) 7 appear? (b) 6 appear?
    - (c) 8 appear?
10. In the number 0.40257, in which place does the
    - (a) 4 appear? (b) 0 appear?
    - (c) 5 appear?
12. Write the names of the following:
    - (a) 0.175 (b) 0.00065 (c) 0.00463
14. The display readout on a piece of test equipment shows the number 304 and the display multiplier knob is set to TEN-THOUSANDTHS. You must record the reading in a test report log as a decimal number. What number would you record?
16. You need to reset a machine in the shop. The machine must be set to remove 273 hundred-thousandths of an inch from a work piece and you must record the new setting in the machine log book as a fractional number. What number do you enter?

END OF CHAPTER PROBLEMS 1–3

1. Convert each of the following decimal numbers to mixed numbers:
   (a) 7.14 (b) 50.02 (c) 710.143
   (d) 9.099 (e) 73.653 (f) 207.7834
   (g) 28.00736 (h) 8.0706
3. Convert each of the mixed numbers to decimal numbers:
   (a) $5\frac{68}{100}$ (b) $25\frac{7}{1000}$
   (c) $7\frac{165}{10{,}000}$ (d) $70\frac{4}{10}$
   (e) $473\frac{25}{1000}$ (f) $80\frac{743}{100{,}000}$
   (g) $2475\frac{35}{1{,}000{,}000}$ (h) $307\frac{8}{100{,}000}$
5. Express the following numbers as decimal numbers, and mixed numbers:
   (a) Ninety-three and seven-tenths
   (b) Thirty and four-hundredths
   (c) Eleven and one ten-thousandth
   (d) Nine hundred five and fifty-two thousandths
   (e) Seventy-eight and thirty-four thousandths
7. Express the following numbers as decimal numbers, and mixed numbers:
   (a) Two hundred seventy-three and twenty-five hundred-thousandths
   (b) Seven hundred four and seven hundred four millionths
   (c) Two thousand forty-four and five hundred four ten-thousandths
   (d) Ten thousand one hundred one and eighty-nine hundred-thousandths
   (e) Ninety and four hundred sixty-six ten-thousandths
   (f) Two hundred seven and one hundred ten-millionths
9. You are instructed to take two current measurements with an ammeter and record the sum of the two measurements as a decimal number in a test log. The first measurement is 4 and the second measurement is 79 thousandths. What number would you record?

2. Convert each of the following decimal numbers to mixed numbers:
   (a) 3.07 (b) 38.4 (c) 17.002
   (d) 170.506 (e) 48.6043 (f) 20.003
   (g) 706.2007 (h) 7.40300
4. Convert each of the following mixed numbers to decimal numbers:
   (a) $4\frac{27}{1000}$ (b) $275\frac{7}{100}$
   (c) $73\frac{4}{10}$ (d) $780\frac{400}{1000}$
   (e) $7\frac{73}{100{,}000}$ (f) $107\frac{703}{1{,}000{,}000}$
   (g) $76\frac{14}{100}$ (h) $45\frac{406}{10{,}000}$
6. Express the following numbers as decimal numbers, and mixed numbers:
   (a) Fourteen and seventeen hundredths
   (b) Seventy and twenty-five thousandths
   (c) Six and seven tenths
   (d) Forty-six and four thousandths
   (e) Four hundred eight and three hundredths
8. Express the following numbers as decimal numbers, and mixed numbers:
   (a) Three hundred seven and eight hundred four hundred-thousandths
   (b) Four and thirty ten-thousandths
   (c) Four hundred fifty-two and seventy-two millionths
   (d) Ninety-four and seventy-three ten-thousandths
   (e) Eight and three thousand four hundred nine millionths
   (f) Seven hundred two and fourteen ten-thousandths
10. You are instructed to take two current measurements with an ammeter and record the sum of the two measurements as a decimal number in a test log. The first measurement is 6 and 3 tenths and the second measurement is 53 hundredths. What number would you record?

END OF CHAPTER PROBLEMS 1–4

Round the following numbers to the nearest ten:

1. 17
3. 45
5. 64
7. 127
9. 874

2. 28
4. 12
6. 86
8. 465
10. 713

Round the following numbers to the nearest ten and hundred:

11. 273
13. 356
15. 1377
17. 1407
19. 8706

12. 904
14. 277
16. 7046
18. 1406
20. 7855

Round the following numbers to the nearest ten, hundred, and thousand:

21. 4817
22. 9616
23. 85,468
24. 44,066
25. 78,673
26. 28,445
27. 27,847
28. 30,073
29. 35,486
30. 98,789
31. 68,448
32. 26,755
33. 73,654
34. 18,506
35. Round 465,432 to the nearest ten-thousand.
36. Round 498,475 to the nearest ten-thousand.
37. You are asked to sort resistors by measuring them with an ohmmeter, rounding the measured value to the nearest hundred, and then placing them in bins marked in increments of 100 ohms. The first resistor measures 2485 ohms. Into what bin would you place this resistor?
38. You are asked to sort resistors by measuring them with an ohmmeter, rounding the measured value to the nearest hundred, and then placing them in bins marked in increments of 100 ohms. The first resistor measures 17,862 ohms. Into what bin would you place this resistor?

END OF CHAPTER PROBLEMS 1–5

Round the following numbers to the nearest hundredth, tenth, one, and ten:

1. 163.782
2. 243.647
3. 9.464
4. 8.746
5. 88.888
6. 44.545
7. 749.493
8. 915.605
9. 39.278
10. 83.655
11. 63.7478
12. 12.4507
13. 478.6706
14. 307.3525
15. 47.474
16. 27.047
17. 16.545
18. 33.554
19. Your lead technician asks you to measure the voltage output of a power supply and record its value rounded to the nearest tenth volt. The voltmeter indicates that the output voltage is 73.82 volts. What value would you record?
20. Your lead technician asks you to measure the voltage output of a power supply and record its value rounded to the nearest hundredth volt. The voltmeter indicates that the output voltage is 82.726 volts. What value would you record?
21. In working on your taxes, your instructions are to round all numbers to the nearest ten. The numbers are 5355, 10,248, and 15,544. What are the rounded numbers?
22. Round the number 65.162 to the nearest tenth and the nearest ten.

END OF CHAPTER PROBLEMS 1–6

1. How many significant digits are in the following numbers?
   (a) 70,940 (b) 0.04320
   (c) 300.0 (d) 9007
   (e) 0.000275 (f) 0.05090
2. How many significant digits are in the following numbers?
   (a) 0.05043 (b) 670.0
   (c) 0.000740 (d) 50,304
   (e) 7040 (f) 0.007040
3. In the number 8042:
   (a) What is the MSD?
   (b) What is the LSD?
4. In the number 2074:
   (a) What is the MSD?
   (b) What is the LSD?
5. In the number 67,403:
   (a) What is the MSD?
   (b) What is the LSD?
6. In the number 75,403:
   (a) What is the MSD?
   (b) What is the LSD?
7. In the number 894,307:
   (a) What is the MSD?
   (b) What is the LSD?
8. In the number 190,348:
   (a) What is the MSD?
   (b) What is the LSD?

9. You are testing the output of a pseudo-random number generator. You must record just the most significant digit and the least significant digit. The first output is 29505. What numbers do you record?
10. You are testing the output of a pseudo-random number generator. You must record just the most significant digit and the least significant digit. The first output is 9014563. What numbers do you record?

END OF CHAPTER PROBLEMS 1–7

Perform the indicated operations:

1. $12 - (+4) + (-3)$
2. $-3 - (+6) - (-2)$
3. $16 + 4 + (-6)$
4. $-12 - (-6) - (+8)$
5. $-20 - (-4) - (-10)$
6. $-20 - (-8) - (+12)$
7. $30 + (-3) + 7$
8. $40 + (-17) + 13$
9. $-10 - (+4) + (-14)$
10. $-16 - (+9) + (-18)$
11. $7 - (-7) - (+7)$
12. $14 + (-5) - (+7)$
13. $-5 - (-3) - (+12)$
14. $-8 - (-10) - (+16)$
15. $14 + (-3) - (-7)$
16. $30 + (-11) - (-8)$
17. $9 - (-4) - (-8)$
18. $7 - (-14) - (-12)$
19. Add the following numbers: 27, −14, 6, and −9.
20. Add the following numbers: 45, −14, −22, and 17.
21. When troubleshooting a circuit you calculate the following node currents: 6 amps, −4 amps, −2 amps, and 3 amps. What is the sum of the calculated node currents?
22. When troubleshooting a circuit you calculate the following node currents: 2 amps, 5 amps, −6 amps, and −8 amps. What is the sum of the calculated node currents?

END OF CHAPTER PROBLEMS 1–8

Perform the indicated operations:

1. $14 \times (-3)$
2. -15×6
3. $-9 \times (-6)$
4. $25 \times (-4)$
5. -12×7
6. $-12 \times (-12)$
7. $-45 \times (-5)$
8. $180 \times (-4)$
9. $-310 \times (-3)$
10. -19×40
11. $28 \div (-4)$
12. $135 \div (-5)$
13. $-144 \div 6$
14. $-512 \div 64$
15. $-72 \div (-6)$
16. $-441 \div (-21)$
17. $512 \div (-16)$
18. $-136 \div (-8)$
19. What is the product of 27 and 14?
20. What is the product of 44 and 33?
21. What is the quotient when 840 is divided by 20?
22. What is the quotient when 450 is divided by 15?
23. Find the product of 63 and 12 to the nearest ten.
24. Find the product of 92 and 18 to the nearest ten.
25. When troubleshooting an electronics circuit, you must calculate the voltage (E) by using the equation $E = IR$. You measure the current (I) to be −8 amps. The resistance (R) is 15 ohms. What is the calculated voltage?
26. When troubleshooting an electronics circuit, you must calculate the voltage (E) by using the equation $E = IR$. You measure the current (I) to be −15 amps. The resistance (R) is 120 ohms. What is the calculated voltage?

END OF CHAPTER PROBLEMS 1–9

Perform the indicated operations:

1. $(-7) \times 3 + 4$
2. $-6 - (-4) + 3 \times 4 - 4$
3. $6 \times (-3) - 6 \times (-3)$
4. $14 - (-9) + 15 \div 3$
5. $6 + 3 \times 5 - 4$
6. $21 + 6 \times 3 - 21 \div 7$
7. $6 \times 4 - 6 \div 2$
8. $-5 + 8 \times (-4) - 3$
9. $(-7) - (-4) \times 6 + 1$
10. $-18 \div 3 + 6 \times 6$
11. $-2 - (-2) \times 4 - 5 \times (-4)$
12. $-4 - (-8) \times 3 + 10 \div (-5)$
13. $16 \div (-4) - 7 \times 3 + 4$
14. $-10 \times (-3) - 6 \times (-10) \div (-5) - 5$
15. $-4 \times (-5) - 3 + 15 \div (-3)$
16. $17 - 5 \times 4 - 24 \div 6$

17. $(-35) \div 5 + 4 + 3 \times 6$

18. $48 \div (-6) - 6 \times (-7) - 3$

19. $7 \times (-3) - 4 \times (-5) + 6 \times (-3)$

20. $-3 \times (-9) - 4 + 27 \div (-3)$

21. When troubleshooting a circuit you must calculate the voltage by using the equation $E = I_1 \times R_1 + I_2 \times R_2$. You measure the currents and find that $I_1 = 6$ amps and $I_2 = 8$ amps. You know that $R_1 = 10$ ohms and $R_2 = 12$ ohms. What is the calculated voltage E?

22. When troubleshooting a circuit you must calculate the voltage by using the equation $E = I_1 \times R_1 + I_2 \times R_2$. You measure the currents and find that $I_1 = 2.7$ amps and $I_2 = 7.5$ amps. You know that $R_1 = 16$ ohms and $R_2 = 6$ ohms. What is the calculated voltage E?

END OF CHAPTER PROBLEMS 1–10

Perform the indicated operations:

1. $-4 - (-2) + 6(6 - 3)$

2. $-9 - (-4) + 6(7 - 9)$

3. $7(-3) - [4(-7)]$

4. $5(-6) + [-3(-5)]$

5. $(5 + 4)(-6 + 2)$

6. $(-8 + 12)(7 - 3)$

7. $5 - 2 - 3[4(3 + 4)] - 2$

8. $-6 - 4 - [8(-3 + 1)] - 7$

9. $4[24 \div (-8 + 4)] + 7 \times 4$

10. $8[33 \div (-9 + 6)] + 6 \times 3$

11. $15 - 4[(-3 + 6)(9 - 12)]$

12. $12 - 7[(4 - 7)(-9 - 3)]$

13. $5[6 - (-4) + (3 - 7)(2 - 6)]$

14. $3[8 - (-8) + (2 - 7)(4 - 8)]$

15. $4[-6 - (-3)(4)] + 7 \times 3$

16. $(4 - 6)(-3 \times 4) - [-9(2 - 4)]$

17. $[9 + (3 \times -7)][18 - (2 \times 3)]$

18. $[24 - (6 \times 5)][14 - (8 \times 4)]$

19. $14 - [(-3) \times 4] + [3(4 - 5)(7 - 3) + 6]$

20. $[-8 - (-4 \times 7)][-15 - (14 \div -2)]$

21. When troubleshooting a circuit you must calculate the voltage by using the equation $E = 10[(5 + 7)/6] + 15[(10 + 6)/32]$. What is the calculated voltage E?

22. When troubleshooting a circuit you must calculate the voltage by using the equation $E = 16[(15 + 7)/44] + 12[(11 + 13)/12]$. What is the calculated voltage E?

CHAPTER 2

Powers of Ten

CHAPTER OBJECTIVES

In this chapter you will learn how to:

1. Convert numbers to powers of ten form.
2. Express numbers in scientific notation rounded to some given number of significant digits.
3. Add, subtract, multiply, and divide numbers in powers of ten form using a calculator.
4. Solve problems with complex denominators using a calculator.
5. Find the reciprocal of numbers using a calculator.
6. Find the square and square root of numbers using a calculator.

MATH IN ACTION

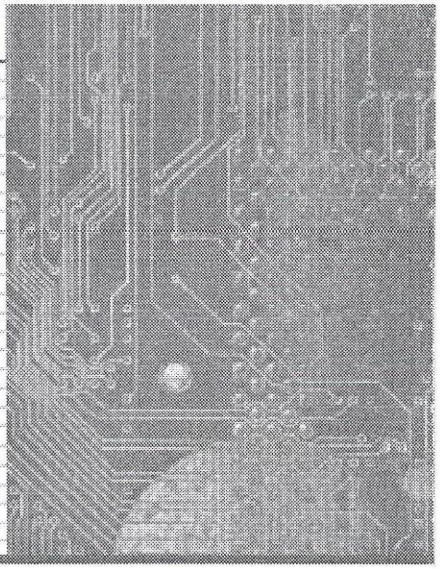
Sandra works in a semiconductor manufacturing plant where specialized microchips are made that are used in computers, cell phones, and many other electronic devices. Sandra is part of a team of engineers and technicians that are testing a new sputtering machine that has recently been purchased by the company. The machine deposits very thin layers of metal on the surface of wafers that will eventually be cut into individual semiconductor chips.

Sandra stops for a moment and thinks about how there are about 4 million transistors (the tiny electronic switches that make the microchips work) crammed into a space only about one-tenth of a square centimeter in area. To remind herself of the approximate size of each transistor, she divides the area by the number of transistors. But when working with such large and small numbers, she finds it easier to first convert the numbers to powers of ten. This way she can work with numbers greater than ten and less than ten, and quickly do the division.

Doing this calculation reminds Sandra of the importance of her current assignment, as the company seeks to make each transistor smaller and smaller. Sandra has been asked to determine how long it takes to deposit a layer of aluminum that is 120×10^{-9} meters thick. After running the machine for 20 seconds, Sandra uses another instrument to measure the thickness of the aluminum layer and determines it is 35×10^{-9} meters thick. After a quick calculation, Sandra decides to run the process for 70 seconds on her next trial.

INTRODUCTION

In our study of computers and electronics, we often work with very large numbers or very small numbers. For example, computer databases can store over 1,000,000,000,000 bits of information. At the other end of the quantity spectrum, some of the components in a computer have leakage currents of less than 0.0000000001 amperes. Mathematical tools have been developed to make it easier to work with these numbers. These tools are a math shorthand called *powers of ten* or *scientific notation*. The use of powers of ten makes working with very large or very small numbers more manageable. In this chapter we will develop an understanding of how to use these tools to help in solving electronic problems.



The concepts of *rounding* covered in Chapter 1 will be used with scientific notation to show how to estimate answers to complex problems. Learning how to estimate answers is important to students because it gives them quick ballpark answers to problems, helps them learn how to follow complex procedures, and helps them to think logically.

Calculator keystrokes for selected problems and some detailed calculator usage instructions are given in Appendix A. It is important for technologists to know how to use calculators, but the ability to quickly and accurately multiply and divide manually and to estimate answers are skills that are harder to acquire and more valued by employers. Pre-employment tests often do not allow calculator usage because employers relate good math skills with good rational thinking skills.

Algorithms will be used where appropriate in the chapter to assist the student in the solution of more complex problems. Technologists who can follow complex procedures, think logically, and make reasonable estimates are usually good problem solvers, are always in demand, and command large salaries.

2–1 CONVERTING NUMBERS TO POWERS OF TEN FORM

To simplify working with large or small numbers, we use a math shorthand called *powers of ten*, which is an easy way to express multiples of ten. Let's see what these terms mean.

In Table 2–1 we have converted numbers that are 10 or multiples of 10 to powers of ten form. This table demonstrates the relationship between the powers of ten from 10^6 (one million) to 10^{-6} (one millionth). It can be expanded to show the largest and smallest numbers imaginable.

In Table 2–1, notice the symmetry between rows. The number in any row is ten times larger than the number below it. Conversely, every number is ten times smaller than the number above it. In the rightmost column, the exponents decrease by one as we go down the rows. Also notice the row where $1 = 10^0$. Any number to the zero power is equal to one. This table should help you visualize why this is true.

What about numbers that are not integer multiples of ten? Later, in the chapters on logarithms, we will show how all numbers can be represented as exponents of ten.

Let's look at $10 \times 10 \times 10 = 1000 = 10^3$. In the expression 10^3, the number 10 is the *base* and 3 is the *exponent*. When a multiple of 10 is expressed in this way, it is said to be in the powers of ten form. 10^3 is read "ten to the third power." 10^3 occurs often in science and engineering and therefore has another name: ten cubed.

The exponent tells us how many times 10 is used as a factor. We may also think of the exponent as telling us how many zeros are included to the right of the digit 1. Thus in the number 1000, 10 is used as a factor 3 times ($10 \times 10 \times 10$) so the exponent is 3. Counting the number of zeros to the right of the 1 in 1000 we again get 3, which also tells us the exponent is 3. $1000 = 10^3$.

POWER OF TEN a number represented by 10 with an exponent, e.g., 10^3.

EXPONENT a superscript of a number or expression that indicates the number of times the number or expression must be multiplied by itself. A negative exponent means the reciprocal, e.g., $10^{-2} = 1/10^2 = 1/(10 \times 10)$.

BASE the number that is multiplied by itself to determine the place value in a numerical system, e.g., ten is multiplied by itself two times to determine the place value of the digit two places to the left of the decimal point.

Table 2-1 Conversions from Numbers That Are Multiples of Ten to Powers of Ten Form.

DECIMAL NUMBER	NUMBER IN POWERS OF TEN FORM
$1{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6
$100{,}000 = 10 \times 10 \times 10 \times 10 \times 10$	10^5
$10{,}000 = 10 \times 10 \times 10 \times 10$	10^4
$1000 = 10 \times 10 \times 10$	10^3
$100 = 10 \times 10$	10^2
$10 = 10$	10^1
$1 = \frac{10}{10}$	10^0
$0.1 = \frac{1}{10}$	$\frac{1}{10^1} = 10^{-1}$
$0.01 = \frac{1}{100} = \frac{1}{10 \times 10}$	$\frac{1}{10^2} = 10^{-2}$
$0.001 = \frac{1}{1000} = \frac{1}{10 \times 10 \times 10}$	$\frac{1}{10^3} = 10^{-3}$
$0.0001 = \frac{1}{10{,}000} = \frac{1}{10 \times 10 \times 10 \times 10}$	$\frac{1}{10^4} = 10^{-4}$
$0.00001 = \frac{1}{100{,}000} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10}$	$\frac{1}{10^5} = 10^{-5}$
$0.000001 = \frac{1}{1{,}000{,}000} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}$	$\frac{1}{10^6} = 10^{-6}$

In the number 100, 10 is used as a factor two times ($10 \times 10 = 100$) so the exponent is 2. Or, in the number 100 there are two zeros following the 1, so the exponent is 2. $100 = 10^2$. 10^2 occurs often in science and engineering and therefore has another name: ten squared. The rule is:

RULE 2-1 In converting some multiple of 10 to powers of ten form, count the number of zeros. This number is the exponent. The answer is 10 raised to that power.

For example, 10,000 has four zeros after the one, therefore $10{,}000 = 10^4$.

PRACTICE PROBLEMS 2-1[1]

Convert the following numbers to powers of ten form:

1. 100
2. 100,000
3. 10
4. 10,000,000
5. 1000
6. 10,000
7. 1,000,000
8. 100,000,000

2-2 CONVERTING FROM POWERS OF TEN FORM TO NUMBERS

Now let's consider a problem in which we wish to convert from powers of ten form to regular decimal notation. Remember that the exponent tells us how many times 10 is used as a factor. Or, said another way, the exponent tells us how many zeros follow the 1. For example, let's change 10^4 to a number. 10^4 is read "ten to the fourth power." 4 is the exponent. Therefore, 10 is multiplied by itself four times: $10 \times 10 \times 10 \times 10 = 10{,}000$. Again, this is the same as saying that the answer is 1 followed by four zeros. $10^4 = 10{,}000$. For positive exponents the rule is:

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[1] At the end of each chapter there are additional problems labeled **End of Chapter Problems**. These problems are additional practice problems. For easy reference these End of Chapter Problems are numbered by section just like the Practice Problems. Thus, Practice Problems 2–1 and End of Chapter Problems 2–1 relate to the material covered in section 2–1. Practice Problems 2–4 and End of Chapter Problems 2–4 relate to the material covered in section 2–4, and so on.

> **RULE 2–2** In converting from powers of ten form to a decimal number, write the digit 1 and follow with as many zeros as indicated by the exponent.

For example: with 10^3 the exponent is three so we place three zeros after the one ($10^3 = 1000$). When the exponent is zero we do not place any zeros after the one ($10^0 = 1$).

PRACTICE PROBLEMS 2–2

Convert the following numbers from powers of ten form to regular numbers.

1. 10^2
2. 10^1
3. 10^6
4. 10^7
5. 10^9

2–3 CONVERTING DECIMAL FRACTIONS TO POWERS OF TEN FORM

When we dealt with numbers that were multiples of 10, we learned that the exponents were 1 or greater. If the number is 1, the exponent is zero. $10^0 = 1$. There are no zeros following the number 1; therefore, in powers of ten form, the exponent is zero.

Numbers whose values are between zero and one can be written in powers of ten form by using negative exponents of ten. Table 2–2, which is a portion of Table 2–1, shows some of these numbers as decimal numbers, decimal fractions, and expressed in powers of ten form. This table could be extended to infinitely small numbers; as the numbers get smaller, the exponent becomes a larger and larger negative number.

From these relationships, we can develop the following rules:

$$10^{-3} = \frac{1}{1000}$$

$$10^{-4} = \frac{1}{10{,}000}$$

$$10^{-3} = 0.001$$

$$10^{-4} = 0.0001$$

> **RULE 2–3** In converting from powers of ten to decimal fractions, the negative exponent tells how many zeros follow the 1 in the denominator.

> **RULE 2–4** In converting from powers of ten to decimal numbers, the negative exponent tells how far the 1 is located to the right of the decimal point.

Table 2–2 Conversions of Numbers That Are between Zero and One to Powers of Ten Form.

DECIMAL NUMBER	DECIMAL FRACTIONS	POWERS OF TEN FORM
0.1	$\frac{1}{10}$	$\frac{1}{10^1} = 10^{-1}$
0.01	$\frac{1}{100} = \frac{1}{10 \times 10}$	$\frac{1}{10^2} = 10^{-2}$
0.001	$\frac{1}{1000} = \frac{1}{10 \times 10 \times 10}$	$\frac{1}{10^3} = 10^{-3}$

PRACTICE PROBLEMS 2–3

Convert the following powers of ten to (a) decimal numbers and (b) decimal fractions:

1. 10^{-4}
2. 10^{-2}
3. 10^{-1}
4. 10^{-8}
5. 10^{-3}
6. 10^{-7}
7. 10^{-6}
8. 10^{-5}

2–4 CONVERTING POWERS OF TEN TO DECIMAL FRACTIONS

Let's turn it around and consider problems in which we wish to convert decimal numbers and decimal fractions to powers of ten form.

RULE 2–5 In converting from a decimal fraction to powers of ten form, count the number of zeros in the denominator. This number is the negative power to which 10 is raised.

RULE 2–6 In converting from a decimal number to powers of ten form, count how far the digit 1 is located to the right of the decimal point. This is the value of the negative exponent.

$$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

$$0.0001 = \frac{1}{10{,}000} = \frac{1}{10^4} = 10^{-4}$$

$$\frac{1}{10{,}000} = 10^{-4}$$

$$\frac{1}{100} = 10^{-2}$$

$$0.0001 = 10^{-4}$$

$$0.01 = 10^{-2}$$

PRACTICE PROBLEMS 2–4

Convert the following decimal numbers to powers of ten form:

1. 0.001
2. 0.1
3. 0.000001
4. 0.01
5. 0.00001
6. 0.0000001
7. 0.0001
8. 0.00000001

Convert the following decimal fractions to powers of ten form:

9. $\frac{1}{1000}$
10. $\frac{1}{100{,}000}$
11. $\frac{1}{10}$
12. $\frac{1}{1{,}000{,}000}$
13. $\frac{1}{100}$
14. $\frac{1}{10{,}000}$

2–5 MULTIPLICATION IN POWERS OF TEN FORM

RULE 2–7 To multiply numbers in powers of ten form, *add the exponents.*

EXAMPLE 2–1

Find $10^6 \times 10^{-2}$.

SOLUTION

$$10^6 \times 10^{-2} = 10^{6+(-2)} = 10^4$$

or
$$1{,}000{,}000 \times 0.01 = 10^4$$

or
$$1{,}000{,}000 \times \frac{1}{100} = 10{,}000 = 10^4$$

EXAMPLE 2–2

Find $10^{-4} \times 10^3$.

SOLUTION

$$10^{-4} \times 10^3 = 10^{-4+3} = 10^{-1}$$

or
$$0.0001 \times 1000 = 0.1 = 10^{-1}$$

or
$$\frac{1}{10{,}000} \times 1000 = \frac{1}{10} = 0.1 = 10^{-1}$$

EXAMPLE 2–3

Find $10^{-5} \times 10^{-2}$.

SOLUTION

$$10^{-5} \times 10^{-2} = 10^{-5+(-2)} = 10^{-7}$$

or
$$\frac{1}{100{,}000} \times \frac{1}{100} = \frac{1}{10{,}000{,}000} = 10^{-7}$$

or
$$0.00001 \times 0.01 = 0.0000001 = 10^{-7}$$

PRACTICE PROBLEMS 2–5

Multiply the following numbers. Express the products in powers of ten form.

1. $10^2 \times 10^3$
2. $10^4 \times 10^6$
3. $10^{-1} \times 10^{-3}$
4. $10^{-3} \times 10^{-5}$
5. $10^5 \times 10^{-2}$
6. $10^6 \times 10^{-4}$
7. $10^2 \times 10^{-3}$
8. $10^3 \times 10^{-3}$
9. $10^{-4} \times 10^7$
10. $10^{-2} \times 10^6$

2–6 DIVISION IN POWERS OF TEN FORM

RULE 2–8 To divide numbers in powers of ten form, *subtract the denominator exponent from the numerator exponent.*

The general equation is $\dfrac{10^m}{10^n} = 10^{m-n}$.

EXAMPLE 2–4

Find $\frac{10^5}{10^2}$.

SOLUTION

$$\frac{10^5}{10^2} = 10^{5-2} = 10^3$$

or

$$\frac{100{,}000}{100} = 1000 = 10^3$$

EXAMPLE 2–5

Find $\frac{10^{-5}}{10^2}$.

SOLUTION

$$\frac{10^{-5}}{10^2} = 10^{-5-2} = 10^{-7}$$

or

$$\frac{0.00001}{100} = 0.0000001 = 10^{-7}$$

EXAMPLE 2–6

Find $\frac{10^5}{10^{-2}}$.

SOLUTION

$$\frac{10^5}{10^{-2}} = 10^{5-(-2)} = 10^{5+2} = 10^7$$

or

$$\frac{100{,}000}{0.01} = 10{,}000{,}000 = 10^7$$

EXAMPLE 2–7

Find $\frac{10^{-5}}{10^{-2}}$.

SOLUTION

$$\frac{10^{-5}}{10^{-2}} = 10^{-5-(-2)} = 10^{-5+2} = 10^{-3}$$

or

$$\frac{0.00001}{0.01} = 0.001 = 10^{-3}$$

PRACTICE PROBLEMS 2-6

Perform the indicated operations. Express your answers in powers of ten form.

1. $\dfrac{10^4}{10^2}$
2. $\dfrac{10^6}{10^3}$
3. $\dfrac{10^2}{10^5}$
4. $\dfrac{10^1}{10^6}$
5. $\dfrac{10^2}{10^{-2}}$
6. $\dfrac{10^3}{10^{-5}}$
7. $\dfrac{10^{-2}}{10^{-4}}$
8. $\dfrac{10^{-3}}{10^1}$
9. $\dfrac{10^{-4}}{10^{-6}}$
10. $\dfrac{10^{-1}}{10^{-4}}$
11. $\dfrac{10^{-5}}{10^{-3}}$
12. $\dfrac{10^{-9}}{10^{-6}}$
13. $\dfrac{1}{10^2}$
14. $\dfrac{1}{10^{-4}}$
15. $\dfrac{1}{10^6}$

2-7 COMBINED MULTIPLICATION AND DIVISION IN POWERS OF TEN FORM

Multiplication and division may be performed in the same problem. We must take care to observe the signs of the exponents as we add or subtract.

$$\frac{10^4 \times 10^2}{10^3} = \frac{10^{4+2}}{10^3} = \frac{10^6}{10^3} = 10^{6-3} = 10^3$$

Notice that we performed the indicated multiplication by adding the exponents. Next we performed the indicated division by subtracting the exponent in the denominator from the exponent in the numerator.

$$\frac{10^{-5} \times 10^3}{10^{-2}} = \frac{10^{-5+3}}{10^{-2}} = \frac{10^{-2}}{10^{-2}} = 10^{-2+2} = 10^0 = 1$$

Let's try two more.

$$\frac{10^3 \times 10^4}{10^{-2} \times 10^6} = \frac{10^{3+4}}{10^{-2+6}} = \frac{10^7}{10^4} = 10^{7-4} = 10^3$$

$$\frac{1}{10^2 \times 10^4} = \frac{1}{10^{2+4}} = \frac{1}{10^6} = 10^{-6}$$

We arrive at the answer 10^{-6} because the numerator (1) can be written in powers of ten form as 10^0. Then we have

$$\frac{10^0}{10^6} = 10^{0-6} = 10^{-6}$$

A very useful rule to remember is:

RULE 2-9 Whenever a power of ten is in the denominator, it can be moved to the numerator by changing the sign of the exponent.

Consider the example above in which we had

$$\frac{10^3 \times 10^4}{10^{-2} \times 10^6}$$

We could have solved by this method:

$$\frac{10^3 \times 10^4}{10^{-2} \times 10^6} = 10^3 \times 10^4 \times 10^2 \times 10^{-6} = 10^3$$

Notice that 10^{-2} in the denominator became 10^2 when it was moved to the numerator. 10^6 in the denominator became 10^{-6} in the numerator. When all powers of ten are in the numerator, we simply add the exponents.

$$10^{3+4+2-6} = 10^3$$

PRACTICE PROBLEMS 2–7

Perform the indicated operations. Express your answers in powers of ten form.

1. $\frac{10^2 \times 10^{-3}}{10^4}$
2. $\frac{10^{-1} \times 10^{-6}}{10^3}$
3. $\frac{10^1 \times 10^0 \times 10^6}{10^{-3}}$
4. $\frac{10^{-3} \times 10^4 \times 10^{-6}}{10^{-3}}$
5. $\frac{10^{-2} \times 10^4}{10^3 \times 10^2}$
6. $\frac{10^2 \times 10^6}{10^3 \times 10^{-1}}$
7. $\frac{1}{10^2 \times 10^4 \times 10^1}$
8. $\frac{1}{10^{-2} \times 10^6 \times 10^{-8}}$
9. $\frac{10^4 \times 10^{-1} \times 10^6}{10^2 \times 10^{-4} \times 10^5}$
10. $\frac{10^{-6} \times 10^{-3} \times 10^3}{10^{-9} \times 10^6 \times 10^{-3}}$

SELF-TEST 2–1 THROUGH 2–7

Convert the following numbers to powers of ten form:

1. 1000
2. 1,000,000
3. 0.001
4. 0.0000001
5. 0.0001
6. 0.00001

Determine the numbers represented by the following powers of ten:

7. 10^3
8. 10^5
9. 10^2
10. 10^0
11. 10^{-4}
12. 10^{-1}
13. 10^{-5}
14. 10^{-8}

Convert the following powers of ten to decimal fractions and to decimal numbers:

15. 10^{-2}
16. 10^{-3}
17. 10^{-5}
18. 10^{-1}

Perform the indicated operations. Express your answers in powers of ten form.

19. $10^2 \times 10^5$
20. $10^3 \times 10^{-4}$
21. $10^6 \times 10^{-2}$
22. $10^{-3} \times 10^{-6}$
23. $\frac{10^2}{10^5}$
24. $\frac{10^4}{10^{-6}}$
25. $\frac{10^{-3}}{10^6}$
26. $\frac{10^{-2}}{10^{-5}}$

27. $\dfrac{10^{-2} \times 10^{4}}{10^{-3}}$

28. $\dfrac{10^{1} \times 10^{-5}}{10^{4} \times 10^{-9}}$

29. $\dfrac{10^{-4} \times 10^{-1} \times 10^{6}}{10^{6} \times 10^{-3} \times 10^{1}}$

30. $\dfrac{10^{-2} \times 10^{-3} \times 10^{9}}{10^{2} \times 10^{3} \times 10^{4}}$

31. $\dfrac{10^{4} \times 10^{-4} \times 10^{6}}{10^{3} \times 10^{-3} \times 10^{-2}}$

32. $\dfrac{10^{1} \times 10^{-7} \times 10^{-2}}{10^{-3} \times 10^{-4} \times 10^{2}}$

33. $\dfrac{1}{10^{2} \times 10^{-3} \times 10^{4}}$

2-8 CONVERTING BETWEEN REGULAR NUMBERS AND POWERS OF TEN NOTATION

Consider the number 470. Let's convert the number to powers of ten notation with positive exponents.

$$470 = 47.0 \times 10^{1} = 4.70 \times 10^{2} = 0.470 \times 10^{3} = 0.0470 \times 10^{4}$$

and so on. This is the same as writing

$$470 = 47.0 \times 10 = 4.70 \times 100 = 0.470 \times 1000 = 0.0470 \times 10{,}000$$

and so on. As we move the decimal point *left*, the exponent becomes more positive.

$$470 = 47.0 \times 10^{1}$$

$$47.0 \times 10^{1} = 4.70 \times 10^{1+1} = 4.70 \times 10^{2}$$

$$4.70 \times 10^{2} = 0.470 \times 10^{2+1} = 0.470 \times 10^{3}$$

$$0.470 \times 10^{3} = 0.0470 \times 10^{3+1} = 0.0470 \times 10^{4}$$

Said another way, as we make the exponent more positive, the decimal point moves left.

RULE 2-10 For each place we move the decimal point left in a number, the exponent increases by 1.

RULE 2-11 For each digit we make the exponent more positive, the decimal point moves one place left.

EXAMPLE 2-8

Change 275 to powers of ten form with an exponent of (a) 2, (b) 4.

SOLUTION

(a) $275 = 2.75 \times 10^{2}$. The exponent changes by 2 in a positive direction, so the decimal point moves two places *left*.

(b) $275 = 0.0275 \times 10^{4}$. The exponent changes by 4 in a positive direction, so the decimal point moves 4 places *left*.

Or, in part (a) we changed the number so that the exponent is 2. If we start from there and change the exponent to 4, we just move the decimal point two places farther left.

$$2.75 \times 10^{2} = 0.0275 \times 10^{4}.$$

Now let's convert the same number to powers of ten form with negative exponents.

$$470 = 4700 \times 10^{-1} = 47{,}000 \times 10^{-2} = 470{,}000 \times 10^{-3}$$

and so on. As we move the decimal point *right*, the exponent becomes more *negative*. For each place we move the point *right*, the exponent decreases by 1 (becomes more negative).

RULE 2–12 For each place we move the decimal point *right* in a number, the exponent decreases by 1.

RULE 2–13 For each digit we make the exponent more negative, the decimal point moves one place *right*.

$$470 = 4700 \times 10^{-1}$$

$$4700 \times 10^{-1} = 47{,}000 \times 10^{-1+(-1)} = 47{,}000 \times 10^{-2}$$

$$47{,}000 \times 10^{-2} = 470{,}000 \times 10^{-2+(-1)} = 470{,}000 \times 10^{-3}$$

Said another way, as we make the exponent more negative, the decimal point moves *right*.

EXAMPLE 2–9

Change 275 to powers of ten form with an exponent of (a) −2, (b) −4.

SOLUTION

(a) $275 = 27{,}500 \times 10^{-2}$. The exponent changes by 2 in a negative direction, so the decimal point moves two places *right*.
(b) $275 = 2{,}750{,}000 \times 10^{-4}$. The exponent changes by 4 in a negative direction, so the decimal point moves four places *right*.

Or, in part (a) we changed the number so that the exponent is −2. To make the exponent −4, we just move the decimal point two places farther right.

$$27{,}500 \times 10^{-2} = 2{,}750{,}000 \times 10^{-4}$$

PRACTICE PROBLEMS 2–8

1. Change 38.6 to powers of ten form with an exponent of (a) 2, (b) 3, and (c) 4.
2. Change 5600 to powers of ten form with an exponent of (a) 2, (b) 3, and (c) 4.
3. Change 0.0000905 to powers of ten form with an exponent of (a) −2, (b) −6, and (c) −8.
4. Change 0.000000277 to powers of ten form with an exponent of (a) −5, (b) −7, and (c) −9.
5. Change 1.80 to powers of ten form with an exponent of (a) 1, (b) 3, (c) −1, and (d) −3.
6. Change 0.833 to powers of ten form with an exponent of (a) 6, (b) 3, (c) −3, and (d) −6.

2–9 EXPRESSING NUMBERS GREATER THAN ONE IN SCIENTIFIC NOTATION

In science and engineering it is common practice to round answers to two, three, or four significant figures and then to change the number to a number between 1 and 10 times some power of ten. When a number is written in this manner, it is said to be expressed in *scientific notation*. The following examples are numbers expressed in scientific notation:

SCIENTIFIC NOTATION a number expressed as a value between 1 and 10 and then multiplied by a power of ten (e.g., 7.32×10^4).

$$1.76 \times 10^4 \qquad 4.067 \times 10^{-2} \qquad 9.2 \times 10^6$$

As we mentioned in Chapter 1, great accuracy in problem solving is not realistic in electronics because we usually work with components whose values are given to two significant figures (color bands on resistors, for example), and they may be as much as ±10% different than that value. In addition, measuring instruments such as voltmeters and ohmmeters, if of the analog type like a multimeter, do not provide accuracy beyond two figures. Even digital meters often provide just 1% accuracies.

Therefore, throughout the text, with few exceptions, we will round to three significant figures, even though your calculator can provide eight or more.

Consider the number 764,832. Let's express this number in scientific notation rounded to three significant digits or simply "three places." Let's first round to three places. The number becomes 765,000. We rounded up because the fourth digit was 8. Now let's change the number to a number between 1 and 10. The answer is 7.65. To get 7.65, we had to move the decimal point five places *to the left*. To have an answer that equals 765,000, we must multiply 7.65 by 100,000 or 10^5. The result is: $765,000 = 7.65 \times 10^5$. For each place we move the decimal point left, we must multiply by 10. Since we moved the decimal point five places to the left, we multiplied by 10^5.

RULE 2–14 To express numbers greater than 1 in scientific notation, count the number of places the decimal point must move left. This number is the exponent and is positive.

EXAMPLE 2–10

Express the number 4703 in scientific notation rounded to three places.

SOLUTION We must move the decimal point three places *left* in order to have a number between 1 and 10. That makes the exponent 3.

$$4703 = 4.703 \times 10^3$$

$$4.703 \times 10^3 = 4.70 \times 10^3, \quad \text{rounded to three places}$$

PRACTICE PROBLEMS 2–9

Round the following numbers to three significant figures and express your answers in scientific notation:

1. 1730
2. 26,745
3. 1,654,736
4. 42.69
5. 173,426
6. 47,394
7. 79.063
8. 678,091
9. 189.83
10. 6746

2–10 EXPRESSING NUMBERS LESS THAN ONE IN SCIENTIFIC NOTATION

Consider the number 0.00715. To change to scientific notation, we must move the decimal point three places to the *right* so that we have a number between 1 and 10. The number is now 7.15. We must multiply this new number by 10^{-3} so that the value of the number in scientific notation equals the value of the original number. For each place we move the decimal point *right*, we must multiply by 0.1 or 10^{-1}. We moved the point *right* three places, so the exponent is -3.

RULE 2–15 To express numbers that are less than 1 in scientific notation, count the number of places the decimal point must move *right*. This number is the exponent and is negative.

EXAMPLE 2–11

Express the number 0.00008706 in scientific notation rounded to three places.

SOLUTION We must move the decimal point five places right to have a number between 1 and 10. That makes the exponent -5.

$$0.00008706 = 8.706 \times 10^{-5}$$

$$8.706 \times 10^{-5} = 8.71 \times 10^{-5}, \quad \text{rounded to three places}$$

PRACTICE PROBLEMS 2–10

Round the following numbers to three significant figures and express your answers in scientific notation:

1. 0.0167
2. 0.640732
3. 0.0003988
4. 0.00607
5. 0.1086
6. 0.00000706
7. 0.00001763
8. 0.009096
9. 0.07646
10. 0.000567

2–11 ADDITION AND SUBTRACTION

RULE 2–16 To add or subtract numbers in powers of ten form, the exponents must be the same.

We can add or subtract numbers in powers of ten form or when using scientific notation *provided the exponents are the same*. For example, let's perform the following addition: $3 \times 10^2 + 4 \times 10^3$. The decimal numbers are 300 and 4000. If we add 300 and 4000, we get 4300, or 4.3×10^3. To perform the addition using powers of ten, we must first move the decimal point of one number (it doesn't matter which) so that the exponents are the same. Let's change 3×10^2 so that the exponent is 3. We are making the exponent one digit more positive, so the decimal point must move one place *left* (Rule 2–11: For each digit we make the exponent more positive, the decimal point moves one place left.)

$$3 \times 10^2 = 0.3 \times 10^3$$

Now we can perform the addition:

$$\begin{array}{r} 0.3 \times 10^3 \\ +4 \times 10^3 \\ \hline 4.3 \times 10^3 \end{array}$$

Addition or subtraction is performed by adding the numbers together and multiplying by the indicated power of ten. The *exponents* are not added. The exponents merely tell us that we are adding 0.3 thousands to 4 thousands to get 4.3 *thousands*. We could have changed 4×10^3 to 40×10^2 and added.

$$\begin{array}{r} 3 \times 10^2 \\ +40 \times 10^2 \\ \hline 43 \times 10^2 \end{array}$$

$$43 \times 10^2 = 4.3 \times 10 \times 10^2 = 4.3 \times 10^3$$

The result is the same but more steps are involved.

EXAMPLE 2–12

Let's subtract 6×10^{-3} from 10^{-2}.

SOLUTION

$$\begin{array}{r} 1 \times 10^{-2} \\ (-)6 \times 10^{-3} \\ \hline \end{array}$$

10^{-2} is written in scientific notation form ($10^{-2} = 1 \times 10^{-2}$) because addition or subtraction of the decimal numbers is to be performed. In this problem we must again change one of the numbers so that both exponents are the same. Let's change 6×10^{-3} so that the exponent is -2.

We are making the exponent one digit more positive, so the decimal point must move one place *left* (Rule 2–11).

$$6 \times 10^{-3} = 0.6 \times 10^{-2}$$

Now we can perform the subtraction:

$$\begin{array}{r} 1 \quad \times 10^{-2} \\ (-)0.6 \times 10^{-2} \\ \hline 0.4 \times 10^{-2} \end{array}$$

$$0.4 \times 10^{-2} = 4 \times 10^{-1} \times 10^{-2} = 4 \times 10^{-3}$$

We could have changed 1×10^{-2} to a number whose exponent is -3. For example: $1 \times 10^{-2} = 0.01 = 10 \times 10^{-3}$. Because we wanted the exponent to equal -3, we had to move the decimal point one place to the right.

$$\begin{array}{r} 10 \times 10^{-3} \\ (-)6 \times 10^{-3} \\ \hline 4 \times 10^{-3} \end{array}$$

We can check our work by performing the subtraction in decimal.

$$\begin{array}{rr} 1 \times 10^{-2} = 0.01 & 0.01 \\ 6 \times 10^{-3} = 0.006 & (-)0.006 \\ \cline{2-2} & 0.004 \end{array}$$

$$0.004 = 4 \times 10^{-3}$$

PRACTICE PROBLEMS 2–11

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $10^2 + 10^3$
2. $10^1 + 10^{-1}$
3. $10^3 - 10^2$
4. $2 \times 10^2 + 3 \times 10^3$
5. $47 \times 10^{-2} + 560 \times 10^{-3}$
6. $40 \times 10^3 - 300 \times 10^2$
7. $3 \times 10^{-2} - 45 \times 10^{-4}$
8. $270 \times 10^{-1} + 46.3 \times 10^1$
9. $6.8 \times 10^3 + 6.8 \times 10^4 + 56{,}000$
10. $213 \times 10^{-6} + 0.000043 + 51.3 \times 10^{-5}$
11. Add the numbers 5830 and 0.245. Express the answer in scientific notation rounded to three places.
12. Subtract 37.8×10^{-3} from 0.000054×10^2.
13. The sides of a playground measure 45.8×10^2 ft, 7.96×10^2 ft, 2738 ft, and 357.5×10 ft. What is the perimeter in feet? Express answer in scientific notation rounded to three places.
14. You have four resistors on your work bench. They are 27,000 Ω, 7.5×10^3 Ω, 3.3×10^4 Ω, and 10×10^3 Ω. What is the total resistance of the four resistors? Express answer in scientific notation rounded to three places.

2–12 ESTIMATING

Students should include estimating as one of their problem solving tools. There are times when a ballpark answer is all that's required. Also, if you can estimate answers, you have the means to quickly check answers as found using the calculator.

Most electronic and computer related problems can be solved to a reasonable degree of accuracy (usually better than 10%) without the use of a calculator by utilizing scientific notation and rounding. Here are a couple of examples.

EXAMPLE 2–13

Multiply 3752×8439. First, estimate the answer and then solve using manual calculations.

SOLUTION When estimating, what we want to do is round to the point where we can perform the multiplication in our heads. In this problem we will round to one place. Using powers of ten and rounding to one place we get

$$3752 \approx 4 \times 10^3$$
$$8439 \approx 8 \times 10^3$$

The resulting multiplication expressed in scientific notation is

$$4 \times 10^3 \times 8 \times 10^3 = 4 \times 8 \times 10^6 = 32 \times 10^6 = 3.2 \times 10^7$$

Using manual calculations: $3752 \times 8439 = 3.17 \times 10^7$ accurate to three places

The estimated answer is within about 1% of the answer to three places found using manual calculations. Typically, we will express estimated answers in scientific notation accurate to two places.

EXAMPLE 2–14

Divide $57{,}950 \times 10^3$ by 0.267. First estimate the answer and then solve using manual calculations.

SOLUTION Round to the point where you can do the division in your head. In this problem we will round to one place. Using powers of ten and rounding to one place we get

$$\frac{57{,}950 \times 10^3}{0.267} = \frac{6 \times 10^7}{3 \times 10^{-1}} = 2 \times 10^{7+1} = 2 \times 10^8$$

Using manual calculations:

$$\frac{57{,}950 \times 10^3}{0.267} = 2.17 \times 10^8 \quad \text{rounded to three places}$$

Sometimes you have to fudge a little bit in estimating. For example, let's estimate the answer to 3.5×3.5. Since both numbers are between 3 and 4, we know that the answer has to be between 9 and 16 because $3 \times 3 = 9$ and $4 \times 4 = 16$. Since both numbers are halfway between 3 and 4, we should expect the answer to be about halfway between 9 and 16 or somewhere around 12. Let's round our numbers to one place, according to the rules of rounding and see what happens. According to the rules of rounding, 3.5 rounds up to 4. But if we round *both* numbers up to 4 our estimated answer ($4 \times 4 = 16$) won't be very close. When the digits to be rounded are close to 5, let's change the rules of rounding so that our estimate will be more accurate. If we round one number up and the other number down we get, to one place, 3 and 4. $3 \times 4 = 12$, which is pretty close to the real answer, which is 12.3 to three places. With practice, you get a feel for when to fudge a little bit with the rule for rounding when estimating answers.

When rounding, we use the following steps:

1. Convert all numbers to scientific notation rounded to one place. Two places is better (results in greater accuracy) if you can still readily find the answer without using the calculator.
2. Perform the indicated multiplication or division.
3. Express the answer in scientific notation accurate to two places.

PRACTICE PROBLEMS 2–12

Estimate the answers to the following problems. Express answers in scientific notation accurate to two places. Then solve the problems using manual calculations. Express answers in scientific notation accurate to three places.

1. 587×43.8
2. 713×8.53
3. 832.51×47.956
4. 397.63×0.00052439
5. $694 \div 137.9$
6. $3870 \div 88.72$
7. $55.374 \div 0.0356$
8. $0.9327 \div 9832.7$
9. A luxury car dealer pays a sound system installer \$33.25 per hour. How much is the sound system installer paid for 37 hours of work? Estimate by rounding to the nearest 10 before multiplying. Then find the exact answer accurate to the nearest cent.
10. A parcel of land measures 1940.7 ft by 620.4 ft. Estimate the area by rounding to the nearest 100, then find the exact area. Express both answers in scientific notation rounded to three places. Express the area in square feet. Area = length × width.
11. Ohm's law states that in a series circuit, $I = \frac{E}{R}$ where I is the circuit current, E is the applied voltage, and R is the total resistance. If $E = 22$ V and $R = 47 \times 10^3\ \Omega$, what is the circuit current? Estimate the answer by rounding the digits to the nearest 10 and then find the actual current in scientific notation rounded to three places.

In the following sections of this chapter and in the following chapters, get into the habit of estimating your answers. Often, mistakes you have made using the calculator (such as keying in the wrong number or selecting the wrong function) can be detected by estimating.

2–13 PROBLEMS WITH COMPLEX DENOMINATORS

A typical electronics problem might look like this:

$$\frac{25 \times 3.3 \times 10^3}{3.3 \times 10^3 + 4.7 \times 10^4}$$

To solve this problem, we must first perform the indicated addition in the denominator.

$$\begin{aligned} 3.3 \times 10^3 &= 0.33 \times 10^4 \\ +4.7 \times 10^4 &= \underline{4.7 \times 10^4} \\ & 5.03 \times 10^4 \end{aligned}$$

Now

$$\frac{25 \times 3.3 \times 10^3}{5.03 \times 10^4} = 1.64$$

Using estimation techniques:

$$25 \times 3.3 \times 10^3 \cong 25 \times 3 \times 10^3 = 75 \times 10^3 \cong 8 \times 10^4$$

$$3.3 \times 10^3 + 4.7 \times 10^4 = 0.33 \times 10^4 + 4.7 \times 10^4 \cong 5 \times 10^4$$

$$\frac{8 \times 10^4}{5 \times 10^4} = 1.6$$

EXAMPLE 2–15

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

(a) $\dfrac{9 \times 18 \times 10^3}{18 \times 10^3 + 56 \times 10^4}$ (b) $\dfrac{150 \times 6800}{6800 + 39 \times 10^3}$

(c) $\dfrac{20 \times 1.2 \times 10^6}{1.2 \times 10^6 + 8.2 \times 10^6}$ (d) $\dfrac{14 \times 56 \times 10^3}{1.5 \times 10^3 + 5.6 \times 10^4}$

SOLUTIONS

(a) 2.80×10^{-1} (b) 2.23×10

(c) 2.55 (d) 1.36×10

PRACTICE PROBLEMS 2–13

Perform the indicated operations. Express your answers in scientific notation.

1. $\dfrac{5 \times 33 \times 10^3}{3.3 \times 10^3 + 33 \times 10^3}$
2. $\dfrac{9 \times 6.8 \times 10^3}{470 + 6.8 \times 10^3}$
3. $\dfrac{20 \times 220 \times 10^3}{75 \times 10^3 + 220 \times 10^3}$
4. $\dfrac{12 \times 15 \times 10^3}{4.7 \times 10^3 + 1.5 \times 10^4}$
5. $\dfrac{9 \times 5.6 \times 10^3}{820 + 5.6 \times 10^3}$
6. $\dfrac{5 \times 2700}{6.8 \times 10^3 + 2700}$
7. $\dfrac{25 \times 4700}{4700 + 47 \times 10^3}$
8. $\dfrac{20 \times 3.3 \times 10^3}{3.3 \times 10^3 + 3.3 \times 10^4}$

2–14 RECIPROCALS

KEY POINT The *reciprocal* of a number is that number divided into 1.

In electronics it is often necessary to find the reciprocal of a number and then perform some other mathematical operation with that reciprocal. The reciprocal of 3 is $\frac{1}{3}$, the reciprocal of 50 is $\frac{1}{50}$, and so on.

EXAMPLE 2–16

Find the reciprocal of 27×10^3. Express the answer in scientific notation rounded to three places.

SOLUTION

$$\frac{1}{27 \times 10^3} = \frac{1}{27} \times 10^{-3} = 0.0370 \times 10^{-3} = 3.70 \times 10^{-5}$$

If we estimate the answer we get

$$\frac{1}{27 \times 10^3} = \frac{1}{2.7 \times 10^4} \approx \frac{1}{3 \times 10^4} = 3.3 \times 10^{-5} \quad \text{carried out to two places}$$

EXAMPLE 2–17

Perform the indicated operation. Express your answer in scientific notation rounded to three places.

$$\frac{1}{22 \times 10^3} + \frac{1}{47 \times 10^3}$$

SOLUTION

$$\frac{1}{22} \times 10^{-3} = 0.0455 \times 10^{-3}$$

$$\frac{1}{47} \times 10^{-3} = 0.0213 \times 10^{-3}$$

$$0.0455 \times 10^{-3} + 0.0213 \times 10^{-3} = 0.0668 \times 10^{-3} = 6.68 \times 10^{-5}$$

If we estimate the answer we get

$$\frac{1}{22 \times 10^3} + \frac{1}{47 \times 10^3} \approx \frac{1}{2 \times 10^4} + \frac{1}{5 \times 10^4}$$

$$= 0.5 \times 10^{-4} + 0.2 \times 10^{-4} = 7 \times 10^{-5}$$

EXAMPLE 2–18

Perform the indicated operation. Express your answer in scientific notation rounded to three places.

$$\frac{1}{\dfrac{1}{3.3 \times 10^3} + \dfrac{1}{6.8 \times 10^3}}$$

SOLUTION First we find the value of the two denominator terms:

$$\frac{1}{3.3 \times 10^3} = \frac{1}{3.3} \times 10^{-3} = 0.303 \times 10^{-3} = 3.03 \times 10^{-4}$$

$$\frac{1}{6.8 \times 10^3} = \frac{1}{6.8} \times 10^{-3} = 0.147 \times 10^{-3} = 1.47 \times 10^{-4}$$

Next we add the two values in the denominator:

$$3.03 \times 10^{-4} + 1.47 \times 10^{-4} = 4.50 \times 10^{-4}$$

To get the final answer we take the reciprocal of the denominator:

$$\frac{1}{4.50 \times 10^{-4}} = \frac{1}{4.50} \times 10^{4} = 0.222 \times 10^{4} = 2.22 \times 10^{3}$$

Using estimation techniques:

$$\frac{1}{3.3 \times 10^3} = \frac{1}{3.3} \times 10^{-3} \cong 0.3 \times 10^{-3} = 3 \times 10^{-4}$$

$$\frac{1}{6.8 \times 10^3} = \frac{1}{6.8} \times 10^{-3} \cong 0.1 \times 10^{-3} = 1 \times 10^{-4}$$

Next we add the two values in the denominator:

$$3 \times 10^{-4} + 1 \times 10^{-4} = 4 \times 10^{-4}$$

To get the final answer we take the reciprocal of the denominator:

$$\frac{1}{4 \times 10^{-4}} = \frac{1}{4} \times 10^{4} = 0.25 \times 10^{4} = 2.5 \times 10^{3}$$

PRACTICE PROBLEMS 2–14

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $\frac{1}{470}$
2. $\frac{1}{33 \times 10^{4}}$
3. $\frac{1}{68 \times 10^{-9}}$
4. $\frac{1}{3.7 \times 10^{-3}}$
5. $\frac{1}{2760}$
6. $\frac{1}{5.6 \times 10^{3}}$
7. $\frac{1}{50 \times 10^{3}}$
8. $\frac{1}{370 \times 10^{-6}}$
9. $\frac{1}{4 \times 10^{-3}}$
10. $\frac{1}{10 \times 10^{3}}$
11. $\frac{1}{2700} + \frac{1}{1800}$
12. $\frac{1}{68 \times 10^{3}} + \frac{1}{47 \times 10^{3}}$
13. $\frac{1}{12 \times 10^{3}} + \frac{1}{7.5 \times 10^{3}}$
14. $\frac{1}{5 \times 10^{4}} + \frac{1}{2.5 \times 10^{4}}$
15. $\frac{1}{\frac{1}{27 \times 10^{3}} + \frac{1}{18 \times 10^{3}}}$
16. $\frac{1}{\frac{1}{6800} + \frac{1}{5600}}$
17. $\frac{1}{27.6 \times 10^{3}} - \frac{1}{47 \times 10^{3}}$
18. $\frac{1}{\frac{1}{270} + \frac{1}{470} + \frac{1}{560}}$
19. The total conductance, G_T, of a circuit consisting of two resistors in parallel is given by the equation $G_T = \frac{1}{R_1} + \frac{1}{R_2}$. Find G_T when $R_1 = 470\ \Omega$ and $R_2 = 560\ \Omega$. Express your answer in scientific notation rounded to three places.

SELF-TEST 2–8 THROUGH 2–14

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. 27×65
2. 0.00173×3900
3. $47{,}000 \times 0.0000423$
4. $1765 \times 24{,}673$
5. $33 \times 10^{4} \times 27 \times 10^{3}$
6. $5.6 \times 10^{3} + 4.7 \times 10^{4}$
7. $147 \times 10^{-7} + 0.833 \times 10^{-5} + 21.3 \times 10^{-6}$
8. $2 \times 3.14 \times 2500 \times 500 \times 10^{-9}$
9. $\frac{47.3 \times 10^{2}}{0.00846}$
10. $\frac{0.025 \times 10^{-6}}{483 \times 10^{-9}}$
11. $\frac{7800 \times 0.00273}{1.28 \times 10^{-4} \times 8000}$
12. $\frac{0.0000064 \times 84 \times 10^{2}}{6730 \times 0.045 \times 10^{-3}}$

13. $\dfrac{12 \times 3900}{56{,}000 + 3900}$

14. $\dfrac{1}{270} + \dfrac{1}{390}$

15. $\dfrac{25 \times 4700}{4700 + 47{,}000}$

16. $\dfrac{1}{3900} + \dfrac{1}{56 \times 10^3} + \dfrac{1}{20 \times 10^3}$

17. $\dfrac{1}{6.28 \times 12 \times 10^3 \times 470 \times 10^{-12}}$

18. $\dfrac{25 \times 2700}{2.7 \times 10^3 + 3.3 \times 10^4}$

19. $\dfrac{1}{\dfrac{1}{560 \times 10^2} + \dfrac{1}{47 \times 10^3}}$

20. $\dfrac{1}{\dfrac{1}{2700} + \dfrac{1}{3300} + \dfrac{1}{4700}}$

2–15 RAISING A POWER OF TEN TO A POWER

When we wrote 10^3, we learned that the exponent (3) told us to multiply 10 by itself three times.

$$10^3 = 10 \times 10 \times 10$$

If we wrote $(10^3)^2$, we would be raising 10^3 to the second power. That is,

$$(10^3)^2 = 10^3 \times 10^3 = 10^6$$

RULE 2–17 When raising a power to a power, multiply the exponents, e.g., $(10^3)^4 = 10^{12}$.

We need the parentheses to identify the quantity we are acting on. Consider the quantity $(10^{-2})^3$.

$$(10^{-2})^3 = 10^{-2} \times 10^{-2} \times 10^{-2} = 10^{-6}.$$

In each case the exponent can be thought of as a multiplier.

$$(10^3)^2 = 10^{3\times2} = 10^6 \quad \text{and} \quad (10^{-2})^3 = 10^{-2\times3} = 10^{-6}$$

EXAMPLE 2–19

Perform the indicated operations.

(a) $(10^4)^{-3}$ (b) $\left(\dfrac{1}{10^4}\right)^2$

SOLUTIONS

(a) $(10^4)^{-3} = 10^{4\times(-3)} = 10^{-12}$ (b) $\left(\dfrac{1}{10^4}\right)^2 = \dfrac{1^2}{10^{4\times2}} = \dfrac{1}{10^8} = 10^{-8}$

When the exponent is in the denominator, we usually move it to the numerator.

PRACTICE PROBLEMS 2–15

Perform the indicated operations. Express your answers in powers of ten form.

1. $(10^2)^3$
2. $(10^4)^2$
3. $(10^3)^3$
4. $(10^{-1})^2$
5. $(10^{-4})^5$
6. $(10^{-6})^2$
7. $(10^{-2})^{-3}$
8. $(10^{-3})^{-1}$
9. $(10^{-1})^{-4}$
10. $(10^3)^{-5}$
11. $(10^2)^{-1}$
12. $(10^6)^{-2}$
13. $\left(\dfrac{1}{10^2}\right)^4$
14. $\left(\dfrac{1}{10^{-3}}\right)^2$
15. $\left(\dfrac{1}{10^3}\right)^{-3}$

2-16 SQUARE AND CUBE ROOTS OF POWERS OF TEN

The square root of a number is one of its two equal factors. For example, the square root of 100 is 10 because $10 \times 10 = 100$. Square roots can be expressed using a radical sign ($\sqrt{\ }$) or an exponent of $\frac{1}{2}$. The square root of 10^4 can be expressed as:

$$\sqrt{(10^4)} = (10^4)^{1/2} = 10^{4 \times 1/2} = 10^2$$

The cube root of a number is one of its three equal factors. For example, the cube root of 1000 is 10 because $10 \times 10 \times 10 = 1000$. Cube roots can be expressed using a radical sign ($\sqrt[3]{\ }$) or an exponent of $\frac{1}{3}$. The cube root of 10^3 can be expressed as:

$$\sqrt[3]{(10^3)} = (10^3)^{1/3} = 10^{3 \times 1/3} = 10$$

PRACTICE PROBLEMS 2-16

Perform the indicated operations. Express your answers in powers of ten form.

1. $(10^6)^{1/2}$
2. $(10^9)^{1/3}$
3. $(10^{-6})^{1/2}$
4. $(10^{-12})^{1/3}$
5. $\left(\frac{1}{10^{-3}}\right)^{1/3}$
6. $(10^{-6})^{1/3}$
7. $(10^3)^{1/3}$
8. $\left(\frac{1}{10^4}\right)^{1/2}$
9. $(10^{-15})^{1/3}$
10. $(10^{10})^{1/2}$

2-17 SQUARING NUMBERS

Let's square some numbers other than powers of ten and express the answers in scientific notation rounded to three places.

EXAMPLE 2-20

Find 14.7^2.

SOLUTION For manual calculation:

$$14.7^2 = 14.7 \times 14.7 = 216.09 = 2.16 \times 10^2$$

To estimate:

$$14.7^2 = (1.47 \times 10) \times (1.47 \times 10)$$

Since 1.47 is about halfway between 1 and 2, we will round one number down and the other up:

$$(1 \times 10) \times (2 \times 10) = 2 \times 10^2$$

EXAMPLE 2-21

Find $(0.0236 \times 10^2)^2$.

SOLUTION For manual calculation:

$$(0.0236 \times 10^2)^2 = 2.36 \times 2.36 = 5.5696 = 5.57$$

To estimate:

Since 2.36 is about halfway between 2 and 3, we will round one number down and the other up:

$$2.36 \times 2.36 = 2 \times 3 = 6$$

EXAMPLE 2–22

Find $\dfrac{278^2}{16}$.

SOLUTION For manual calculation:

$$\frac{278^2}{16} = \frac{77284}{16} = 4830 = 4.83 \times 10^3$$

To estimate:

$$\frac{278^2}{16} = \frac{(3 \times 10^2)^2}{2 \times 10} = \frac{9 \times 10^4}{2 \times 10} = 4.5 \times 10^3$$

PRACTICE PROBLEMS 2–17

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. 6.3^2
2. 43^2
3. 0.0372^2
4. 0.00047^2
5. $(6.73 \times 10^2)^2$
6. $(1.04 \times 10^{-3})^2$
7. $(796 \times 10^{-2})^2$
8. 8764^2
9. 0.973^2
10. $(0.00567 \times 10^{-4})^2$
11. $(0.0000873 \times 10^4)^2$
12. $(1260 \times 10^{-3})^2$
13. $0.0073^2 \times 4700$
14. $(27 \times 10^{-6})^2 \times 68{,}000$
15. $20^2 \div 3300$
16. $9^2 \div 270$
17. A playground is laid out in the shape of a square. Therefore the area equals any side squared. $A = S^2$. What is the area of a playground whose sides equal 78.4 yards? Express your answer in scientific notation rounded to three places.
18. A 36 in. × 36 in. ceramic tile shower stall is being installed. How many 4 in. × 4 in. tiles are needed to cover the floor? Disregard the drain opening and grout spaces.
19. In an electrical circuit, $P = I^2R$. Find the power, in watts, if the current, I, equals 48.7×10^{-4} amps and the resistance, R, equals 5.6×10^3 ohms. Express your answer in scientific notation rounded to three places.

2–18 SQUARE ROOTS OF NUMBERS

As explained earlier, the square root of a number is one of its two equal factors; for example, the square root of 100 is 10. When the number is a perfect square like 100 or 4 or 9, we can find the square root by recalling the squares in the multiplication table that we memorized in school. Most of the time, however, the number is not a perfect square, so we have to resort to other techniques. Although there are algorithms for determining square roots, the two most common techniques are using a calculator or using estimation techniques. The steps for using a calculator are very simple: Enter the number, hit the "square root" key, and hit the "equals" key. Using estimation techniques to find a square root requires recalling and using rules for rounding numbers, powers of ten, rules for exponents, and the squares from the multiplication table. The mental gymnastics practiced in using estimation techniques are very similar to the mental activity used to solve all kinds of technical problems. The indirect and, perhaps, the greatest value in learning math skills is to improve general problem-solving skills.

The estimation technique we will use to find the square root of a number is demonstrated below:

Find the square root of 179,432:

Step 1. Convert 179,432 to scientific notation with three significant digits:

$$179{,}432 = 1.79 \times 10^5$$

Step 2. Convert this number to a number between 4 and 400 so that the exponent is even:

$$1.79 \times 10^5 = 17.9 \times 10^4$$

Step 3. Estimate the square root:

$$(17.9 \times 10^4)^{1/2} = (17.9)^{1/2} \times (10^4)^{1/2} \cong (18)^{1/2} \times 10^2$$

At this point we have reduced the problem from finding the square root of a six-digit number (179,432) to estimating the square root of a two-digit number (18). We can make this estimate by comparing 18 to memorized squares. If you remember the 12 × 12 multiplication table, you already know the first 11 columns in the table below:

N	2	3	4	5	6	7	8	9	10	11	12	16	20
N^2	4	9	16	25	36	49	64	81	100	121	144	256	400

The square root of N^2 is N, so the square root of each number in the second row is directly above it in the first row. The next-to-last column indicates that $16 \times 16 = 256$, which is something you probably never memorized, but is useful to know since it is a power of two and occurs often in computer-related topics. The last column can be easily calculated ($20 \times 20 = 400$). The value of remembering this table is that the square root of any number between two numbers in the second row will be between the two numbers above in the first row.

For our example, 18 is between 16 and 25 in the second row, so its square root is between 4 and 5 in the first row. Since 18 is closer to 16 than 25 we can assume that the answer is closer to 4 than to 5; a good estimate would be 4.2. The complete estimate is: The square root of $179{,}532 \cong 4.2 \times 10^2$.

To summarize, an estimation technique to find the square root of a number is:

1. Rewrite the number in scientific notation with three significant digits.
2. Rewrite the number in powers of ten notation so that the exponent of ten is even and the significant digits fall within the range of 4 to 400.
3. Estimate the answer by taking the square of the significant digits multiplied by a power of ten with an exponent that is one half the value found in step 2.

After some practice you may find that you can skip the first step, but it is easier for beginners to start with step 1. Step 2 uses the range from 4 to 400 because it moves numbers between 1 and 4 to the range 100 to 400; there are few recognizable squares between 1 and 4, but several between 100 and 400. For example, if we had to find the square root of 1.3×10^4, we should first convert it to 130×10^2. It would be difficult to directly estimate the square root of 1.3, but it is easy to estimate that the square root of 130 is about 12 because it is near 144, which is a known square ($12 \times 12 = 144$).

EXAMPLE 2–23

Find the square root of 376.

SOLUTION For this problem the number 376 is already between the range of 4 to 400, so we can skip the first two steps of the estimation technique and go to the third step. The square root of 376 will be between the square root of two known squares, 256 and 400, so the answer will be between 16 and 20. Since 376 is much closer to 400 than to 256 you might estimate that the answer is 19. But if for some reason you knew or calculated that $19^2 = 361$, then you might more accurately estimate that the answer is 19.4.

EXAMPLE 2–24

Find the square root of 0.00002689.

SOLUTION Convert to scientific notation:

$$0.00002689 = 2.69 \times 10^{-5}$$

Convert to an even power of ten so that the number is between 4 and 400:

$$2.69 \times 10^{-5} = 26.9 \times 10^{-6}$$

Estimate the square root:

$$(26.9 \times 10^{-6})^{1/2} \cong (27)^{1/2} \times 10^{-3}$$

Since 27 is between the two squares 25 and 36, the square root must be between 5 and 6. Since 27 is closer to 5, the estimate is 5.2 and the final answer is 5.2×10^{-3}.

PRACTICE PROBLEMS 2–18

Perform the indicated operations. Express your answer in scientific notation rounded to three places.

1. $36.5^{1/2}$
2. $0.925^{1/2}$
3. $(635 \times 10^{-2})^{1/2}$
4. $(1870 \times 10^{3})^{1/2}$
5. $(0.000675 \times 10^{2})^{1/2}$
6. $(27.3 \times 10^{2})^{1/2}$
7. $(5.46 \times 10^{-4})^{1/2}$
8. $\left(\dfrac{734 \times 10^{2}}{423}\right)^{1/2}$
9. $\left(\dfrac{0.00726 \times 10^{-3}}{77 \times 10^{-6}}\right)^{1/2}$
10. $\dfrac{1}{6.28(0.2 \times 3 \times 10^{-9})^{1/2}}$
11. The current, I, in a series circuit is found using the equation $I = (\frac{P}{R})^{1/2}$. Find the circuit current when the power dissipated, P, equals 38.1×10^{-2} watts, and the resistance, R, equals 91×10^{2} ohms. Express your answer in scientific notation rounded to three places.

SELF-TEST 2–15 THROUGH 2–18

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $(10^{3})^{5}$
2. $(10^{2})^{-4}$
3. $\left(\dfrac{1}{10^{-2}}\right)^{3}$
4. $(10^{-6})^{1/2}$
5. $(10^{6})^{1/3}$
6. $(10^{-3})^{1/3}$
7. 78.3^{2}
8. 0.000293^{2}
9. $(8796 \times 10^{3})^{2}$
10. $0.0173^{2} \times 1800$
11. $(39 \times 10^{-4})^{2} \times 120$
12. $\dfrac{15^{2}}{1500}$

13. $273^{1/2}$

14. $(2.65 \times 10^{-4})^{1/2}$

15. $\left(\dfrac{0.0000467 \times 10^{3}}{63}\right)^{1/2}$

16. $\left(\dfrac{645 \times 10^{2}}{14}\right)^{1/2}$

17. $\dfrac{(24.5 \times 10^{-3})^{1/2}}{17}$

18. $(300 \times 10^{-3} \times 220)^{1/2}$

19. $\left(\dfrac{273 \times 10^{4}}{0.0012}\right)^{1/2}$

20. $\dfrac{1}{6.28(50 \times 10^{-3} \times 2 \times 10^{-6})^{1/2}}$

CHAPTER 2 AT A GLANCE

PAGE	KEY POINT OR RULE	EXAMPLE
27	*Power of Ten:* A number represented by 10 with an exponent.	10^3
27	*Exponent:* A superscript of a number or expression that indicates the number of times the number or expression must be multiplied by itself. A negative exponent means the reciprocal.	$10^{-2} = \dfrac{1}{10^2} = \dfrac{1}{(10 \times 10)}$
27	*Base:* The number that is multiplied by itself to determine the place value in a numerical system, e.g., ten is multiplied by itself two times to determine the place value of the digit two places to the left of the decimal point.	10 is the base in the decimal number system.
28	*Rule 2–1:* In converting some multiple of 10 to powers of ten form, count the number of zeros. This number is the exponent. The answer is 10 raised to that power.	$1000 = 10^3$
29	*Rule 2–2:* In converting from powers of ten form to a decimal number, write the digit 1 and follow with as many zeros as indicated by the exponent.	$10^3 = 1000$
29	*Rule 2–3:* In converting from powers of ten to decimal fractions, the negative exponent tells how many zeros follow the 1 in the denominator.	$10^{-2} = \dfrac{1}{100}$
29	*Rule 2–4:* In converting from powers of ten to decimal numbers, the negative exponent tells how far the 1 is located to the right of the decimal point.	$10^{-2} = 0.01$
30	*Rule 2–5:* In converting from a decimal fraction to powers of ten form, count the number of zeros in the denominator. This number is the negative power to which 10 is raised.	$\dfrac{1}{100} = 10^{-2}$
30	*Rule 2–6:* In converting from a decimal number to powers of ten form, count how far the digit 1 is located to the right of the decimal point. This is the value of the negative exponent.	$0.01 = 10^{-2}$

30	*Rule 2–7:* To multiply numbers in powers of ten form, add the exponents.	$10^2 \times 10^3 = 10^{2+3} = 10^5$ $10^2 \times 10^{-3} = 10^{2+(-3)}$ $= 10^{-1}$
31	*Rule 2–8:* To divide numbers in powers of ten form, subtract the denominator exponent from the numerator exponent. The general equation is $\frac{10^m}{10^n} = 10^{m-n}$	$\frac{10^4}{10^2} = 10^{4-2} = 10^2$
33	*Rule 2–9:* Whenever a power of ten is in the denominator, it can be moved to the numerator by changing the sign of the exponent.	$\frac{10^2}{10^{-3}} = 10^2 \times 10^3 = 10^{2+3} = 10^5$
35	*Rule 2–10:* For each place we move the decimal point *left,* the exponent increases by 1.	$1750 = 175 \times 10$ $= 17.5 \times 10^2$ $= 1.75 \times 10^3$
35	*Rule 2–11:* For each digit we make the exponent more positive, the decimal point moves 1 place left.	
36	*Rule 2–12:* For each place we move the decimal point *right,* the exponent decreases by 1.	$0.00425 = 0.0425 \times 10^{-1}$ $= 0.425 \times 10^{-2}$ $= 4.25 \times 10^{-3}$
36	*Rule 2–13:* For each digit we make the exponent more negative, the decimal point moves 1 place *right.*	
36	*Scientific Notation:* A number expressed as a value between 1 and 10 and then multiplied by a power of ten.	7.32×10^4
37	*Rule 2–14:* To express numbers greater than 1 in scientific notation, count the number of places the decimal point must move left. This number is the exponent and is positive.	$704 = 7.04 \times 10^2$ $37{,}654 = 3.77 \times 10^4$ (rounded to three places)
37	*Rule 2–15:* To express numbers that are less than 1 in scientific notation, count the number of places the decimal point must move right. This number is the exponent and is negative.	$0.00156 = 1.56 \times 10^{-3}$ $0.00027643 = 2.76 \times 10^{-4}$ (rounded to three places)
38	*Rule 2–16:* To add or subtract numbers in powers of ten form, the exponents must be the same.	$3 \times 10^4 = 3 \times 10^4$ $\underline{+3 \times 10^3 = 0.3 \times 10^4}$ $= 3.3 \times 10^4$
42	*Key Point:* The reciprocal of a number is that number divided into 1.	Find the reciprocal of 150. $\frac{1}{150} = 6.67 \times 10^{-3}$
45	*Rule 2–17:* When raising a power to a power, multiply the exponents.	$(10^2)^3 = 10^{2\times 3} = 10^6$ $(10^6)^{1/2} = 10^{6\times 1/2} = 10^3$ $(3.6 \times 10^3)^2 = 1.30 \times 10^7$ $(746 \times 10^{-3})^{1/2} = 8.64 \times 10^{-1}$

END OF CHAPTER PROBLEMS 2–1

When working **End of Chapter Problems,** if you need to go back to the section where the material was presented, just look at the problem set number. For example, if you are working problems in *End of Chapter Problems 2–13* and you need help, go back to section 2–13 and review the material and examples presented there.

Convert the following numbers to powers of ten form.

1. 10
2. 100,000
3. 1000
4. 10,000,000
5. 100
6. 1,000,000

END OF CHAPTER PROBLEMS 2–2

Convert the following powers of ten numbers to decimal numbers.

1. 10^3
2. 10^5
3. 10^0
4. 10^8
5. 10^6
6. 10^4

END OF CHAPTER PROBLEMS 2–3

Convert the following powers of ten numbers to (a) decimal fractions and (b) decimal numbers.

1. 10^{-2}
2. 10^{-5}
3. 10^{-3}
4. 10^{-6}
5. 10^{-1}
6. 10^{-4}

END OF CHAPTER PROBLEMS 2–4

Convert the following decimal numbers to powers of ten form.

1. 0.0001
2. 0.01
3. 0.000001
4. 0.001

Convert the following decimal fractions to powers of ten form.

5. $\frac{1}{10{,}000}$
6. $\frac{1}{10{,}000{,}000}$
7. $\frac{1}{1000}$
8. $\frac{1}{1{,}000{,}000}$

END OF CHAPTER PROBLEMS 2–5

Perform the indicated operations. Express your answers in powers of ten form.

1. $10^4 \times 10^6$
2. $10^3 \times 10^2$
3. $10^2 \times 10^4$
4. $10^3 \times 10^5$
5. $10^5 \times 10^{-1}$
6. $10^6 \times 10^{-3}$
7. $10^3 \times 10^{-7}$
8. $10^2 \times 10^{-6}$
9. $10^{-1} \times 10^4$
10. $10^{-2} \times 10^6$
11. $10^{-6} \times 10^{-4}$
12. $10^{-7} \times 10^{-3}$
13. $10^{-3} \times 10^{-9}$
14. $10^{-2} \times 10^{-5}$
15. $10^{-4} \times 10^{-5}$
16. $10^{-3} \times 10^{-6}$

END OF CHAPTER PROBLEMS 2–6

Perform the indicated operations. Express your answers in powers of ten form.

1. $\frac{10^7}{10^3}$
2. $\frac{10^9}{10^4}$
3. $\frac{10^3}{10^8}$
4. $\frac{10^2}{10^5}$
5. $\frac{10^4}{10^{-4}}$
6. $\frac{10^7}{10^{-3}}$

7. $\frac{10^{3}}{10^{-9}}$
8. $\frac{10^{4}}{10^{-6}}$
9. $\frac{10^{-2}}{10^{-9}}$
10. $\frac{10^{-1}}{10^{-7}}$
11. $\frac{10^{-8}}{10^{-5}}$
12. $\frac{10^{-6}}{10^{-5}}$
13. $\frac{10^{-3}}{10^{3}}$
14. $\frac{10^{-6}}{10^{1}}$
15. $\frac{10^{-2}}{10^{5}}$
16. $\frac{10^{-4}}{10^{7}}$
17. $\frac{1}{10^{-3}}$
18. $\frac{1}{10^{-7}}$
19. $\frac{1}{10^{4}}$
20. $\frac{1}{10^{6}}$

END OF CHAPTER PROBLEMS 2–7

Perform the indicated operations. Express your answers in powers of ten form.

1. $\frac{10^{-3} \times 10^{-6}}{10^{-2}}$
2. $\frac{10^{2} \times 10^{6}}{10^{4}}$
3. $\frac{10^{-6} \times 10^{4}}{10^{5}}$
4. $\frac{10^{-4} \times 10^{9}}{10^{3}}$
5. $\frac{1}{10^{2} \times 10^{-4} \times 10^{-9}}$
6. $\frac{1}{10^{-2} \times 10^{4} \times 10^{-7}}$
7. $\frac{1}{10^{-3} \times 10^{-6} \times 10^{-1}}$
8. $\frac{1}{10^{2} \times 10^{6} \times 10^{-3}}$
9. $\frac{10^{-4}}{10^{6} \times 10^{-2} \times 10^{4}}$
10. $\frac{10^{3}}{10^{4} \times 10^{6} \times 10^{-1}}$
11. $\frac{10^{-3} \times 10^{2} \times 10^{4}}{10^{2} \times 10^{-7}}$
12. $\frac{10^{-1} \times 10^{2} \times 10^{4}}{10^{4} \times 10^{2}}$
13. $\frac{10^{4} \times 10^{6} \times 10^{3}}{10^{2} \times 10^{-7} \times 10^{5}}$
14. $\frac{10^{-1} \times 10^{-7} \times 10^{6}}{10^{1} \times 10^{0} \times 10^{-4}}$
15. $\frac{10^{3} \times 10^{-5} \times 10^{6}}{10^{4} \times 10^{-3} \times 10^{7}}$
16. $\frac{10^{-2} \times 10^{-6} \times 10^{3}}{10^{3} \times 10^{6} \times 10^{-4}}$
17. $\frac{10^{-3} \times 10^{-6} \times 10^{-9}}{10^{3} \times 10^{6} \times 10^{9}}$
18. $\frac{10^{-9} \times 10^{3} \times 10^{-3}}{10^{4} \times 10^{-1} \times 10^{5}}$

END OF CHAPTER PROBLEMS 2–8

Change the following numbers to powers of ten form with an exponent of (a) 2, (b) 4, and (c) 6.

1. 475
2. 1500
3. 65.9
4. 478
5. 93,400
6. 180,000
7. 4780
8. 4700
9. 41,800
10. 18,000

Change the following numbers to powers of ten form with an exponent of (a) -2, (b) -4, and (c) -6.

11. 0.00465
12. 0.000758
13. 0.00000555
14. 0.0000906
15. 0.000673
16. 0.00825
17. 0.0000000108
18. 0.00000745

Change the following numbers to powers of ten form with an exponent of (a) 3, and (b) −3.

19. 325
20. 470
21. 722
22. 0.0157
23. 2700
24. 4.39
25. 180
26. 8060
27. Change the number 0.0257 to powers of ten form with an exponent of (a) 3, (b) −3, and (c) −6.
28. Change the number 0.00786 to powers of ten form with an exponent of (a) 3, (b) −3, and (c) −6.
29. Change the number 680 to powers of ten form with exponents of 3 and –3.
30. Change the number 0.0642 to powers of ten form with exponents of 3 and –3.

END OF CHAPTER PROBLEMS 2–9

Round the following numbers to three significant figures and express your answers in scientific notation.

1. 27,640
2. 43,966
3. 47.8
4. 277.7
5. 1,765,400
6. 3.716,500
7. 273.46
8. 35.986
9. 173,460
10. 406,600
11. 78.88
12. 15.756
13. 1,670,000
14. 12,673
15. 57,945
16. 83,876
17. 80,975
18. 40,713
19. Round 92,151 to three significant figures and express your answer in scientific notation.
20. Round 126,458 to three significant figures and express your answer in scientific notation.

END OF CHAPTER PROBLEMS 2–10

Round the following numbers to three significant figures and express your answers in scientific notation.

1. 0.004783
2. 0.017600
3. 0.7474
4. 0.002087
5. 0.01637
6. 0.001037
7. 0.0045446
8. 0.0078449
9. 0.000050085
10. 0.000030067
11. 0.25465
12. 0.056374
13. 0.040045
14. 0.0740347
15. 0.0008964
16. 0.0006794
17. 0.0170983
18. 0.07096
19. Round 0.00020507 to three significant figures and express your answer in scientific notation.
20. Round 0.005648 to three significant figures and express your answer in scientific notation.

END OF CHAPTER PROBLEMS 2–11

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $10^3 + 10^4$
2. $10^2 + 10$
3. $10^{-1} + 10^{-1}$
4. $10^{-1} + 10^{-2}$
5. $33 \times 10^2 + 56 \times 10$
6. $56 \times 10^3 + 33 \times 10^4$
7. $470 \times 10^3 + 1.2 \times 10^4$
8. $750 \times 10^3 + 1.5 \times 10^4$
9. $27 \times 10^{-2} + 180 \times 10^{-4}$
10. $56 \times 10^{-2} + 470 \times 10^{-4}$
11. $45 \times 10^{-3} - 63 \times 10^{-4}$
12. $75 \times 10^{-3} - 47 \times 10^{-4}$
13. $8.2 \times 10^3 + 8.2 \times 10^4 + 9100$
14. $2.7 \times 10^2 + 6.8 \times 10^3 + 1200$
15. Add 470×10^3 and 33×10^4.
16. Add 2500×10^{-4} and 3530×10^{-5}.

END OF CHAPTER PROBLEMS 2–12

In the following problems, first estimate the answer by rounding each term to one-place accuracy and using scientific notation. Then calculate the answer using a calculator set to scientific notation with three-place accuracy.

1. 737×53.9
2. 3.15×0.324
3. 0.0059×0.005892
4. 0.0932×0.00337
5. 5957.32×0.01317
6. 384.7×0.846
7. $620 \times 270 \times 190$
8. $434 \times 597 \times 850$
9. $32572 \div 0.329$
10. $5227.8 \div 0.0322$
11. $0.0987 \div 0.00475$
12. $0.0529 \div 0.00289$
13. $392.73 \div 8388.2$
14. $213.71 \div 7569.3$
15. $283 \times 489 \div 511$
16. $849 \times 334 \div 731$

END OF CHAPTER PROBLEMS 2–13

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $$\frac{12 \times 47 \times 10^3}{4.7 \times 10^3 + 4.7 \times 10^4}$$
2. $$\frac{12 \times 56 \times 10^3}{3.3 \times 10^3 + 5.6 \times 10^4}$$
3. $$\frac{5 \times 4700}{470 + 4700}$$
4. $$\frac{5 \times 5600}{680 + 5600}$$
5. $$\frac{15 \times 82 \times 10^4}{10 \times 10^3 + 82 \times 10^4}$$
6. $$\frac{15 \times 33 \times 10^4}{10 \times 10^3 + 33 \times 10^4}$$
7. $$\frac{25 \times 750 \times 10^3}{91 \times 10^3 + 750 \times 10^3}$$
8. $$\frac{25 \times 680}{82 \times 10 + 680}$$
9. $$\frac{18 \times 12 \times 10^6}{12 \times 10^6 + 8.2 \times 10^6}$$
10. $$\frac{18 \times 18 \times 10^6}{18 \times 10^6 + 4.7 \times 10^6}$$
11. $$\frac{25 \times 3300}{3300 + 56{,}000}$$
12. $$\frac{20 \times 4700}{39{,}000 + 4700}$$
13. $$\frac{12 \times 5600}{5600 + 33{,}000}$$
14. $$\frac{5 \times 680}{680 + 1500}$$
15. $$\frac{12 \times 4.7 \times 10^3}{4.7 \times 10^4 + 4.7 \times 10^3}$$
16. $$\frac{15 \times 1.8 \times 10^3}{4.7 \times 10^4 + 1.8 \times 10^3}$$
17. $$\frac{20 \times 6.8 \times 10^3}{6.8 \times 10^3 + 4.7 \times 10^4}$$
18. $$\frac{5 \times 1.2 \times 10^3}{1.2 \times 10^3 + 1.8 \times 10^4}$$
19. A luxury car dealer pays a sound system installer \$27.50 per hour. How much is the sound system installer paid for 47 hours of work? Estimate by rounding to the nearest 10 before multiplying. Then find the exact answer accurate to the nearest cent.
20. A luxury car dealer pays a sound system installer \$28.75 per hour. How much is the sound system installer paid for 33 hours of work? Estimate by rounding to the nearest 10 before multiplying. Then find the exact answer accurate to the nearest cent.
21. A parcel of land measures 1786 ft by 814.4 ft. Estimate the area by rounding to the nearest hundred, then find the exact area. Express both answers in scientific notation rounded to three places. Express the area in square feet. Area = length × width.
22. A parcel of land measures 1289.3 ft by 1865.4 ft. Estimate the area by rounding to the nearest hundred, then find the exact area. Express both answers in scientific notation rounded to three places. Express the area in square feet. Area = length × width.
23. Ohm's law states that in a series circuit, $I = \frac{E}{R}$, where I is the circuit current, E is the applied voltage, and R is the total resistance. If $E = 9$ V and $R = 33 \times 10^4\ \Omega$, what is the circuit current? Estimate the answer by rounding to the nearest ten, then find the actual current in scientific notation rounded to three places.
24. Ohm's law states that in a series circuit, $I = \frac{E}{R}$, where I is the circuit current, E is the applied voltage, and R is the total resistance. If $E = 15$ V and $R = 8.2 \times 10^3\ \Omega$, what is the circuit current? Estimate the answer by rounding to the nearest ten, then find the actual current in scientific notation rounded to three places.

END OF CHAPTER PROBLEMS 2–14

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $\dfrac{1}{6.28 \times 50 \times 10^3 \times 30 \times 10^{-9}}$
2. $\dfrac{1}{6.28 \times 600 \times 10 \times 10^{-6}}$
3. $\dfrac{1}{6.28 \times 25 \times 10^3 \times 50 \times 10^{-3}}$
4. $\dfrac{1}{6.28 \times 2 \times 10^3 \times 7 \times 10^{-6}}$
5. $\dfrac{1}{680} + \dfrac{1}{560}$
6. $\dfrac{1}{470} + \dfrac{1}{560}$
7. $\dfrac{1}{3300} + \dfrac{1}{2700} + \dfrac{1}{1000}$
8. $\dfrac{1}{6800} + \dfrac{1}{5600} + \dfrac{1}{9100}$
9. $\dfrac{1}{56 \times 10^3} + \dfrac{1}{47 \times 10^3}$
10. $\dfrac{1}{27 \times 10^5} + \dfrac{1}{10 \times 10^5}$
11. $\dfrac{1}{470 \times 10^3} + \dfrac{1}{330 \times 10^3}$
12. $\dfrac{1}{68 \times 10^3} + \dfrac{1}{75 \times 10^3}$
13. $\dfrac{1}{27 \times 10^3} + \dfrac{1}{56 \times 10^3} + \dfrac{1}{18 \times 10^3}$
14. $\dfrac{1}{820 \times 10^3} + \dfrac{1}{390 \times 10^3} + \dfrac{1}{270 \times 10^3}$
15. $\dfrac{1}{\dfrac{1}{4700} + \dfrac{1}{5600}}$
16. $\dfrac{1}{\dfrac{1}{270} + \dfrac{1}{560}}$
17. $\dfrac{1}{\dfrac{1}{270} + \dfrac{1}{180} + \dfrac{1}{390}}$
18. $\dfrac{1}{\dfrac{1}{7500} + \dfrac{1}{5600} + \dfrac{1}{3300}}$
19. $\dfrac{1}{\dfrac{1}{56 \times 10^3} + \dfrac{1}{68 \times 10^3}}$
20. $\dfrac{1}{\dfrac{1}{2.2 \times 10^3} + \dfrac{1}{810}}$
21. $\dfrac{1}{\dfrac{1}{9.1 \times 10^3} + \dfrac{1}{8.2 \times 10^3} + \dfrac{1}{5.6 \times 10^3}}$
22. $\dfrac{1}{\dfrac{1}{18 \times 10^3} + \dfrac{1}{39 \times 10^3} + \dfrac{1}{56 \times 10^3}}$
23. The total conductance, G_T, of a circuit consisting of two resistors in parallel is given by the equation $G_T = \frac{1}{R_1} + \frac{1}{R_2}$. Find G_T when $R_1 = 330\ \Omega$ and $R_2 = 750\ \Omega$. Express your answer in scientific notation rounded to three places.
24. The total conductance, G_T, of a circuit consisting of two resistors in parallel is given by the equation $G_T = \frac{1}{R_1} + \frac{1}{R_2}$. Find G_T when $R_1 = 2.2 \times 10^4\ \Omega$ and $R_2 = 3.9 \times 10^4\ \Omega$. Express your answer in scientific notation rounded to three places.

END OF CHAPTER PROBLEMS 2–15

Perform the indicated operations.

1. $(10^2)^3$
2. $(10^2)^5$
3. $(10^5)^2$
4. $(10^{-2})^{-3}$
5. $(10^3)^{-4}$
6. $(10^2)^{-7}$
7. $(10^{-4})^{-3}$
8. $\left(\dfrac{1}{10^3}\right)^3$
9. $\left(\dfrac{1}{10^{-4}}\right)^{-2}$
10. $\left(\dfrac{1}{10^{-4}}\right)^{-2}$

END OF CHAPTER PROBLEMS 2–16

Perform the indicated operations.

1. $(10^6)^{1/2}$
2. $(10^8)^{1/2}$
3. $(10^{-4})^{1/2}$
4. $(10^{-2})^{1/2}$
5. $\left(\dfrac{1}{10^4}\right)^{1/2}$
6. $\left(\dfrac{1}{10^8}\right)^{1/2}$

7. $\left(\frac{1}{10^{-10}}\right)^{1/2}$
8. $\left(\frac{1}{10^{-6}}\right)^{1/2}$
9. $\left(\frac{10^{2}}{10^{-4}}\right)^{1/2}$
10. $\left(\frac{10^{5}}{10^{-1}}\right)^{1/2}$

END OF CHAPTER PROBLEMS 2–17

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. 79.3^2
2. 446^2
3. 1780^2
4. 5600^2
5. 0.04758^2
6. 0.0652^2
7. 0.00873^2
8. 0.00165^2
9. $(45 \times 10^3)^2$
10. $(264 \times 10^2)^2$
11. $(300 \times 10^3)^2$
12. $(400 \times 10^3)^2$
13. $(0.296 \times 10^{-3})^2$
14. $(525 \times 10^{-4})^2$
15. $(0.00903 \times 10^{-2})^2$
16. $(0.033 \times 10^{-2})^2$
17. $(50 \times 10^{-3})^2 \times 2700$
18. $(375 \times 10^{-9})^2 \times 120$
19. $(0.0765 \times 10^2)^2 \times 0.037$
20. $(0.000821 \times 10^3)^2 \times 0.976$
21. A playground is laid out in the shape of a square. Therefore the area equals any side squared. $A = S^2$. What is the area of a playground whose sides equal 53.7 yards? Express your answer in scientific notation rounded to three places.
22. A playground is laid out in the shape of a square. Therefore the area equals any side squared. $A = S^2$. What is the area of a playground whose sides equal 127.4 yards? Express your answer in scientific notation rounded to three places.
23. A 48 in. × 48 in. ceramic tile shower stall is being installed. How many 4 in. × 4 in. tiles are needed to cover the floor? Disregard the drain opening and grout spaces.
24. A 42 in. × 42 in. ceramic tile shower stall is being installed. How many 6 in. × 6 in. tiles are needed to cover the floor? Disregard the drain opening and grout spaces.
25. In an electrical circuit, $P = I^2R$. Find the power in watts if the current, I, equals 734×10^{-3} amps and the resistance, R, equals 47 ohms. Express your answer in scientific notation rounded to three places.
26. In an electrical circuit, $P = I^2R$. Find the power in watts if the current, I, equals 17.9×10^{-5} amps and the resistance, R, equals 27×10^4 ohms. Express your answer in scientific notation rounded to three places.

END OF CHAPTER PROBLEMS 2–18

Perform the indicated operations. Express your answers in scientific notation rounded to three places.

1. $93^{1/2}$
2. $783^{1/2}$
3. $237^{1/2}$
4. $5756^{1/2}$
5. $0.00705^{1/2}$
6. $0.0085^{1/2}$
7. $0.000105^{1/2}$
8. $0.000187^{1/2}$
9. $(270 \times 10^5)^{1/2}$
10. $(432 \times 10^4)^{1/2}$
11. $(4500 \times 10^4)^{1/2}$
12. $(74.5 \times 10^3)^{1/2}$
13. $(0.00923 \times 10^{-3})^{1/2}$
14. $(0.00176 \times 10^4)^{1/2}$
15. $(0.000346 \times 10^5)^{1/2}$
16. $(0.000906 \times 10^{-6})^{1/2}$
17. $\left(\frac{0.289 \times 10^{-1}}{254}\right)^{1/2}$
18. $\left(\frac{0.00573 \times 10^{-4}}{330}\right)^{1/2}$
19. $\frac{1}{6.28(20 \times 10^{-3} \times 20 \times 10^{-9})^{1/2}}$
20. $\frac{1}{6.28(0.0047 \times 560 \times 10^{-12})^{1/2}}$
21. $\frac{1}{6.28(0.150 \times 680 \times 10^{-12})^{1/2}}$
22. $\frac{1}{6.28(100 \times 10^{-3} \times 20 \times 10^{-9})^{1/2}}$
23. The current, I, in a series circuit is found using the equation $I = (\frac{P}{R})^{1/2}$. Find the circuit current when the power dissipated, P, equals 83.2×10^{-2} watts, and the resistance, R, equals 75×10^2 ohms. Express your answer in scientific notation rounded to three places.
24. The current, I, in a series circuit is found using the equation $I = (\frac{P}{R})^{1/2}$. Find the circuit current when the power dissipated, P, equals 278×10^{-3} watts, and the resistance, R, equals 180×10^2 ohms. Express your answer in scientific notation rounded to three places.

CHAPTER 3

Units and Prefixes

CHAPTER OBJECTIVES

In this chapter you will learn how to:

1. Convert numbers to engineering notation.
2. Apply the various prefixes used in the electronics field.
3. Convert from one prefix to another.
4. Affix the correct prefix to electrical quantities.
5. Use the calculator in the engineering notation mode.
6. Apply the various prefixes used in the metric system.
7. Convert between the English and metric systems of measurements.

MATH IN ACTION

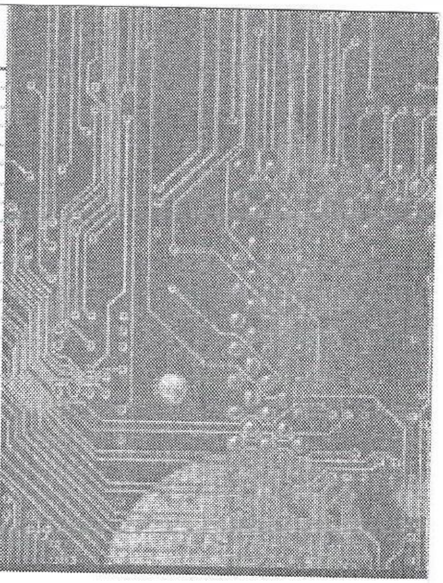
"I hope you can help me," said Professor Tatum to the computer technician, Sam, who had just arrived at his office. "My computer won't let me send any e-mail." Sam quickly determined that Professor Tatum's mailbox exceeded the 50-megabyte storage limit, and Professor Tatum must delete at least 7 megabytes of old e-mail before he can send any new e-mail. Professor Tatum is a little confused, and Sam can understand why. The file size is not shown in megabytes, but in bytes. Professor Tatum is not familiar with this use of units and prefixes. Sam explains how to convert between base numbers and numbers with prefixes and helps Professor Tatum understand what a megabyte is. Sam helps Professor Tatum free up 16 megabytes of space.

INTRODUCTION

In this chapter the use of prefixes is introduced, and we show how the use of prefixes makes reading large and small quantities easy. In Chapter 2, we learned how to express answers in scientific notation. In this chapter, we will learn how to express answers in *engineering* notation. Engineering notation makes converting to prefixes easier, since both prefixes and engineering notation move the decimal point in increments of three. In the study of computers and electronics, we will find that prefixes are used to express the values of memory size, current, resistance, capacitance, frequency, and many other quantities. Because prefixes are so widely used, it is essential that you understand how to use them.

The metric system is the international system of weights and measurements. Because of the need for a single internationally accepted system, the metric system is gradually replacing the English system in this country.

In electronics, we already use the international system (SI) of measurements for most quantities, such as voltage, current, capacitance, and conductance, to name a few; but some measurements, such as distance, velocity, and weight, may be specified in either the English system or the metric system. For this reason, we must be able to convert from one system to the other.



3–1 UNITS

Most of the units we deal with are either very large or very small. To deal with these quantities more easily, prefixes are used. The prefixes relate to powers of ten that are multiples of 3 or −3. *Engineering notation* is the process of moving the decimal point in a number so that the resulting power of ten is changed to a multiple of 3 or −3. Some examples are 10^3, 10^6, 10^{-3}, and 10^{-6}. Once we have changed a number into engineering notation, we can replace the power of ten with the prefix that represents that power of ten.

Before we begin to use engineering notation and these prefixes, let's review powers of ten by changing a number to powers of ten notation with positive exponents. The rule is:

RULE 3–1 If we make the exponent more positive, the decimal point moves to the left.

Consider the number 6700. Let's change the number using powers of ten notation with positive exponents.

$$6700 = 670 \times 10^1 = 67 \times 10^2 = 6.7 \times 10^3 = 0.67 \times 10^4$$

and so on. Notice that as we make the exponent more positive, the decimal point moves farther left.

Now let's convert some numbers to powers of ten form using engineering notation. We will use Rule 3–1 and work some example problems.

EXAMPLE 3–1

Change 4700 to a number times 10^3.

SOLUTION We are changing the number to powers of ten form and we want the exponent to be 3, so the decimal point moves *left* three places.

$$4700 = 4.7 \times 10^3$$

EXAMPLE 3–2

Change 670×10^2 to a number times 10^3.

SOLUTION We can solve this two ways. First, we could change 670×10^2 to a regular number. $670 \times 10^2 = 67{,}000$. Then, applying the same rule as before, $67{,}000 = 67 \times 10^3$.

Another approach would be to note that the exponent is already 2. The difference between 10^2 and 10^3 is 10^1. Therefore, we need to move the decimal point one place to the left.

$$670 \times 10^2 = 67 \times 10^3$$

EXAMPLE 3–3

Change 330,000 to some number times 10^3 and then to some number times 10^6.

SOLUTION Again, we move the decimal point three places to the left because the exponent is 3. Then we move the decimal point six places to the left because the exponent is 6.

$$330{,}000 = 330 \times 10^3$$

$$330{,}000 = 0.330 \times 10^6$$

Once we have 330×10^3, we can do this:

$$330 \times 10^3 = 0.330 \times 10^6$$

In this case, the exponent increases from 3 to 6, a change of 3 in a positive direction. Therefore, the decimal point moves three places to the left.

Now let's change numbers to powers of ten form with negative exponents. The rule is:

RULE 3–2 If the exponent is made more negative, the decimal point moves to the right.

Consider the number 0.00275. Let's change to powers of ten notation using negative exponents and following Rule 3–2.

$$0.00275 = 0.0275 \times 10^{-1} = 0.275 \times 10^{-2} = 2.75 \times 10^{-3}$$

and so on. Notice that as we make the exponent more negative, the decimal point moves farther right.

Now, let's convert some numbers to powers of ten form using engineering notation. We will use Rule 3–2 and work some example problems.

EXAMPLE 3–4

Change 1.73 to some number times 10^{-3}.

SOLUTION The exponent is -3. Therefore, we move the decimal point three places to the *right.*

$$1.73 = 1730 \times 10^{-3}$$

EXAMPLE 3–5

Change 27.7×10^{-4} to some number times 10^{-6}.

SOLUTION We can solve the problem by changing 27.7×10^{-4} to a regular number and then moving the decimal point six places to the right.

$$27.7 \times 10^{-4} = 0.00277 = 2770 \times 10^{-6}$$

We could also solve the problem by noting that the exponent is already -4. Increasing its value to -6 is a change of 2 in a negative direction. Therefore, the decimal point moves two places to the right. Remember, if we make the exponent more negative, the decimal point moves to the *right.*

$$27.7 \times 10^{-4} = 2770 \times 10^{-6}$$

Now let's consider numbers for which we make the exponent more positive and then more negative.

EXAMPLE 3–6

Change 0.05×10^{-7} to some number times 10^{-9} and then to some number times 10^{-6}.

SOLUTION When we change the power of ten from 10^{-7} to 10^{-9}, we are making the exponent more negative. The difference is -2. That is, $10^{-7-2} = 10^{-9}$. Therefore, the decimal point must move to the *right.*

$$0.05 \times 10^{-7} = 5 \times 10^{-9}$$

Next we need to change the exponent so that it is -6. We must add 1 to -7. $-7 + 1 = -6$. That is, $10^{-7+1} = 10^{-6}$. According to our first rule, if we *add* 1 to the exponent, we must move the decimal point to the *left.*

$$0.05 \times 10^{-7} = 0.005 \times 10^{-6}$$

EXAMPLE 3–7

Change 4.7 to some number times 10^{3} and then to some number times 10^{-3}.

SOLUTION

$$4.7 = 0.0047 \times 10^{3}$$

Since the exponent is 3, the decimal point moves to the *left.*

$$4.7 = 4700 \times 10^{-3}$$

Since the exponent is -3, the decimal point moves to the *right.*

KEY POINT

Expressing numbers as numbers between 1 and 10 times the proper power of ten is called *scientific* notation. Expressing numbers as numbers times powers of ten that are multiples of 3 is called *engineering* notation.

PRACTICE PROBLEMS 3–1

Change the following numbers to numbers times 10^{3} and to numbers times 10^{6}:

1. 27,000
2. 330,000
3. 5600
4. 390×10^{2}
5. 68×10^{4}
6. 1,200,000
7. 180×10^{4}
8. 1500×10^{5}
9. 1800×10^{2}
10. 51×10^{5}

Change the following numbers to numbers times 10^{-3} and to numbers times 10^{-6}:

11. 0.000423
12. 0.00716
13. 0.000014
14. 28.3×10^{-4}
15. 173×10^{-2}
16. 8.3×10^{-1}
17. 173×10^{-5}
18. 1.73×10^{-1}
19. 17.3×10^{-4}
20. 0.0706

Change the following numbers to numbers times 10^{-9} and to numbers times 10^{-12}:

21. 0.0046×10^{-4}
22. 0.413×10^{-7}
23. 6.73×10^{-10}
24. 6.73×10^{-8}
25. 67.3×10^{-11}
26. 0.0173×10^{-6}
27. 0.0563×10^{-8}
28. $76,500 \times 10^{-15}$
29. 6.43×10^{-10}
30. 906×10^{-10}

Change the following numbers to regular numbers, to numbers times 10^{-3}, and to numbers times 10^3:

31. 17×10^{-1}
32. 0.637×10^2
33. 716×10^{-2}
34. 47×10
35. 27.3×10^1
36. 3.93×10^2
37. 3.93×10^{-2}
38. 0.043×10^4
39. 0.706×10^{-1}
40. 89.1×10^2

3–2 PREFIXES

Table 3–1 is a partial list of the International System of Units (SI). Only those units of interest to electronics students are included. Prefixes used with these electrical units are listed in Table 3–2. Prefixes are used to make it easier for us to communicate with each other. For example, suppose the value of a capacitance was 0.000000003 F. This is a very clumsy number to work with. If we express it in scientific notation, we get 3×10^{-9} F. That's better, but still awkward. Looking at our table of prefixes, we see that 10^{-9} is a *nano* unit and the symbol is n (lowercase n). We would write the value as 3 nF. "Three nanofarads" is the way we would say it.

2,700,000 Ω would be simpler to express if we used the prefix equal to 10^6, which is *mega* and is symbolized as M.

$$2{,}700{,}000\ \Omega = 2.7 \times 10^6\ \Omega = 2.7\ \text{M}\Omega$$

Suppose we calculate a circuit current to be 5.6×10^{-4} A. We need to replace the power of ten with some prefix to make the quantity easier to read. Since our prefixes are multiples of 10^3 or 10^{-3}, we have to change 10^{-4} to either 10^{-3} or 10^{-6}. Let's do both. First change 5.6×10^{-4} to a number times 10^{-3}.

$$5.6 \times 10^{-4}\ \text{A} = 0.56 \times 10^{-3}\ \text{A}$$

Table 3–1 Partial List of the International System of Units (SI).

SI PHYSICAL CHARACTERISTIC	NAME OF SI UNIT	SYMBOL
Capacitance (*C*)	farad	F
Admittance (*Y*)	siemens	S
Conductance (*G*)	siemens	S
Current (*I*, *j*)	ampere	A
Energy (*w*)	joule	J
Frequency (*f*)	hertz	Hz
Impedance (*Z*)	ohm	Ω
Inductance (*L*)	henry	H
Power (*P*)	watt	W
Reactance (*X*)	ohm	Ω
Resistance (*R*)	ohm	Ω
Susceptance (*B*)	siemens	S
Time (*t*)	second	s
Wavelength (λ)	meter	m
Electromotive force (*E*)	volt	V
Difference in potential (*V*)	volt	V

Table 3–2 Prefixes Used with Electrical Units Listed in Table 3–1.

PREFIX	SYMBOL	POWER OF TEN
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
units	none	10^{0}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

To change the exponent from −4 to −3, we must add one ($10^{-4+1} = 10^{-3}$). Because we made the exponent more positive, the decimal point moved to the *left* (Rule 3–1). The prefix for 10^{-3} is *milli* (m). Then

$$5.6 \times 10^{-4}\ \text{A} = 0.56 \times 10^{-3}\ \text{A} = 0.56\ \text{mA}$$

Now let's change 5.6×10^{-4} to a number times 10^{-6}.

$$5.6 \times 10^{-4}\ \text{A} = 560 \times 10^{-6}\ \text{A}$$

To change the exponent from −4 to −6, we had to add −2. $10^{-4+(-2)} = 10^{-6}$. Because we made the exponent more negative, the decimal point moved to the *right* (Rule 3–2). The prefix for 10^{-6} is *micro* (the Greek letter μ is the symbol).

$$5.6 \times 10^{-4}\ \text{A} = 560 \times 10^{-6}\ \text{A} = 560\ \mu\text{A}$$

PRACTICE PROBLEMS 3–2

Using the indicated prefixes, change the following quantities:

1. 2700 Ω = ____________ kΩ = ____________ MΩ
2. 12,000 Hz = ____________ kHz = ____________ MHz
3. 0.00076 A = ____________ mA = ____________ μA
4. 0.000023 A = ____________ mA = ____________ μA
5. 0.0000002 F = ____________ μF = ____________ nF
6. 0.000004 F = ____________ μF = ____________ nF
7. 68,000 Ω = ____________ kΩ = ____________ MΩ
8. 120,000 Ω = ____________ kΩ = ____________ MΩ
9. 3,500,000 Hz = ____________ kHz = ____________ MHz
10. 0.00037 S = ____________ mS = ____________ μS
11. 5×10^{-4} S = ____________ mS = ____________ μS
12. 5.6×10^{4} Ω = ____________ kΩ = ____________ MΩ
13. 21×10^{-2} A = ____________ mA = ____________ μA
14. 68×10^{-10} F = ____________ nF = ____________ pF
15. 45×10^{-4} H = ____________ mH = ____________ μH
16. 0.15×10^{-2} V = ____________ mV = ____________ μV

3–3 CONVERTING NUMBERS WITH PREFIXES TO BASIC UNITS

Next, let's change quantities to basic units. The rule is:

> **RULE 3–3** To change numbers with prefixes to regular numbers, replace the prefix with the power of ten that it represents and then change to a regular number.

Some examples are:

$$2200\ \mu\text{S} = 2200 \times 10^{-6}\ \text{S} = 0.0022\ \text{S}$$

$$68\ \text{k}\Omega = 68 \times 10^{3}\ \Omega = 68{,}000\ \Omega$$

$$30\ \text{mA} = 30 \times 10^{-3}\ \text{A} = 0.03\ \text{A}$$

$$25\ \text{kHz} = 25 \times 10^{3}\ \text{Hz} = 25{,}000\ \text{Hz}$$

PRACTICE PROBLEMS 3–3

Perform the indicated operations:

1. 46.7 mA = ______________ A
2. 407 mA = ______________ A
3. 68 kΩ = ______________ Ω
4. 4.7 kΩ = ______________ Ω
5. 2.73 kHz = ______________ Hz
6. 300 kHz = ______________ Hz
7. 670 μS = ______________ S
8. 37 mS = ______________ S
9. 55 mH = ______________ H
10. 465 μH = ______________ H
11. 120 kΩ = ______________ Ω
12. 2 MΩ = ______________ Ω
13. 150 mA = ______________ A
14. 0.43 mA = ______________ A
15. 0.68 kΩ = ______________ Ω

3–4 CHANGING PREFIXES

Now let's change from one prefix to another.

EXAMPLE 3–8

20 mA = ______________ μA

SOLUTION In changing from mA to μA, we are really changing the exponent from -3 to -6; so let's rewrite the problem substituting powers of ten for prefixes.

$$20 \times 10^{-3} = \underline{\qquad\qquad} \times 10^{-6}$$

We are making the exponent more *negative*. Remember Rule 3–2: *If we make the exponent more negative, the decimal point moves to the right.* The change in the value of the exponent from 10^{-3} to 10^{-6} is a change of -3. Therefore, the decimal point moves *three places to the right.*

$$20\ \text{mA} = 20{,}000\ \mu\text{A}$$

EXAMPLE 3–9

Change 450 μA to mA.

SOLUTION In changing from μA to mA, we are really changing the exponent from -6 to -3; so let's rewrite the problem substituting powers of ten for prefixes.

$$450 \times 10^{-6}\text{ A} = \underline{\hspace{3cm}} \times 10^{-3}\text{ A}$$

We are making the exponent more *positive*. Remember Rule 3–1: *If we make the exponent more positive, the decimal point moves to the left.* The change in the value of the exponent from 10^{-6} to 10^{-3} is a change of $+3$. Therefore, the decimal point moves *three places to the left.*

$$450 \times 10^{-6}\text{ A} = 0.450 \times 10^{-3}\text{ A}$$

$$450\ \mu\text{A} = 0.450\text{ mA}$$

EXAMPLE 3–10

Change 680 kΩ to MΩ.

SOLUTION Substituting powers of ten for prefixes, we get

$$680 \times 10^{3}\ \Omega = \underline{\hspace{3cm}} \times 10^{6}\ \Omega$$

In going from kΩ to MΩ, $10^3\ \Omega$ to $10^6\ \Omega$, we are making the exponent more positive. Therefore, the decimal point moves to the *left*. The change from 10^3 to 10^6 is a change in the exponent of $+3$. Using Rule 3–1, the decimal point moves three places to the left.

$$680\text{ k}\Omega = 0.680\text{ M}\Omega$$

EXAMPLE 3–11

Change 0.01 μF to pF.

SOLUTION Substituting powers of ten for prefixes, we get

$$0.01 \times 10^{-6}\text{F} = \underline{\hspace{3cm}} \times 10^{-12}\text{ F}$$

In going from μF to pF, 10^{-6} to 10^{-12}, we are making the exponent more negative. Therefore, the decimal point moves to the *right*. The change from 10^{-6} to 10^{-12} is a change in the exponent of -6. Using Rule 3–2, the decimal point moves six places to the right.

$$0.01\ \mu\text{F} = 10{,}000\text{ pF}$$

In the following practice problems you will either change prefixes or change to basic units.

PRACTICE PROBLEMS 3–4

Perform the indicated operations.

1. 20 mA = ____________ μA = ____________ A
2. 0.01 μF = ____________ nF = ____________ pF
3. 2 H = ____________ mH = ____________ μH
4. 2.5 kΩ = ____________ Ω = ____________ MΩ
5. 680 Ω = ____________ kΩ = ____________ MΩ
6. 1.8 V = ____________ mV = ____________ μV
7. 20 μA = ____________ mA = ____________ A
8. 1.5 MΩ = ____________ kΩ = ____________ Ω
9. 0.02 S = ____________ mS = ____________ μS
10. 50 nF = ____________ μF = ____________ pF

11. $4.7 \times 10^4\ \Omega$ = ______ kΩ = ______ MΩ
12. 16×10^{-7} F = ______ μF = ______ nF
13. 2.4×10^{-2} A = ______ mA = ______ μA
14. $8.4 \times 10^2\ \Omega$ = ______ kΩ = ______ MΩ
15. $56 \times 10^5\ \Omega$ = ______ kΩ = ______ MΩ
16. 27.3 mS = ______ S = ______ μS
17. 5600 μS = ______ S = ______ mS
18. 170 mW = ______ W = ______ μW
19. 0.000642 V = ______ μV = ______ mV
20. 30 kHz = ______ MHz = ______ Hz

SELF-TEST 3–1 THROUGH 3–4

Express your answers in scientific notation:

1. 4760 = ______ $\times 10^3$ = ______ $\times 10^6$
2. 32.4×10^4 = ______ $\times 10^3$ = ______ $\times 10^6$
3. 2.71×10^{-1} = ______ $\times 10^{-3}$ = ______ $\times 10^3$
4. 46.7×10^{-2} = ______ $\times 10^{-3}$ = ______ $\times 10^3$
5. 76.3 mA = ______ A = ______ μA
6. 0.0055 μF = ______ nF = ______ pF
7. 8340 μS = ______ mS = ______ S
8. 20 kΩ = ______ Ω = ______ MΩ
9. 75,000 Hz = ______ MHz = ______ kHz
10. 146 mS = ______ S = ______ μS

3–5 APPLICATIONS

In problem solving we perform many different operations and express answers in many different forms. In this section we will do some of these operations and express answers by using prefixes whenever practical.

For example, when resistors are connected in parallel, we usually convert from resistance to conductance for problem-solving purposes. (Refer to Table 3–1 for the symbols and units of resistance and conductance.)

EXAMPLE 3–12

Two resistors are in parallel. $R_1 = 27\ \text{k}\Omega$ and $R_2 = 18\ \text{k}\Omega$. Find the total conductance (G_T) and the total resistance (R_T). Round answers to three places.

SOLUTION To find the total resistance, we use the equation

$$R_T = \frac{1}{G_T} \tag{3–1}$$

where

$$G_T = G_1 + G_2$$

$$G_1 = \frac{1}{R_1}$$

$$G_2 = \frac{1}{R_2}$$

Since

$$G_1 = \frac{1}{R_1} \quad \text{and} \quad G_2 = \frac{1}{R_2}$$

we can write the equation like this:

$$R_T = \frac{1}{G_1 + G_2} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} \tag{3–2}$$

$$= \frac{1}{\dfrac{1}{27\ \text{k}\Omega} + \dfrac{1}{18\ \text{k}\Omega}} = \frac{1}{37.0\ \mu\text{S} + 55.6\ \mu\text{S}}$$

$$= \frac{1}{92.6\ \mu\text{S}} = 10.8\ \text{k}\Omega$$

The total conductance is 92.6 μS. The total resistance is 10.8 kΩ.

Example 3–13 offers an alternate method of solution.

EXAMPLE 3–13

When two resistors are connected in parallel, their total resistance can be calculated with the equation:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \tag{3–3}$$

If $R_1 = 10\ \text{k}\Omega$ and $R_2 = 5\ \text{k}\Omega$, what is R_T in ohms? in kilohms?

SOLUTION

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{10\text{k} \times 5\text{k}}{10\text{k} + 5\text{k}}\ \Omega = \frac{10 \times 10^3 \times 5 \times 10^3}{10 \times 10^3 + 5 \times 10^3}\ \Omega = \frac{50 \times 10^6}{15 \times 10^3}\ \Omega$$

$$= \frac{50}{15} \times 10^3\ \Omega = 3.33 \times 10^3\ \Omega = 3.33\ \text{k}\Omega$$

EXAMPLE 3–14

Three resistors are connected in parallel. $R_1 = 470\ \Omega$, $R_2 = 220\ \Omega$, and $R_3 = 680\ \Omega$. Find the total conductance and the total resistance. Round answers to three places.

SOLUTION

$$R_T = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \tag{3–4}$$

Since $R_1 = 470\ \Omega$, $R_2 = 220\ \Omega$, and $R_3 = 680\ \Omega$, then

$$R_T = \frac{1}{\dfrac{1}{470\ \Omega} + \dfrac{1}{220\ \Omega} + \dfrac{1}{680\ \Omega}} = \frac{1}{2.13\ \text{mS} + 4.55\ \text{mS} + 1.47\ \text{mS}}$$

$$= \frac{1}{8.14\ \text{mS}} = 123\ \Omega \quad \text{(rounded to three places)}$$

The total conductance is 8.14 mS. The total resistance is 123 Ω.

To add units together when the prefixes are different, it is necessary to convert so that all prefixes are the same. For example, let's add 6300 μS and 15.3 mS. We cannot add milliunits and microunits together, just as we could not add 10^{-3} and 10^{-6} together. We must convert 6300 μS to milliunits or convert 15.3 mS to microunits.

$$6300\ \mu\text{S} + 15.3\ \text{mS} =$$

$$6.30\ \text{mS} + 15.3\ \text{mS} = 21.6\ \text{mS}$$

or

$$6300\ \mu\text{S} + 15.3\ \text{mS} =$$

$$6300\ \mu\text{S} + 15{,}300\ \mu\text{S} = 21{,}600\ \mu\text{S}$$

In this case, converting to milliunits is better because it is easier to work with that number.

Example 3–15 offers an alternate solution.

EXAMPLE 3–15

When three resistors are connected in parallel, their total resistance can be calculated with the equation:

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \tag{3–5}$$

If $R_1 = 10\ \text{k}\Omega$, $R_2 = 5\ \text{k}\Omega$, and $R_3 = 10\ \text{k}\Omega$, what is R_T in ohms? in kilohms?

SOLUTION

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{10\text{k} \times 5\text{k} \times 10\text{k}}{10\text{k} \times 5\text{k} + 10\text{k} \times 10\text{k} + 5\text{k} \times 10\text{k}}\ \Omega$$

$$= \frac{10 \times 10^3 \times 5 \times 10^3 \times 10 \times 10^3}{10 \times 10^3 \times 5 \times 10^3 + 10 \times 10^3 \times 10 \times 10^3 + 5 \times 10^3 \times 10 \times 10^3}\ \Omega$$

$$= \frac{500 \times 10^9}{50 \times 10^6 + 100 \times 10^6 + 50 \times 10^6}\ \Omega = \frac{500 \times 10^9}{200 \times 10^6}\ \Omega$$

$$= 2.5 \times 10^3\ \Omega = 2.5\ \text{k}\Omega$$

PRACTICE PROBLEMS 3–5

Perform the indicated operations:

1. $27\ \text{mA} + 0.037\ \text{A} =$ ________ A = ________ mA
2. $370\ \mu\text{S} + 0.060\ \text{mS} =$ ________ mS = ________ μS
3. $\dfrac{1}{68\ \text{k}\Omega} + \dfrac{1}{47\ \text{k}\Omega} =$ ________ mS = ________ μS
4. $\dfrac{1}{\dfrac{1}{4.7\ \text{k}\Omega} + \dfrac{1}{1.8\ \text{k}\Omega}} =$ ________ Ω = ________ kΩ
5. $\dfrac{20\ \text{V}}{27\ \text{k}\Omega} =$ ________ A = ________ mA = ________ μA

6. $\frac{25\text{ V}}{2.36\text{ mA}} =$ ______________ Ω = ______________ kΩ = ______________ MΩ

7. $\frac{1}{6.28 \times 12\text{ kHz} \times 50\text{ nF}} =$ ______________ Ω = ______________ kΩ

8. $\frac{1}{2.7\text{ k}\Omega} + \frac{1}{3.3\text{ k}\Omega} + \frac{1}{4.7\text{ k}\Omega} =$ ______________ mS = ______________ μS = ______________ S

9. $\frac{1}{\frac{1}{22\text{ k}\Omega} + \frac{1}{33\text{ k}\Omega} + \frac{1}{56\text{ k}\Omega}} =$ ______________ Ω = ______________ kΩ = ______________ MΩ

10. $\frac{1}{6340\ \mu\text{S}} =$ ______________ Ω = ______________ kΩ

11. $2.73\text{ k}\Omega \times 0.43\text{ mA} =$ ______________ V = ______________ mV

12. $1.27\text{ V} + 48\text{ mV} + 5630\ \mu\text{V} =$ ______________ V = ______________ mV = ______________ μV

When two resistors are connected in parallel, their total resistance can be calculated. Using equation (3–2) or (3–3):

13. If $R_1 = 2\text{ M}\Omega$ and $R_2 = 2\text{ M}\Omega$, what is R_T in kilohms and in megohms?

14. If $R_1 = 2\text{ M}\Omega$ and $R_2 = 4\text{ M}\Omega$, what is R_T in kilohms and in megohms?

When three resistors are connected in parallel, their total resistance can be calculated. Using equation (3–4) or (3–5):

15. If $R_1 = 10\text{ k}\Omega$, $R_2 = 20\text{ k}\Omega$, and $R_3 = 30\text{ k}\Omega$, what is R_T in ohms and in kilohms?

16. If $R_1 = 10\text{ k}\Omega$, $R_2 = 20\text{ k}\Omega$, and $R_3 = 30\text{ M}\Omega$, what is R_T in kilohms and in megohms?

17. The capacitive reactance, X_C, of a capacitor is measured in ohms and is found using the equation $X_C = \frac{1}{2\pi f C}$ where π is a constant (3.14 rounded to three places), *f* is the frequency in hertz, and *C* is the capacitance in farads. If the frequency is 5 kHz and the capacitance is 75 nF, what is the capacitive reactance in ohms, kilohms, and megohms, rounded to three places.

18. The total resistance of three resistors connected in parallel can be found using the equation $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$. Given that $R_1 = 22\text{ k}\Omega$, $R_2 = 3.9 \times 10^4\ \Omega$, and $R_3 = 180 \times 10^2\ \Omega$, what is the total resistance in ohms and kilohms, rounded to three places?

3–6 SYSTEMS OF MEASUREMENT

The systems of measurement with which we must become familiar are the metric system and the English system. The metric system uses the meter for linear measurement and the gram for mass or weight. The English system uses the yard for linear measurement and the pound for weight.

The metric system is the international system of weights and measurements. Because of the need for a single, internationally accepted system, the metric system is gradually replacing the English system in this country.

In electronics, we already use the international system (SI) of measurements for most quantities, such as voltage, current, capacitance, and conductance, to name a few; but some measurements, such as distance, velocity, and weight, may be specified in either the English system or the metric system. For this reason, we must be able to convert from one system to the other.

The most commonly used units of linear measurement in metric are the millimeter (mm), centimeter (cm), meter (m), and kilometer (km). The relationships between these units are

$$1\text{ millimeter} = 10^{-3}\text{ meter} = 0.001\text{ meter}$$

$$1\text{ centimeter} = 10^{-2}\text{ m} = 0.01\text{ m}$$

$$1\text{ kilometer} = 10^{3}\text{ m} = 1000\text{ m}$$

In the English system, we have

$$12 \text{ inches (in)} = 1 \text{ foot (ft)}$$

$$3 \text{ ft} = 1 \text{ yard (yd)}$$

$$5280 \text{ ft} = 1760 \text{ yd} = 1 \text{ mile (mi)}$$

To convert from English to metric,

$$1 \text{ in} = 2.540 \text{ cm}$$

$$1 \text{ yd} = 0.9144 \text{ m}$$

$$1 \text{ mi} = 1.609 \text{ km}$$

To convert from metric to English,

$$1 \text{ cm} = 0.3937 \text{ in}$$

$$1 \text{ m} = 39.37 \text{ in}$$

$$1 \text{ m} = 1.094 \text{ yd}$$

$$1 \text{ km} = 0.6214 \text{ mi}$$

EXAMPLE 3–16

Convert the following: (a) 14 cm to inches; (b) 40 cm to inches; (c) 1.7 m to inches and yards; (d) 3 km to yards and miles. Round answers to four places.

SOLUTIONS

(a) $14 \text{ cm} \times 0.3937 \text{ in/cm} = 5.512 \text{ in}$

(b) $40 \text{ cm} \times 0.3937 \text{ in/cm} = 15.75 \text{ in}$

(c) $1.7 \text{ m} \times 39.37 \text{ in/m} = 66.93 \text{ in}$
$1.7 \text{ m} \times 1.094 \text{ yd/m} = 1.860 \text{ yd}$

(d) $3000 \text{ m} \times 1.094 \text{ yd/m} = 3282 \text{ yd}$
$3000 \text{ m} = 3 \text{ km}; 3 \text{ km} \times 0.6214 \text{ mi/km} = 1.864 \text{ mi}$

EXAMPLE 3–17

Convert the following: (a) 20 in to cm; (b) 60 in to cm and m; (c) 100 yd to m; (d) 2000 yd to m and km. Round answers to four places.

SOLUTIONS

(a) $20 \text{ in} \times 2.54 \text{ cm/in} = 50.80 \text{ cm}$

(b) $60 \text{ in} \times 2.54 \text{ cm/in} = 152.4 \text{ cm} = 1.524 \text{ m}$

(c) $100 \text{ yd} \times 0.9144 \text{ m/yd} = 91.44 \text{ m}$

(d) $2000 \times 0.9144 \text{ m/yd} = 1829 \text{ m} = 1.829 \text{ km}$

The units of velocity are meters/second or kilometers/hour in metric and feet/second or miles/hour in English.

EXAMPLE 3–18

Convert the following: (a) 300 meters/second (m/s) to feet/second (ft/s); (b) 35 miles/hour (mi/h) to kilometers/hour (km/h); (c) 700 feet/second (ft/s) to meters/second (m/s); (d) 108 kilometers/hour (km/h) to miles/hour (mi/h). Round answers to four places.

SOLUTIONS

(a) 39.37 in = 1 m
    39.37 in/m ÷ 12 in/ft = 3.281 ft/m
    3.281 ft/m × 300 m/s = 984.3 ft/s
(b) 35 mi/h × 1.609 km/mi = 56.32 km/h
(c) 0.3048 ft = 1 m
    700 ft/s × 0.3048 m/ft = 213.4 m/s
(d) 108 km/h × 0.6214 mi/km = 67.11 mi/h

The most commonly used units of mass or weight in metric are the milligram, gram, and kilogram. In the English system we have the ounce, pound, and ton.

16 ounces (oz) = 1 pound (lb)

2000 pounds = 1 ton (t)

To convert from metric to English,

1 gram (g) = 0.03527 ounces

1 kilogram (kg) = 35.27 ounces = 2.205 pounds

To convert from English to metric,

1 ounce = 28.35 grams

1 pound = 453.6 grams = 0.4536 kilogram

EXAMPLE 3–19

Convert the following: (a) 30 g to ounces; (b) 700 g to ounces and pounds; (c) 2.35 kg to ounces and pounds; (d) 0.255 oz to grams; (e) 7.5 oz to kilograms; (f) 1.5 lb to kilograms. Round answers to four places.

SOLUTIONS

(a) 30 g × 0.03527 oz/g = 1.058 oz
(b) 700 g × 0.03527 oz/g = 24.69 oz
    24.69 oz ÷ 16 oz/lb = 1.543 lb
(c) 2.35 kg × 35.27 oz/kg = 82.88 oz
    2.35 kg × 2.204 lb/kg = 5.180 lb
(d) 0.255 oz × 28.35 g/oz = 7.229 g
(e) 7.5 oz × 0.02835 kg/oz = 0.2126 kg (1 oz = 28.35 g = 0.02835 kg)
(f) 1.5 lb × 0.4536 kg/lb = 0.6804 kg

PRACTICE PROBLEMS 3–6

Make the following conversions. Round answers to four places.

1. 27 centimeters to inches
2. 3.4 meters to inches and yards
3. 5000 meters to yards and miles
4. 27 inches to centimeters
5. 440 yards to meters and kilometers
6. 2 miles to meters and kilometers
7. 400 meters/second to feet/second
8. 200 kilometers/hour to miles/hour
9. 300 feet/second to meters/second
10. 65 miles/hour to kilometers/hour

11. 70 grams to ounces
12. 1000 grams to ounces and pounds
13. 3.2 kilograms to ounces and pounds
14. 2 pounds to kilograms
15. 2.45 ounces to grams
16. Two packages to be sent air express weigh 5 lb 4 oz each. What is the shipping weight of the two packages in ounces? In pounds? In grams, rounded to four places?
17. How many feet of wire are needed to fence a property line $1\frac{1}{4}$ mi long? How many yards is this? How many kilometers? Round answers to four places.
18. How many 25-gram packages of seed can be made from 2 kg of seed?
19. How many liters of weed killer are contained in a 55-gallon drum? How many pints? (1 gal = 3.785 L)
20. A 49-foot section of highway is to be resurfaced. How many yards is this? How many meters?
21. If gasoline costs $2.12/gal, what is the cost per liter?
22. If an automobile is traveling 60 mi/hr, how fast is it traveling in km/hr?
23. Twenty boxes, each weighing 42 kg, are to be moved. How much weight must be moved? How much is the total weight in pounds?
24. A recipe calls for 5 mL of vanilla flavoring and 24 cL of milk. How much liquid is this in mL? In pints?

CHAPTER 3 AT A GLANCE

PAGE	KEY POINT OR RULE	EXAMPLE
59	*Rule 3–1:* If we make the exponent more positive, the decimal point moves to the *left.*	$6700 = 670 \times 10$ $= 67.0 \times 10^2$ $= 6.70 \times 10^3$
60	*Rule 3–2:* If the exponent is made more negative, the decimal point moves to the *right.*	$0.00275 = 0.0275 \times 10^{-1}$ $= 0.275 \times 10^{-2}$ $= 2.75 \times 10^{-3}$
61	*Key Point:* Expressing numbers as numbers between 1 and 10 times the proper power of ten is called *scientific* notation. Expressing numbers as numbers times powers of ten that are multiples of 3 is called *engineering* notation.	$47{,}000 = 4.70 \times 10^4$ in scientific notation to three places. $47{,}000 = 47.0 \times 10^3$ in engineering notation to three places.
64	*Rule 3–3:* To change numbers with prefixes to regular numbers, replace the prefix with the power of ten that it represents and then change to a regular number.	$30\text{ mA} = 30 \times 10^{-3}\text{ A}$ $= 0.030\text{ A}$ $33\text{ k}\Omega = 33 \times 10^3\ \Omega$ $= 33{,}000\ \Omega$
70	Some conversions of linear measure between English and metric are: 1 in = 2.54 cm 1 cm = 0.3937 in 1 yd = 0.9144 m 1 m = 1.094 yd 1 mi = 1.609 km 1 km = 0.6214 mi	20 cm = 7.874 in 12 in = 30.48 cm 100 m = 109.4 yd 5 mi = 8.045 km
70	The units of velocity are meters/second or kilometers/hour in metric and feet/second or miles/hour in English.	15 m/s = 49.22 ft/s
71	Common units of weight in metric are the milligram, gram, and kilogram. In the English system, we have the ounce, pound, and ton. 16 oz = 1 lb 1 kg = 35.27 oz 2000 lb = 1 ton 1 lb = 453.6 gm	100 lb = 45.36 kg 100 kg = 220.5 lb

END OF CHAPTER PROBLEMS 3–1

Change the following numbers to numbers times 10^3 and to numbers times 10^6:

1. 56,000
2. 15,000
3. 220,000
4. 270,000
5. 39×10^4
6. 730×10^4
7. 1.8×10^5
8. 6.8×10^4
9. 4300×10
10. 3300×10

Change the following numbers to numbers times 10^{-3} and to numbers times 10^{-6}:

11. 0.00022
12. 0.00012
13. 0.00213
14. 0.0000179
15. 0.00556×10^{-2}
16. 0.0133×10^{-1}
17. 1.22×10^{-4}
18. 40×10^{-4}
19. 25.6×10^{-5}
20. 11×10^{-2}

Change the following numbers to numbers times 10^{-9} and to numbers times 10^{-12}:

21. 0.00667×10^{-4}
22. 0.00233×10^{-4}
23. 0.179×10^{-7}
24. 0.00041×10^{-8}
25. 67.4×10^{-10}
26. 273×10^{-10}
27. 1.77×10^{-11}
28. 3.75×10^{-11}
29. 700×10^{-10}
30. 0.034×10^{-7}

Change the following numbers to regular numbers, to numbers times 10^{-3}, and to numbers times 10^3:

31. 73×10^{-1}
32. 4.20×10^2
33. 0.567×10
34. 17.4×10^{-2}
35. 1.78×10^{-2}
36. 92.5×10^{-1}
37. 783×10^{-1}
38. 0.000945×10^2
39. 0.0845×10^4
40. 32.5×10^{-2}

END OF CHAPTER PROBLEMS 3–2

Using the indicated prefixes, change the following quantities:

1. 0.00026 A = ______________ mA = ______________ μA
2. 0.0736 A = ______________ mA = ______________ μA
3. 0.000632 S = ______________ mS = ______________ μS
4. 0.00024 S = ______________ mS = ______________ μS
5. 7630 Ω = ______________ kΩ = ______________ MΩ
6. 56,000 Ω = ______________ kΩ = ______________ MΩ
7. 17.3×10^{-3} A = ______________ mA = ______________ μA
8. 64×10^{-6} A = ______________ mA = ______________ μA
9. 71.3×10^4 Ω = ______________ kΩ = ______________ MΩ
10. 6.8×10^4 Ω = ______________ kΩ = ______________ MΩ
11. 5.63×10^6 Hz = ______________ kHz = ______________ MHz
12. 48.7×10^2 Hz = ______________ kHz = ______________ MHz
13. 2,000,000 Ω = ______________ kΩ = ______________ MΩ
14. 470,000 Ω = ______________ kΩ = ______________ MΩ
15. 23.7×10^{-5} S = ______________ mS = ______________ μS
16. 5.63×10^{-2} S = ______________ mS = ______________ μS
17. 30×10^{-8} F = ______________ μF = ______________ nF
18. 1.2×10^{-7} F = ______________ μF = ______________ nF
19. 0.000062 A = ______________ mA = ______________ μA
20. 0.0075 A = ______________ mA = ______________ μA

END OF CHAPTER PROBLEMS 3–3

Using the indicated prefixes, change the following quantities.

1. 800 mA = ______________ A
2. 7.03 mA = ______________ A
3. 2.5 mS = ______________ S
4. 400 μS = ______________ S

5. 33 kΩ = ______________ Ω
7. 0.47 kΩ = ______________ Ω
9. 12.5 kHz = ______________ Hz
11. 100 μF = ______________ F
13. 0.25 mA = ______________ A
15. 900 μS = ______________ S
17. 750 kΩ = ______________ Ω
19. 30 mA = ______________ A

6. 680 kΩ = ______________ Ω
8. 0.56 MΩ = ______________ Ω
10. 73 kHz = ______________ Hz
12. 500 μF = ______________ F
14. 50 mA = ______________ A
16. 2.5 mS = ______________ S
18. 12 kΩ = ______________ Ω
20. 30.5 mA = ______________ A

END OF CHAPTER PROBLEMS 3–4

Using the indicated prefixes, change the following quantities.

1. 0.00065 A = ______________ mA = ______________ μA
2. 0.0175 A = ______________ mA = ______________ μA
3. 0.00000805 S = ______________ mS = ______________ μS
4. 0.00417 A = ______________ mA = ______________ μA
5. 56.2×10^{-3} S = ______________ mS = ______________ μS
6. 245×10^{-3} A = ______________ mA = ______________ μA
7. 613×10^{-6} S = ______________ mS = ______________ μS
8. 2.27×10^{-6} A = ______________ mA = ______________ μA
9. 7500 Ω = ______________ kΩ = ______________ MΩ
10. 1250 Hz = ______________ kHz = ______________ MHz
11. 510,000 Ω = ______________ kΩ = ______________ MΩ
12. 1,800,000 Ω = ______________ kΩ = ______________ MΩ
13. 12.7×10^{4} Hz = ______________ kHz = ______________ MHz
14. 77.5×10^{6} Hz = ______________ kHz = ______________ MHz
15. 713×10^{3} Ω = ______________ kΩ = ______________ MΩ
16. 910×10^{5} Ω = ______________ kΩ = ______________ MΩ
17. 46.3 mA = ______________ μA = ______________ A
18. 415 mS = ______________ S = ______________ μS
19. 0.05 nF = ______________ μF = ______________ pF
20. 157 nF = ______________ pF = ______________ μF
21. 3.2 kΩ = ______________ Ω = ______________ MΩ
22. 35.9 kHz = ______________ Hz = ______________ MHz
23. 270 kΩ = ______________ MΩ = ______________ Ω
24. 4.7 kΩ = ______________ MΩ = ______________ Ω
25. 403 μS = ______________ S = ______________ mS
26. 704 mS = ______________ S = ______________ μS
27. 1.03 MHz = ______________ Hz = ______________ kHz
28. 42.3 kHz = ______________ MHz = ______________ Hz
29. 1.43 nF = ______________ pF = ______________ μF
30. 0.025 μF = ______________ pF = ______________ nF
31. 55×10^{-4} S = ______________ mS = ______________ μS
32. 4.67×10^{-4} S = ______________ mS = ______________ μS
33. 106×10^{-2} nF = ______________ μF = ______________ pF
34. 7.36×10^{3} nF = ______________ μF = ______________ pF
35. 4.63×10^{2} kΩ = ______________ Ω = ______________ MΩ
36. 66.7×10^{2} kΩ = ______________ Ω = ______________ MΩ
37. 9.63×10^{4} Hz = ______________ kHz = ______________ MHz
38. 82.3×10^{2} Hz = ______________ kHz = ______________ MHz
39. 0.0078 A = ______________ mA = ______________ μA
40. 0.00104 A = ______________ mA = ______________ μA
41. 176 mV = ______________ V = ______________ μV
42. 0.000425 V = ______________ mV = ______________ μV
43. 1.73 W = ______________ mW = ______________ kW
44. 1.67 W = ______________ mW = ______________ kW

45. 0.025 H = ______ mH = ______ μH
46. 0.0005 H = ______ mH = ______ μH
47. 173 mS = ______ S = ______ μS
48. 146 μS = ______ mS = ______ S
49. 25×10^{-7} F = ______ μF = ______ nF
50. 250 mH = ______ H = ______ μH

END OF CHAPTER PROBLEMS 3–5

Perform the indicated operations. Round answers to three places.

1. 367 μA + 1.67 mA = ______ mA = ______ μA
2. 0.417 mA + 630 μA = ______ mA = ______ μA
3. 0.0173 mA + 78.4 μA = ______ mA = ______ μA
4. 650 μA + 0.235 mA = ______ mA = ______ μA
5. $\frac{1}{10\ \text{k}\Omega} + \frac{1}{15\ \text{k}\Omega} =$ ______ mS = ______ μS
6. $\frac{1}{6.8\ \text{k}\Omega} + \frac{1}{4.7\ \text{k}\Omega} =$ ______ mS = ______ μS
7. $\frac{1}{33\ \text{k}\Omega} + \frac{1}{56\ \text{k}\Omega} + \frac{1}{100\ \text{k}\Omega} =$ ______ mS = ______ μS
8. $\frac{1}{33\ \text{k}\Omega} + \frac{1}{100\ \text{k}\Omega} + \frac{1}{68\ \text{k}\Omega} =$ ______ mS = ______ μS
9. $\frac{1}{1.2\ \text{k}\Omega} + \frac{1}{4.7\ \text{k}\Omega} + \frac{1}{2.7\ \text{k}\Omega} =$ ______ mS = ______ μS
10. $\frac{1}{270\ \Omega} + \frac{1}{470\ \Omega} + \frac{1}{680\ \Omega} =$ ______ mS = ______ μS
11. $\frac{1}{\frac{1}{2.7\ \text{k}\Omega} + \frac{1}{1.5\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ
12. $\frac{1}{\frac{1}{8.2\ \text{k}\Omega} + \frac{1}{6.8\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ
13. $\frac{1}{\frac{1}{100\ \text{k}\Omega} + \frac{1}{220\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ = ______ MΩ
14. $\frac{1}{\frac{1}{470\ \text{k}\Omega} + \frac{1}{680\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ = ______ MΩ
15. $\frac{1}{\frac{1}{39\ \text{k}\Omega} + \frac{1}{12\ \text{k}\Omega} + \frac{1}{27\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ = ______ MΩ
16. $\frac{1}{\frac{1}{1.2\ \text{M}\Omega} + \frac{1}{2.7\ \text{M}\Omega} + \frac{1}{1\ \text{M}\Omega}} =$ ______ Ω = ______ kΩ = ______ MΩ
17. $\frac{1}{\frac{1}{180\ \Omega} + \frac{1}{470\ \Omega} + \frac{1}{1\ \text{k}\Omega}} =$ ______ Ω = ______ kΩ
18. $\frac{1}{\frac{1}{330\ \Omega} + \frac{1}{820\ \Omega} + \frac{1}{910\ \Omega}} =$ ______ Ω = ______ kΩ
19. $\frac{1}{213\ \mu\text{S}} =$ ______ Ω = ______ kΩ
20. $\frac{1}{2.75\ \text{mS}} =$ ______ Ω = ______ kΩ

21. $500\ \mu\text{S} + 1.76\ \text{mS} + 0.000043\ \text{S} =$ ________ mS = ________ μS
22. $670\ \mu\text{S} + 2\ \text{mS} + 0.00002\ \text{S} =$ ________ mS = ________ μS
23. $\dfrac{9\ \text{V}}{33\ \text{k}\Omega} =$ ________ mA = ________ μA
24. $\dfrac{12\ \text{V}}{2.7\ \text{k}\Omega} =$ ________ mA = ________ μA
25. $\dfrac{14\ \text{V}}{200\ \mu\text{A}} =$ ________ Ω = ________ kΩ
26. $\dfrac{10\ \text{V}}{2.5\ \text{mA}} =$ ________ Ω = ________ kΩ
27. $\dfrac{50\ \text{V}}{1.2\ \text{M}\Omega} =$ ________ mA = ________ μA
28. $\dfrac{40\ \text{V}}{15\ \mu\text{A}} =$ ________ kΩ = ________ MΩ
29. $68\ \text{k}\Omega \times 140\ \mu\text{A} =$ ________ V = ________ mV
30. $270\ \text{k}\Omega \times 0.17\ \text{mA} =$ ________ V = ________ mV
31. $1.2\ \text{k}\Omega \times 3.73\ \text{mA} =$ ________ V = ________ mV
32. $4.7\ \text{k}\Omega \times 634\ \mu\text{A} =$ ________ V = ________ mV
33. $\dfrac{1}{6.28 \times 75\ \text{kHz} \times 25\ \text{pF}} =$ ________ Ω = ________ kΩ
34. $\dfrac{1}{6.28 \times 2.5\ \text{kHz} \times 20\ \text{nF}} =$ ________ Ω = ________ kΩ
35. $\dfrac{1}{6.28 \times 10\ \text{kHz} \times 0.05\ \mu\text{F}} =$ ________ Ω = ________ kΩ
36. $\dfrac{1}{6.28 \times 37.2\ \text{kHz} \times 200\ \text{nF}} =$ ________ Ω = ________ kΩ
37. $\dfrac{1}{6.28 \times 12\ \text{kHz} \times 2.7\ \text{k}\Omega} =$ ________ μF = ________ nF
38. $\dfrac{1}{6.28 \times 200\ \text{Hz} \times 600\ \Omega} =$ ________ μF = ________ nF
39. $\dfrac{1}{6.28(150\ \text{mH} \times 200\ \text{nF})^{1/2}} =$ ________ Hz = ________ kHz
40. $\dfrac{1}{6.28(0.75\ \text{mH} \times 0.5\ \text{nF})^{1/2}} =$ ________ Hz = ________ kHz

When two resistors are connected in parallel, their total resistance can be calculated. Using equation (3–2) or (3–3):

41. If $R_1 = 10\ \text{k}\Omega$ and $R_2 = 10\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
42. If $R_1 = 10\ \text{k}\Omega$ and $R_2 = 9\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
43. If $R_1 = 10\ \text{M}\Omega$ and $R_2 = 10\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
44. If $R_1 = 1\ \text{M}\Omega$ and $R_2 = 5\ \text{M}\Omega$, then R_T equals ________ kΩ = ________ MΩ?

When three resistors are connected in parallel, their total resistance can be calculated. Using equation (3–4) or (3–5):

45. If $R_1 = 10\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, and $R_3 = 5\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
46. If $R_1 = 500\ \Omega$, $R_2 = 1\ \text{k}\Omega$, and $R_3 = 2\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
47. If $R_1 = 1\ \text{k}\Omega$, $R_2 = 2\ \text{k}\Omega$, and $R_3 = 4\ \text{k}\Omega$, then R_T equals ________ Ω = ________ kΩ?
48. If $R_1 = 1\ \text{M}\Omega$, $R_2 = 5\ \text{M}\Omega$, and $R_3 = 25\ \text{M}\Omega$, then R_T equals ________ Ω = ________ kΩ?

49. The capacitive reactance, X_C, of a capacitor is measured in ohms and is found using the equation $X_C = \frac{1}{2\pi fC}$ where π is a constant (3.14 rounded to three places), f is the frequency in hertz, and C is the capacitance in farads. If the frequency is 10 kHz and the capacitance is 350 pF, what is the capacitive reactance in ohms, kilohms, and megohms, rounded to three places?

50. The capacitive reactance, X_C, of a capacitor is measured in ohms and is found using the equation $X_C = \frac{1}{2\pi fC}$ where π is a constant (3.14 rounded to three places), f is the frequency in hertz, and C is the capacitance in farads. If the frequency is 255 kHz and the capacitance is 200 pF, what is the capacitive reactance in ohms, kilohms, and megohms, rounded to three places?

51. The total resistance of three resistors connected in parallel can be found using the equation $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$. Given that $R_1 = 33\ \text{k}\Omega$, $R_2 = 5.6 \times 10^4\ \Omega$, and $R_3 = 820 \times 10^2\ \Omega$, what is the total resistance in ohms and kilohms, rounded to three places?

52. The total resistance of three resistors connected in parallel can be found using the equation $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$. Given that $R_1 = 2.7\ \text{M}\Omega$, $R_2 = 15 \times 10^5\ \Omega$, and $R_3 = 910 \times 10^3\ \Omega$, what is the total resistance in ohms and kilohms, rounded to three places?

END OF CHAPTER PROBLEMS 3–6

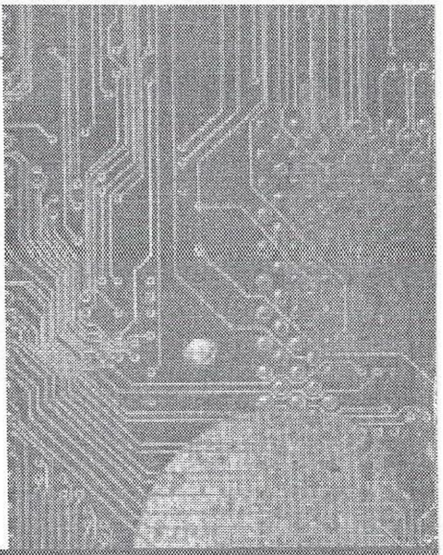
Make the following conversions. Round answers to four places.

1. 100 centimeters to inches
3. 40 centimeters to inches
5. 2.3 meters to inches and yards
7. 10 meters to yards
9. 10,000 meters to yards and miles
11. 10 inches to centimeters
13. 35.6 inches to centimeters
15. 880 yards to meters and km
17. 55 mi/h to km/h
19. 100 mi/h to km/h
21. 500 meters/s to feet/s
23. 100 meters/s to feet/s
25. 60 grams to ounces
27. 150 grams to ounces
29. 800 grams to ounces and pounds
31. 5 kg to oz and lb
33. 1.35 lb to kg
35. 4 oz to g
37. 0.275 oz to g
39. Two packages to be sent air express weigh 3 lb 7 oz each. What is the shipping weight of the two packages in ounces? In pounds? In grams, rounded to four places?
41. How many feet of wire are needed to fence a property line $2\frac{3}{8}$ mi long? How many yards is this? How many kilometers? Round answers to four places.
43. How many 30-gram packages of seed can be made from 2 kg of seed?
45. How many liters of weed killer are contained in two 55-gallon drums? How many quarts? (1 gal = 3.785 L)
47. A 75-foot section of highway is to be resurfaced. How many yards is this? How many meters?
49. If gasoline costs \$2.45/gal, what is the cost per liter?
51. If an automobile is traveling 70 mi/hr, how fast is it traveling in km/hr?
53. Fifteen boxes, each weighing 35 kg, are to be moved. How much weight must be moved? How much is the total weight in pounds?
55. A recipe calls for 5 mL of vanilla flavoring and 28 cL of milk. How much liquid is this in mL? In pints?

2. 9 centimeters to inches
4. 130 centimeters to inches
6. 2.25 meters to inches and yards
8. 27 meters to yards
10. 15,000 meters to yards and miles
12. 15 inches to centimeters
14. 50.3 inches to centimeters
16. 1000 yards to meters and kilometers
18. 65 mi/h to km/h
20. 75 mi/h to km/h
22. 200 meters/s to feet/s
24. 400 meters/s to feet/s
26. 85 grams to ounces
28. 225 grams to ounces
30. 1400 grams to ounces and pounds
32. 7.5 kg to oz and lb
34. 0.85 lb to kg
36. 55 oz to g
38. 0.45 oz to g
40. Two packages to be sent air express weigh 2 lb 14 oz each. What is the shipping weight of the two packages in ounces? In pounds? In grams, rounded to four places?
42. How many feet of wire are needed to fence a property line $3\frac{3}{4}$ mi long? How many yards is this? How many kilometers? Round answers to four places.
44. How many 40-gram packages of seed can be made from 3 kg of seed?
46. How many liters of weed killer are contained in half a 55-gallon drum? How many quarts?
48. A 110-foot section of highway is to be resurfaced. How many yards is this? How many meters?
50. If gasoline costs \$1.85/gal, what is the cost per liter?
52. If an automobile is traveling 65 mi/hr, how fast is it traveling in km/hr?
54. Twenty-five boxes, each weighing 56 kg, are to be moved. How much weight must be moved? How much is the total weight in pounds?
56. A recipe calls for 7 mL of vanilla flavoring and 36 cL of milk. How much liquid is this in mL? In pints?

SECTION SIX

Math for Digital Electronics



24 Computer Number Systems
25 Boolean Algebra

Digital electronics is the branch of electronics that uses the ON and OFF states of transistor switches to perform specified tasks. Just as a wall switch can turn electrical lights ON or OFF by interrupting current, a transistor switch can interrupt the current in a digital circuit. Since transistors were invented over 50 years ago, their size and cost have decreased every year. The decreased cost has allowed engineers to design and develop hundreds of digital electronic circuit applications. Common examples are digital watches, TV remote controls, high definition TV, wireless telephones, calculators, and computers.

One of the keys to the understanding of digital electronics is understanding the mathematics that is the foundation upon which digital circuits are built.

In Chapter 24, students are introduced to computer number systems. Chapter 25 describes the mathematical properties of three basic digital circuits (AND, OR, and NOT) that are the elements of all other digital circuits. The interaction of these three digital functions can be described by the rules of Boolean algebra.

CHAPTER 24

Computer Number Systems

CHAPTER OBJECTIVES

In this chapter you will learn how to:

1. Read and write numbers in the binary, octal, and hexadecimal number systems.
2. Convert between the decimal, binary, octal, and hexadecimal number systems.
3. Add numbers in the various number systems.
4. Subtract numbers in the various number systems using both the direct and complement methods.

INTRODUCTION

The basic tools for understanding computers are an understanding of the number systems used and an understanding of basic logic functions. Number systems are covered in this chapter and logic functions in Chapter 25.

We all know and understand the decimal number system, the number system we use daily, but the computer world makes use of the *binary* number system. All data are stored and manipulated inside the computer in binary. That is, within the computer all data are reduced to 1s and 0s. These 1s and 0s are stored in logic circuits called *registers* or they are stored in *memory*. The size of a register or memory location varies according to the kind of computer used. Microcomputers store 8 or 16 binary digits in each location. Large computers may store as many as 128 binary digits in each location.

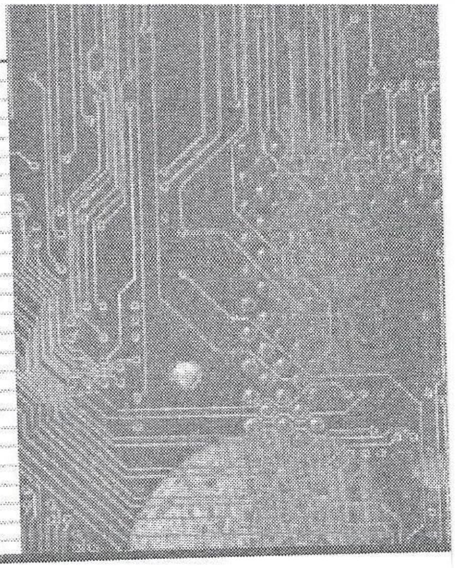
We normally enter data into the computer in some number system other than binary because entering data in binary is too time-consuming and too prone to error. There are too many 1s and 0s. Data are entered into the computer by means of the decimal, octal, or hexadecimal number system. Octal and hexadecimal are used most often because they are more closely related to binary than is the decimal system.

In this chapter we will work with numbers in the binary, octal, decimal, and hexadecimal number systems. We use subscripts to denote the base we are working in. This must be done when working with different bases to avoid errors. For example, the number 11 is a valid number in all the bases we study, but it has a different value in each. If it is a binary number, we write it this way: 11_2. If it is an octal number, it is written this way: 11_8. The importance of indicating the base is apparent when you consider that 11_2 has a decimal value of 3, whereas 11_8 has a decimal value of 9!

Because all these number systems are used in the computer world in one way or another, it is necessary that we understand how they are related to each other.

24–1 BINARY NUMBER SYSTEM

In Chapter 1, we talked about the decimal number system and how the value or *weight* of a digit depended on its position in the number. For example, in the number 7037, the 7 in the ones position has a weight of 7, whereas the 7 in the thousands position has a weight of 7000. Furthermore, because our number system has ten digits (0 through 9), the next number after 9 causes a *carry* into the next position. In the decimal number system, we have no special symbol or character for numbers greater than 9, so we use combinations



MSB	4SB	3SB	2SB	LSB
2^4	2^3	2^2	2^1	2^0
16	8	4	2	1

FIGURE 24–1 Positional value of the first five positions for the binary number system.

of these numbers in our *place value system* to indicate quantities greater than 9. Thus, the next number after 9 is 10. The one in the tens position has a weight of ten.

In powers of ten form in base ten, numbers in the ones positions are raised to the zero power ($10^0 = 1$), numbers in the tens position are raised to the first power (10^1), numbers in the hundreds position are raised to the second power (10^2), and so on. Using the powers of ten form in the various positions, our number looks like this:

$$(7 \times 10^3) + (0 \times 10^2) + (3 \times 10^1) + (7 \times 10^0)$$
$$= 7000 + 0 + 30 + 7 = 7037$$

Those concepts, developed in Chapter 1 and reviewed here, apply to *all* number systems. The only difference is that other number systems have fewer (or more) characters.

In the binary number system, there are only two digits, 0 and 1. Digits in the ones position can have a value of only 0 or 1 since that's all the digits there are. Numbers greater than 1 cause a carry into the next position. Just as in base 10, each position represents the base raised to a power. The ones position has a power of 2^0, the next position is 2^1, and so on, as illustrated in Figure 24–1. In decimal, the weight or value of each position is a multiple of 10. In binary, the weight or value of each position is a multiple of 2.

Binary digits are called *bits* (a contraction of binary digits). Therefore, the digit in the ones position is called the least significant *bit* (LSB), and so on, until the most significant bit (MSB) is reached.

24–1–1 Binary to Decimal Conversions

Converting from base two to base ten is easy. If a 1 is present in a given position, the weight of that bit is added. If a 0 is present, the weight of that bit is not added. Consider the number 11010 in Example 24–1. The decimal equivalent is 26.

EXAMPLE 24–1

$11010_2 = (?)_{10}$

SOLUTION

$$(1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) =$$
$$16 \quad + \quad 8 \quad + \quad 0 \quad + \quad 2 \quad + \quad 0 \quad = 26$$

Because there are only 1s and 0s in binary, each bit position either equals the weight of that digit position or it equals 0. In Example 24–1, the values of the bit positions are added to get the decimal equivalent. In the 2^4 or 16's position, there is a 1. Therefore, that bit position converts to 16. In the 2^3 or 8's position, there is a 1. That bit position converts to 8. In the 2^2 or 4's position, there is a 0. That bit position converts to 0. In the 2's position, there is a 1. That bit position converts to 2. There is a 0 in the ones position. If we add the numbers together, we get decimal 26.

> **RULE 24–1** In converting from binary to decimal, find the value or weight of the MSB. Work down to the LSB, adding the weight of that position if a 1 is present, or a 0 if a 0 is present.

Table 24–1 is a conversion table for all the computer number systems.

Table 24–1 Computer Numbering System.

Numbering system		Decimal		Binary						Octal		Hexa-decimal	
Base		10		2						8		16	
Position weight	Power	10^1	10^0	2^5	2^4	2^3	2^2	2^1	2^0	8^1	8^0	16^1	16^0
	Value	10	1	32	16	8	4	2	1	8	1	16	1
											(LSD)		(LSD)
		0	0	0	0	0	0	0	0	0	0	0	0
Example: $1_{(10)} = 1_{(2)} = 1_{(8)} = 1_{(16)}$		↓	1	0	0	0	0	0	1	↓	1	↓	1
			2	0	0	0	0	1	0		2		2
			3	0	0	0	0	1	1		3		3
			4	0	0	0	1	0	0		4		4
			5	0	0	0	1	0	1		5		5
			6	0	0	0	1	1	0		6		6
			7	0	0	0	1	1	1		7		7
			8	0	0	1	0	0	0	1	0		8
			9	0	0	1	0	0	1	1	1		9
Example: $10_{(10)} = 1010_{(2)} = 12_{(8)} = 0A_{(16)}$		1	0	0	0	1	0	1	0	1	2		A
		1	1	0	0	1	0	1	1	1	3		B
		1	2	0	0	1	1	0	0	1	4		C
		1	3	0	0	1	1	0	1	1	5		D
		1	4	0	0	1	1	1	0	1	6		E
		1	5	0	0	1	1	1	1	1	7		F
		1	6	0	1	0	0	0	0	2	0	1	0
		1	7	0	1	0	0	0	1	2	1	1	1
		1	8	0	1	0	0	1	0	2	2	1	2
		1	9	0	1	0	0	1	1	2	3	1	3
		2	0	0	1	0	1	0	0	2	4	1	4
		2	1	0	1	0	1	0	1	2	5	1	5
		2	2	0	1	0	1	1	0	2	6	1	6
		2	3	0	1	0	1	1	1	2	7	1	7
		2	4	0	1	1	0	0	0	3	0	1	8
Example: $25_{(10)} = 11001_{(2)} = 31_{(8)} = 19_{(16)}$		2	5	0	1	1	0	0	1	3	1	1	9
		2	6	0	1	1	0	1	0	3	2	1	A
		2	7	0	1	1	0	1	1	3	3	1	B
		2	8	0	1	1	1	0	0	3	4	1	C
		2	9	0	1	1	1	0	1	3	5	1	D
		3	0	0	1	1	1	1	0	3	6	1	E
		3	1	0	1	1	1	1	1	3	7	1	F
		3	2	1	0	0	0	0	0	4	0	2	0

EXAMPLE 24–2

Change the following binary numbers to decimal numbers:

(a) 1100 (b) 10001 (c) 101011 (d) 111101

SOLUTIONS

(a) $1100_2 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) =$

$8 + 4 + 0 + 0 = 12_{10}$

(b) $10001_2 = (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$16 + 0 + 0 + 0 + 1 = 17_{10}$

(c) $101011_2 = (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) +$

$(1 \times 2^0) = 32 + 0 + 8 + 0 + 2 + 1$

$= 43_{10}$

(d) $111101_2 = (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) +$

$(1 \times 2^0) = 32 + 16 + 8 + 4 + 0 + 1$

$= 61_{10}$

24–1–2 Decimal to Binary Conversions

Let's turn the process around and find the binary equivalent of a decimal number.

EXAMPLE 24–3

Convert 11_{10} to binary:

SOLUTION

Step 1. $\frac{11}{2} = 5 + 1$

Step 2. $\frac{5}{2} = 2 + 1$

Step 3. $\frac{2}{2} = 1 + 0$

Step 4. $\frac{1}{2} = 0 + 1$ – MSB → 1 0 1 1

LSB

Since we are converting to base 2, we first divide the decimal number by 2 (step 1). The answer is 5 with a remainder of 1. This remainder is the LSB in the answer. The whole number left after this initial division (5) is again divided by 2 (step 2). The answer is 2 with a remainder of 1. This 1 is the bit in the 2^1 position. The whole number left (2) is divided by 2 (step 3). The result is 1 and a remainder of 0. This 0 is the bit in the 2^2 position. Step 4 is the final step because no further divisions are possible. The remainder in the final step is the MSB.

A second method of converting from decimal to binary is shown in the following example.

EXAMPLE 24–4

Convert 167 to binary.

SOLUTION The highest multiple of 2 contained in 167 is 128 (2^7). Therefore, we need a 128. So we start writing our binary number with a 1 in the MSB position, remembering that this "1" has a value of 128. Next we determine if we need a 64. If we do, we put a 1 in the next bit position. If we don't need a 64, we put a 0 in this position. Since 128 + 64 is greater than 167, we must put a zero in this position. At this point we have 10, which is 128 + 0.

This procedure is followed for each bit position. Can we use 32? Yes. We now have 101. Remember, we started at the MSB position and are working down to the LSB position. Can we use a 16? No; a 16 is too much. Put a zero in the 16 (2^4) position. We now have 1010. Can we use an 8? No; so put a zero in the 2^3 position. Now we have 10100. Can we use a 4? Yes. Our number is now 101001. Can we use a 2? Yes. We now have 1010011. Do we need a 1 or a 0 in the LSB position? We need a 1. Our binary number is 10100111. Any decimal number can be converted to binary in this manner. With practice, this method is faster than the method shown previously. The process is detailed in Figure 24–2.

EXAMPLE 24–5

Convert the following decimal numbers to binary numbers:

(a) 7 (b) 34 (c) 89 (d) 203

SOLUTIONS

(a) $\frac{7}{2} = 3 + 1$

$\frac{3}{2} = 1 + 1$

$\frac{1}{2} = 0 + 1 \rightarrow 1 \quad 1 \quad 1$

$7_{10} = 111_2$

FIGURE 24–2
Conversion of 167_{10} to binary.

MSB							LSB
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1	0	1	0	0	1	1	1

$128 + 0 + 32 + 0 + 0 + 4 + 2 + 1 = 167_{10}$

Binary No.		
	167	Original decimal number
1	−128	2^7 is the largest binary number that we can subtract from 167, so the MSB is in the 2^7 position.
	39	Remainder
0	−0	The 2^6 position must equal 0 (64 is greater than 39), so we must put a 0 in the 2^6 position.
1	−32	We can subtract 32 from 39, so there is a 1 in the 2^5 position.
	7	Remainder
0	−0	The 2^4 position must equal 0 (can't subtract 16 from 7).
0	−0	The 2^3 position must equal 0 (can't subtract 8 from 7).
1	−4	We can subtract 4 from 7, so there is a 1 in the 2^4 position.
	3	Remainder
1	−2	We can subtract 2 from 3, so there is a 1 in the 2^1 position.
1	1	Remainder: There is a 1 in the LSB position. (If there had been no remainder, the LSB would equal 0.)

(b)
$$\frac{34}{2} = 17 + 0$$
$$\frac{17}{2} = 8 + 1$$
$$\frac{8}{2} = 4 + 0$$
$$\frac{4}{2} = 2 + 0 \qquad 34_{10} = 100010_2$$
$$\frac{2}{2} = 1 + 0$$
$$\frac{1}{2} = 0 + 1 \rightarrow 1\quad 0\quad 0\quad 0\quad 1\quad 0$$

(c)
$$\frac{89}{2} = 44 + 1$$
$$\frac{44}{2} = 22 + 0$$
$$\frac{22}{2} = 11 + 0 \qquad 89_{10} = 1011001_2$$
$$\frac{11}{2} = 5 + 1$$
$$\frac{5}{2} = 2 + 1$$
$$\frac{2}{2} = 1 + 0$$
$$\frac{1}{2} = 0 + 1 \rightarrow 1\quad 0\quad 1\quad 1\quad 0\quad 0\quad 1$$

(d)
$$\frac{203}{2} = 101 + 1$$
$$\frac{101}{2} = 50 + 1$$
$$\frac{50}{2} = 25 + 0$$
$$\frac{25}{2} = 12 + 1 \qquad 203_{10} = 11001011_2$$
$$\frac{12}{2} = 6 + 0$$
$$\frac{6}{2} = 3 + 0$$
$$\frac{3}{2} = 1 + 1$$
$$\frac{1}{2} = 0 + 1 \rightarrow 1\quad 1\quad 0\quad 0\quad 1\quad 0\quad 1\quad 1$$

PRACTICE PROBLEMS 24–1

Convert the following binary numbers to decimal numbers:

1. 101_2
2. 111_2
3. 1001_2
4. 1110_2

5. 10110_2
6. 11001_2
7. 100101_2
8. 111011_2
9. 1010100_2
10. 1111001_2

Convert the following decimal numbers to binary numbers:

11. 4
12. 8
13. 10
14. 27
15. 39
16. 57
17. 78
18. 100
19. 150
20. 200

24–2 OCTAL NUMBER SYSTEM

Octal, or base eight, is an important computer number system. Since there are eight digits in octal (0, 1, 2, 3, 4, 5, 6, 7), the digit of greatest value is 7, which is 1 less than the base. In counting, the sequence is 0 to 7. On the next count, we return to 0 in the ones position (just as we do after reaching 9 in decimal) and a carry is generated. The resulting number is 10 and is read "one-zero." This number is equal to eight because the carry is equal to eight. This process continues: the sequence goes from 0 to 7 and back to 0 again. Each time a count greater than 7 is reached, a carry is generated.

In the octal number system, digits in the ones position have the indicated value. Digits in the next position have *place value*. That is, digits in this position indicate the number of 8s present. This, then, is the 8's position. The next position indicates the number of 64s present, and so on, as shown in Figure 24–3.

Again, the concept of *place value* must be understood so that we can work in this (or any) base. Consider the number 567_{10}. The five has a value of 500 because it is in the hundreds or 10^2 position ($5 \times 10^2 = 500$). The six has a value of 60 because it is in the tens position ($6 \times 10 = 60$). If the number were 567_8, the five would have a value of 320 in decimal because it is in the 8^2 or 64s position ($5 \times 8^2 = 320$). The six has a value of 48, in decimal, because it is in the eights (8^1) position, $6 \times 8 = 48$. In both bases the seven is in the ones position and has a value of 7.

$$567_8 = (5 \times 8^2) + (6 \times 8) + 7 = 375_{10}$$

24–2–1 Octal to Decimal Conversions

Let's examine the number 1025 in octal. We might be tempted to read this as "one thousand twenty-five." Such language implies base ten. The number should be read "one-zero-two-five," which does not imply any base. We normally think in decimal, and decimal is the number system we use most; therefore, let's convert the number to base ten. There is a 1 in the 512's position + a 0 in the 64's position + a 2 in the 8's position + a 5 in the ones position.

RULE 24–2 In converting from octal to decimal, find the weight of the digit in each position. Add the values of the digits in each position to determine the decimal equivalent.

$$1025_8 = (1 \times 8^3) + (0 \times 8^2) + (2 \times 8^1) + (5 \times 8^0) = 533_{10}$$

$$1025_8 = 512 + 0 + 16 + 5 = 533_{10}$$

EXAMPLE 24–6

Change the following octal numbers to decimal:

(a) 73_8 (b) 432_8 (c) 600_8 (d) 1234_8

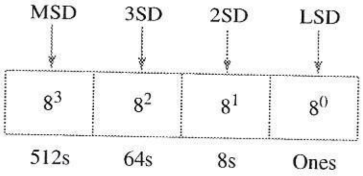


FIGURE 24–3 Positional value of the first four positions.

SOLUTIONS

(a) $73_8 = (7 \times 8^1) + (3 \times 8^0) = 56 + 3 = 59_{10}$
(b) $432_8 = (4 \times 8^2) + (3 \times 8^1) + (2 \times 8^0) = 256 + 24 + 2 = 282_{10}$
(c) $600_8 = (6 \times 8^2) + (0 \times 8^1) + (0 \times 8^0) = 384 + 0 + 0 = 384_{10}$
(d) $1234_8 = (1 \times 8^3) + (2 \times 8^2) + (3 \times 8^1) + (4 \times 8^0) = 512 + 128 + 24 + 4 = 668_{10}$

24-2-2 Decimal to Octal Conversions

Let's reverse the process. In Example 24–7, the number 189 is converted from decimal to octal.

EXAMPLE 24–7

$189_{10} = (?)_8$

SOLUTION

Step 1. $\dfrac{189}{8} = 23 + 5$

Step 2. $\dfrac{23}{8} = 2 + 7$

Step 3. $\dfrac{2}{8} = 0 + 2 -$ MSD → $2 \quad 7 \quad 5 \qquad 189_{10} = 275_8$

Since we are converting to base eight, we first divide the decimal number by 8 (step 1). The answer is 23 with a remainder of 5. This remainder is the LSD (least significant digit) of our base eight number. The whole number left after this initial division (23) is again divided by 8 to determine the value of the digit in the next (8^1) position. This division leaves a remainder of 7, which becomes the digit in the 8^1 position. The whole number resulting from this division (2) is again divided by 8, and the remainder (2) is the digit in the 8^2 position. This digit is the MSD (most significant digit) since no further division is possible.

EXAMPLE 24–8

Convert the following decimal numbers to octal numbers:

(a) 78 (b) 376 (c) 1463

SOLUTIONS

(a) $\dfrac{78}{8} = 9 + 6$

$\dfrac{9}{8} = 1 + 1$

$\dfrac{1}{8} = 0 + 1 \rightarrow 1 \quad 1 \quad 6 \qquad 78_{10} = 116_8$

(b) $\dfrac{376}{8} = 47 + 0$

$\dfrac{47}{8} = 5 + 7$

$\dfrac{5}{8} = 0 + 5 \rightarrow 5 \quad 7 \quad 0 \qquad 376_{10} = 570_8$

(c) $\dfrac{1463}{8} = 182 + 7$

$$\frac{182}{8} = 22 + 6$$

$$\frac{22}{8} = 2 + 6$$

$$\frac{2}{8} = 0 + 2 \rightarrow 2 \quad 6 \quad 6 \quad 7 \qquad 1463_{10} = 2667_{8}$$

PRACTICE PROBLEMS 24–2

1. Convert the following numbers from base eight to base ten:

   (a) 46_8 (b) 111_8 (c) 204_8 (d) 400_8 (e) 777_8

2. Convert the following numbers from base ten to base eight:

   (a) 27_{10} (b) 89_{10} (c) 100_{10} (d) 256_{10} (e) 400_{10}

24–3 HEXADECIMAL NUMBER SYSTEM

Another important computer number system is the hexadecimal system (base sixteen). How many digits exist in hexadecimal? Sixteen, because the base is sixteen. What is the value of the largest digit in hexadecimal? Fifteen, one less than the base. However, 15 is a decimal number and requires *two digits* (15). In the hexadecimal number system, sixteen *different* symbols must be used. Zero through 9 are used for the first ten, and A through F are used for the remaining six, as shown in Table 24–2. (Also refer to Table 24–1 for the complete conversion table.)

Table 24–2 Hexadecimal to Decimal Conversions.

DECIMAL	HEXADECIMAL	DECIMAL	HEXADECIMAL
0	0	9	9
1	1	10	A
2	2	11	B
3	3	12	C
4	4	13	D
5	5	14	E
6	6	15	F
7	7	16	10
8	8		

Let's look at the number 567 again. Recall from the previous section that, in the number 567_{10}, the 5 is in the 10^2 position and so has a value of 500 ($5 \times 10^2 = 500$). Six is in the 10s position and so has a value of 60 ($6 \times 10 = 60$). In the number 567_{16}, the 5 is in the 16^2 position, so it has a value of 1280 in decimal ($5 \times 16^2 = 1280$). The 6 is in the 16's position and so has a value of 96 ($6 \times 16 = 96$). Again, the 7 is in the ones position and so has a value of 7.

$$567_{16} = (5 \times 16^2) + (6 \times 16) + 7 = 1383$$

24–3–1 Hexadecimal to Decimal Conversions

The hexadecimal number 3AC2 may be converted to base ten as follows (see Figure 24–4):

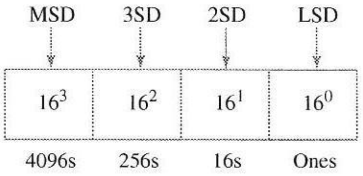


FIGURE 24–4 Positional value of a four-digit hexadecimal number.

Three 4096s, which equal	12,288
+ ten 256s, which equal	2,560
+ twelve 16s, which equal	192
+ two 1s, which equal	2
SUM	$15{,}042_{10}$

or

$$(3 \times 16^3) + (10 \times 16^2) + (12 \times 16^1) + (2 \times 16^0) = 15{,}042_{10}$$

RULE 24–3 In converting from hexadecimal to decimal, find the weight of the digit in each position. Add the values of the digits in each position to determine the decimal equivalent.

EXAMPLE 24–9

Convert the following hexadecimal numbers to decimal numbers:

(a) 72 (b) C29 (c) 12AB (d) 1A2A

SOLUTIONS

(a) $72_{16} = (7 \times 16^1) + (2 \times 16^0) = 112 + 2 = 114_{10}$
(b) $C29_{16} = (12 \times 16^2) + (2 \times 16^1) + (9 \times 16^0) = 3113_{10}$
(c) $12AB_{16} = (1 \times 16^3) + (2 \times 16^2) + (10 \times 16^1) + (11 \times 16^0) = 4779_{10}$
(d) $1A2A_{16} = (1 \times 16^3) + (10 \times 16^2) + (2 \times 16^1) + (10 \times 16^0) = 6698_{10}$

24–3–2 Decimal to Hexadecimal Conversions

The conversion of decimal numbers to hexadecimal is identical to conversion using other bases. In Example 24–10, the number 1324 is converted from decimal to hexadecimal. Since the hexadecimal number system has a base of sixteen, division is by 16. In working with remainders greater than 9, we substitute the characters A, B, C, D, E, and F as required.

EXAMPLE 24–10

$1324_{10} = (?)_{16}$

SOLUTION

Step 1. $\frac{1324}{16} = 82 + 12$

Step 2. $\frac{82}{16} = 5 + 2$ LSD

Step 3. $\frac{5}{16} = 0 + 5$ – MSD → 5 2 C $1324_{10} = 52C_{16}$

EXAMPLE 24–11

Convert the following decimal numbers to hexadecimal numbers:

(a) 672 (b) 1763 (c) 12,760

SOLUTIONS

(a)
$$\frac{672}{16} = 42 + 0$$
$$\frac{42}{16} = 2 + 10$$
$$\frac{2}{16} = 0 + 2 \rightarrow 2 \quad A \quad 0 \qquad 672_{10} = 2A0_{16}$$

(b)
$$\frac{1763}{16} = 110 + 3$$
$$\frac{110}{16} = 6 + 14$$
$$\frac{6}{16} = 0 + 6 \rightarrow 6 \quad E \quad 3 \qquad 1763_{10} = 6E3_{16}$$

(c)
$$\frac{12760}{16} = 797 + 8$$
$$\frac{797}{16} = 49 + 13$$
$$\frac{49}{16} = 3 + 1$$
$$\frac{3}{16} = 0 + 3 \rightarrow 3 \quad 1 \quad D \quad 8 \qquad 12760_{10} = 31D8_{16}$$

PRACTICE PROBLEMS 24–3

1. Convert the following hexadecimal numbers to decimal numbers:

   (a) D_{16} (b) $3F_{16}$ (c) $A4_{16}$ (d) $1CD_{16}$ (e) $10BE_{16}$

2. Convert the following decimal numbers to hexadecimal numbers:

   (a) 27_{10} (b) 85_{10} (c) 100_{10} (d) 256_{10} (e) 1500_{10}

24–4 BINARY TO OCTAL TO HEXADECIMAL CONVERSIONS

24–4–1 Binary to Octal Conversions

Many 1s and 0s are required when we use binary numbers to represent large quantities. If we convert from binary to octal, the number of digits required is reduced by a factor of 3 because one octal digit equals 3 bits. That is, all octal digits, 0 through 7, can be represented by 3 bits.

$0 = 000 \qquad 4 = 100$
$1 = 001 \qquad 5 = 101$
$2 = 010 \qquad 6 = 110$
$3 = 011 \qquad 7 = 111$

KEY POINT To convert from binary to octal, separate the bits into 3-bit groups, starting with the LSB and moving left to the MSB.

EXAMPLE 24–12

Convert 110111100001_2 to octal:

SOLUTION

110 111 100 001 (binary number divided into groups of three)
6 7 4 1 (octal substitution for each 3-bit group)

EXAMPLE 24–13

Convert 11101001_2 to octal:

SOLUTION 011 101 001
3 5 1 $11101001_2 = 351_8$

Notice that a zero was added to the leftmost group of bits to remind us that 3 bits represent one octal digit.

The procedure may be reversed to convert from octal to binary.

EXAMPLE 24–14

Convert 417_8 to binary:

SOLUTION 4 1 7 $417_8 = 100001111_2$
100 001 111

EXAMPLE 24–15

Convert 362_8 to binary:

SOLUTION 3 6 2 $362_8 = 11110010_2$
011 110 010

In each example, 3 bits are substituted for each octal digit. A zero in the leftmost bit position has no meaning and need not be written in the answer.

24-4-2 Binary to Hexadecimal Conversions

If we convert from binary to hexadecimal, the number of digits required is reduced by a factor of 4 because one hexadecimal digit equals 4 bits. That is, all hexadecimal digits (0 through F) can be represented by 4 bits.

0 = 0000
1 = 0001
2 = 0010
3 = 0011
4 = 0100
5 = 0101
6 = 0110
7 = 0111
8 = 1000
9 = 1001
A = 1010
B = 1011
C = 1100
D = 1101
E = 1110
F = 1111

KEY POINT To convert from binary to hexadecimal, separate the bits into 4-bit groups, starting with the LSB and moving left to the MSB.

EXAMPLE 24–16

Convert 110110011011_2 to hexadecimal:

SOLUTION 1101 1001 1011 (binary number divided into 4-bit groups)
D 9 B (hexadecimal substitution for each 4-bit group)

PRACTICE PROBLEMS 24–4

Convert the following binary numbers to (a) octal numbers and then (b) hexadecimal numbers:

1. 1101001101_2
2. 1000011001_2
3. 1100110001_2
4. 111001110101_2
5. $11110101100011 0_2$

Convert the following numbers to binary numbers:

6. 73_8
7. 127_8
8. 617_8
9. 506_8
10. 1207_8
11. $A6_{16}$
12. $4C_{16}$
13. $C0D_{16}$
14. $BA1_{16}$
15. $F23_{16}$

24–5 BINARY TO OCTAL TO DECIMAL TO HEXADECIMAL CONVERSIONS

24–5–1 The Calculator

Conversion between number systems is possible with many calculators. However, use the calculator only after you have a thorough understanding of the concepts.

24–5–2 Converting between the Various Number Systems

The simplest octal to hexadecimal conversion is to first convert the number to binary and then make a conversion from binary to the other number system. We have converted from decimal to the other systems, and we have converted between binary, octal, and hexadecimal. Now let's practice some conversions among all the number systems we have studied. For example, let's convert 200_{10} to binary, octal, and hexadecimal.

EXAMPLE 24–17

$$200_{10} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{16}$$

SOLUTION There are many ways to solve this problem. We either could convert from 200_{10} to each of the other systems, or we could convert to base two and then convert to base eight and base sixteen from base two. Another way, and probably the one requiring fewest steps, is to convert from base ten to base sixteen, then from base sixteen to base two, and then from base two to base eight. Remember, fewer steps mean fewer chances for error.

$$\frac{200}{16} = 12 + 8$$

$$\frac{12}{16} = 0 + 12 \longrightarrow C \quad 8$$

$$200_{10} = C8_{16}$$

Convert to binary:

$$\begin{array}{cc} \text{C} & 8 \\ 1100 & 1000 \end{array} \quad \text{C8}_{16} = 11001000_2$$

Convert to octal:

$$\begin{array}{ccc} 011 & 001 & 000 \\ 3 & 1 & 0 \end{array} \quad 11001000_2 = 310_8$$

$$200_{10} = 11001000_2 = 310_8 = \text{C8}_{16}$$

EXAMPLE 24–18

$\text{C27}_{16} =$ ________ $_2 =$ ________ $_8 =$ ________ $_{10}$

SOLUTION In this example, we have a straight conversion to binary and then to octal. Convert to binary:

$$\begin{array}{ccc} \text{C} & 2 & 7 \\ 1100 & 0010 & 0111 \end{array} \quad \text{C27}_{16} = 110000100111_2$$

Convert to octal:

$$\begin{array}{cccc} 110 & 000 & 100 & 111 \\ 6 & 0 & 4 & 7 \end{array} \quad 110000100111_2 = 6047_8$$

The conversion to decimal can be made from any of the other systems, but conversion from base sixteen requires fewer steps.

$$\begin{aligned} \text{C27}_{16} &= (12 \times 16^2) + (2 \times 16) + (7 \times 16^0) \\ &= 3072 + 32 + 7 \\ &= 3111_{10} \end{aligned}$$

$$\text{C27}_{16} = 110000100111_2 = 6047_8 = 3111_{10}$$

You will find that some conversions in the following problems will exceed the limits of the calculator. Consult the user's manual and become familiar with the maximum numbers that can be stored and displayed in systems other than decimal.

PRACTICE PROBLEMS 24–5

1. $73_8 =$ ________ $_{16} =$ ________ $_2 =$ ________ $_{10}$
2. $273_8 =$ ________ $_{16} =$ ________ $_2 =$ ________ $_{10}$
3. $\text{A3}_{16} =$ ________ $_8 =$ ________ $_2 =$ ________ $_{10}$
4. $\text{12E}_{16} =$ ________ $_8 =$ ________ $_2 =$ ________ $_{10}$
5. $1101001_2 =$ ________ $_8 =$ ________ $_{16} =$ ________ $_{10}$
6. $10111100_2 =$ ________ $_8 =$ ________ $_{16} =$ ________ $_{10}$

SELF-TEST 24–1 THROUGH 24–5

Convert the following numbers to decimal numbers:

1. 46_8
2. 276_8
3. F6_{16}
4. C3A_{16}
5. 1011011_2
6. 11001010_2

Convert the following decimal numbers to (a) binary, (b) octal, and (c) hexadecimal:

7. 10
8. 28
9. 187
10. 625

Convert the following to the bases indicated.

11. 93_{10} = ______________ $_2$ = ______________ $_8$ = ______________ $_{16}$
12. E1A_{16} = ______________ $_2$ = ______________ $_8$ = ______________ $_{10}$
13. 11010010110_2 = ______________ $_8$ = ______________ $_{10}$ = ______________ $_{16}$
14. 573_8 = ______________ $_2$ = ______________ $_{10}$ = ______________ $_{16}$

24-6 DECIMAL AND OCTAL ADDITION

24-6-1 Adding Decimal Numbers

Add the numbers 23 and 45. When numbers are added, they are added in columns; one number is placed below the other. Either number may be placed first. In Example 24–19, the numbers 23 and 45 are placed in the position for addition. The digits are separated to emphasize positional differences. Eight is in the ones position (3 + 5), and 6 is in the tens position (2 + 4). The answer is 68 (six 10s and eight 1s).

EXAMPLE 24–19

$$23 + 45 = ?$$

$$\begin{array}{cc} 2 & 3 \\ 4 & 5 \\ \hline 6 & 8 \end{array}$$

EXAMPLE 24–20

Add the numbers 64 and 87 in base ten:

SOLUTION

$$\begin{array}{rccc} \text{carry} & 1 & 1 & \\ & & 6 & 4 \\ & & 8 & 7 \\ \hline & 1 & (15) & (11) \\ \text{(subtract value of carry)} & & 10 & 10 \\ \hline & 1 & 5 & 1 \end{array} \quad \text{(remainders taken as decimal sum)}$$

In Example 24–20, there is an 11 in the ones position. This number is greater than 9, which tells us there must be a carry (10). The digit remaining in the ones position is the difference of 10 and 11, or 1. The tens position now contains 6 + 8 + a carry of 1, or 15. Again, since the number is greater than 9, a carry is indicated. Subtracting 10 from 15, we get 5 with a carry of 1. The third position contains only the carry. Since we have been adding for years in decimal, our experience allows us to perform these operations automatically. We have taken time to discuss the arithmetic steps because this is the addition process in all bases.

EXAMPLE 24–21

Add the following decimal numbers using the procedure given in Example 24–20:

(a) 738 + 417 (b) 9706 + 463 (c) 1774 + 7268

SOLUTIONS

(a)
$$\begin{array}{cccc}
1 & & 1 & \\
 & 7 & 3 & 8 \\
 & 4 & 1 & 7 \\
\hline
1 & (11) & 5 & (15) \\
 & 10 & & 10 \\
\hline
1 & 1 & 5 & 5
\end{array}
\qquad 738 + 417 = 1155$$

(b)
$$\begin{array}{ccccc}
1 & 1 & & & \\
 & 9 & 7 & 0 & 6 \\
 & & 4 & 6 & 3 \\
\hline
1 & (10) & (11) & 6 & 9 \\
 & 10 & 10 & & \\
\hline
1 & 0 & 1 & 6 & 9
\end{array}
\qquad 9706 + 463 = 10{,}169$$

(c)
$$\begin{array}{cccc}
1 & 1 & 1 & \\
1 & 7 & 7 & 4 \\
7 & 2 & 6 & 8 \\
\hline
9 & (10) & (14) & (12) \\
 & 10 & 10 & 10 \\
\hline
9 & 0 & 4 & 2
\end{array}
\qquad 1774 + 7268 = 9042$$

24-6-2 Adding Octal Numbers

EXAMPLE 24–22

Add the octal numbers 736 and 215.

SOLUTION

$$\begin{array}{rcccc}
\text{(carry)} & 1 & & 1 & \\
 & & 7 & 3 & 6 \\
 & & 2 & 1 & 5 \\
\cline{3-5}
\text{(decimal sum)} & 1 & (9) & 5 & (11) \\
\text{(subtract value of carry)} & & 8 & & 8 \\
\cline{3-5}
 & 1 & 1 & 5 & 3
\end{array}
\qquad 736_8 + 215_8 = 1153_8$$

Again, the LSD or ones position is added first. The sum of 6 and 5 is 11. The number is greater than 7 (7 is the largest digit in base 8), which tells us there must be a carry. Subtracting 8 (the value of the carry) leaves 3 with a carry of 1. The 8^1 position now contains 3 + 1 + a carry of 1, or 5. Since 5 is less than 7, no carry exists. In the 8^2 position, 7 + 2 is 9. Since the number is greater than 7, a carry is generated. Subtracting the carry (8) yields a remainder of 1 with a carry of 1. Since the 8^3 position contains only the carry, the answer is 1153_8.

EXAMPLE 24–23

Add the following octal numbers:

$$\text{(a)}\ \begin{array}{r} 243 \\ \underline{172} \end{array} \qquad \text{(b)}\ \begin{array}{r} 764 \\ \underline{414} \end{array} \qquad \text{(c)}\ \begin{array}{r} 604 \\ \underline{777} \end{array}$$

SOLUTIONS

(a)
$$\begin{array}{ccl} 1 & & \\ 2 & 4 & 3 \\ 1 & 7 & 2 \\ \hline 4 & (11) & 5 \\ & 8 & \\ \hline 4 & 3 & 5 = 435_8 \end{array}$$

(b)
$$\begin{array}{cccl} 1 & 1 & 1 & \\ & 7 & 6 & 4 \\ & 4 & 1 & 4 \\ \hline 1 & (12) & (8) & (8) \\ & 8 & 8 & 8 \\ \hline 1 & 4 & 0 & 0 = 1400_8 \end{array}$$

(c)
$$\begin{array}{cccl} 1 & 1 & 1 & \\ & 6 & 0 & 4 \\ & 7 & 7 & 7 \\ \hline 1 & (14) & (8) & (11) \\ & 8 & 8 & 8 \\ \hline 1 & 6 & 0 & 3 = 1603_8 \end{array}$$

PRACTICE PROBLEMS 24–6

Add the following octal numbers.

1. $\begin{array}{r} 204 \\ \underline{316} \end{array}$
2. $\begin{array}{r} 447 \\ \underline{173} \end{array}$
3. $\begin{array}{r} 605 \\ \underline{726} \end{array}$
4. $\begin{array}{r} 517 \\ \underline{567} \end{array}$
5. $\begin{array}{r} 176 \\ \underline{761} \end{array}$

24–7 ADDING HEXADECIMAL NUMBERS

EXAMPLE 24–24

Add the hexadecimal numbers 7AF and 579.

SOLUTION

$$\begin{array}{rcccl} \text{(carry)} & 1\leftarrow & 1\leftarrow & & \\ & 7 & A & F & \\ & 5 & 7 & 9 & \\ \hline \text{(decimal sum)} & (13) & (18) & (24) & \\ \text{(subtract value of carry)} & & -16 & -16 & \\ \hline & D & 2 & 8 & = D28_{16} \end{array}$$

Adding the ones position yields 24. Twenty-four is greater than the largest digit (F), which is equal to decimal 15. This indicates that a carry should exist. Subtracting the value of a carry (16) leaves a remainder of 8 with a carry of 1. In the 16^1 position, the sum of A (decimal 10) + 7 + a carry of 1 is 18. Again, a carry is generated, leaving a remainder of 2 with a carry of 1. In the 16^2 position, the carry + 7 + 5 equals decimal 13. In hexadecimal, 13 is D; therefore, the answer is $D28_{16}$.

EXAMPLE 24–25

Add the following hexadecimal numbers:

(a) $\begin{array}{r} 4D3 \\ \underline{818} \end{array}$ (b) $\begin{array}{r} 789 \\ \underline{C47} \end{array}$ (c) $\begin{array}{r} F347 \\ \underline{E006} \end{array}$

SOLUTIONS

(a)
$$\begin{array}{rrrl} 4 & D & 3 & \\ 8 & 1 & 8 & \\ \hline (12) & (14) & (11) & \\ \hline C & E & B & = CEB_{16} \end{array}$$

(b)
$$\begin{array}{rrrrl} 1\leftarrow & & 1\leftarrow & & \\ & 7 & 8 & 9 & \\ & C & 4 & 7 & \\ \hline 1 & (19) & (13) & (16) & \\ & -16 & & -16 & \\ \hline 1 & 3 & D & 0 & = 13D0_{16} \end{array}$$

(c)
$$\begin{array}{rrrrrl} 1\leftarrow & & & & & \\ & F & 3 & 4 & 7 & \\ & E & 0 & 0 & 6 & \\ \hline 1 & (29) & 3 & 4 & (13) & \\ & -16 & & & & \\ \hline 1 & D & 3 & 4 & D & = 1D34D_{16} \end{array}$$

PRACTICE PROBLEMS 24–7

Add the following hexadecimal numbers:

1. $\begin{array}{r} C4 \\ \underline{1B} \end{array}$
2. $\begin{array}{r} A4 \\ \underline{B7} \end{array}$
3. $\begin{array}{r} A28 \\ \underline{6F9} \end{array}$
4. $\begin{array}{r} 9B5 \\ \underline{4C8} \end{array}$
5. $\begin{array}{r} FACE \\ \underline{1701} \end{array}$

24–8 ADDING BINARY NUMBERS

EXAMPLE 24–26

Add the binary numbers 1101 and 1001.

SOLUTION

$$\begin{array}{rrrrrrl} & 1\leftarrow & & & 1\leftarrow & & \text{(carry)} \\ & & 1 & 1 & 0 & 1 & \\ & & 1 & 0 & 0 & 1 & \\ \hline \text{(decimal sum)} & & (2) & 1 & 1 & (2) & \\ \text{(subtract value of carry)} & & -2 & & & -2 & \\ \hline & 1 & 0 & 1 & 1 & 0 & = 10110_2 \end{array}$$

The method of addition is the same as that used with other bases. Notice that 1 + 1 equals decimal 2. Since 2 is invalid in base two, a carry is generated into the next position. We see from the example that a carry is generated whenever a sum greater than 1 results from the addition of numbers in a column.

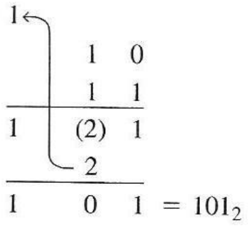
EXAMPLE 24–27

Add the following binary numbers.

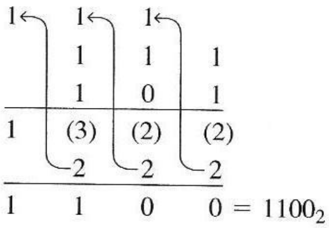
(a) $\begin{array}{r} 10 \\ \underline{11} \end{array}$ (b) $\begin{array}{r} 111 \\ \underline{101} \end{array}$ (c) $\begin{array}{r} 1001 \\ \underline{1010} \end{array}$ (d) $\begin{array}{r} 11100 \\ \underline{01010} \end{array}$

SOLUTIONS

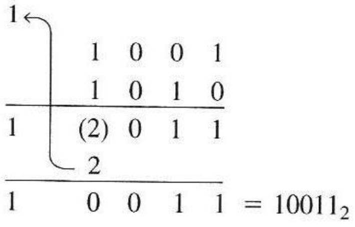
(a)



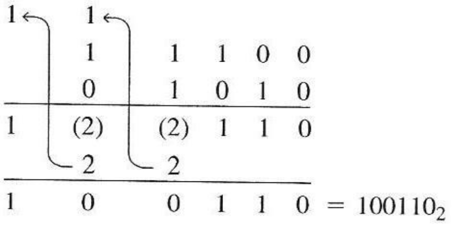
(b)



(c)



(d)



From working with these and other examples of binary addition, we see that:

1. 0 + 0 always equals 0.
2. 0 + 1 always equals 1.
3. 1 + 1 always equals 0 and a carry.
4. 1 + 1 + a carry always equals 1 and a carry.

PRACTICE PROBLEMS 24–8

Add the following binary numbers:

1. 1011
   1100
2. 1001
   1111
3. 10111
   10001
4. 11001
   11110
5. 101110
   111100

SELF-TEST 24–6 THROUGH 24–8

Add the following numbers:

1. AF_{16}
   36_{16}
2. $C4A_{16}$
   $17E_{16}$
3. $F4A9_{16}$
   $FE7A_{16}$
4. 73_8
   66_8
5. 63_8
   41_8
6. 73_8
   46_8

7. $\begin{array}{r} 101_2 \\ \underline{111_2} \end{array}$

8. $\begin{array}{r} 1101_2 \\ \underline{1011_2} \end{array}$

9. $\begin{array}{r} 10110_2 \\ \underline{11110_2} \end{array}$

24–9 DECIMAL AND OCTAL SUBTRACTION

24–9–1 Subtracting Decimal Numbers

Subtraction in base ten is so automatic for us that we may do it without thinking of the rules involved. Since these rules in base ten are identical to those for other bases, and since we are more familiar with base ten than with other bases, let's review the process of subtraction in base ten.

EXAMPLE 24–28

Subtract 25 from 43:

SOLUTION

$$\begin{array}{lccc} & 3 & & \text{borrow 1} \\ \text{(minuend)} & \cancel{4} & \leftarrow\!\rightarrow (1) & 3 \\ \text{(subtrahend)} & 2 & & 5 \\ \hline \text{(difference)} & 1 & & 8 \end{array}$$

As you know, we start with the least significant position and subtract. Since we cannot subtract 5 from 3, we *borrow* from the next significant position of the minuend. We may now interpret the 3 as 13 and find the difference, which is 8. Moving to the next position, we must reduce the minuend by 1 because of the borrow. We now subtract the number in the subtrahend from the new minuend and the difference is 1. Remember, the borrow carries with it a weight equal to that of the base.

EXAMPLE 24–29

Subtract the following decimal numbers. Show the process of borrowing.

$$\text{(a)}\ \begin{array}{r} 624 \\ \underline{-276} \end{array} \qquad \text{(b)}\ \begin{array}{r} 854 \\ \underline{-235} \end{array} \qquad \text{(c)}\ \begin{array}{r} 8534 \\ \underline{-6748} \end{array} \qquad \text{(d)}\ \begin{array}{r} 705 \\ \underline{-528} \end{array}$$

SOLUTIONS

$$\text{(a)}\ \begin{array}{ccc} 5 & (1)1 & \\ \cancel{6} & \cancel{2} & (1)4 \\ -2 & 7 & 6 \\ \hline 3 & 4 & 8 \end{array} \qquad \text{(b)}\ \begin{array}{ccc} & 4 & \\ 8 & \cancel{5} & (1)4 \\ -2 & 3 & 5 \\ \hline 6 & 1 & 9 \end{array}$$

$$\text{(c)}\ \begin{array}{cccc} 7 & (1)4 & (1)2 & \\ \cancel{8} & \cancel{5} & \cancel{3} & (1)4 \\ -6 & 7 & 4 & 8 \\ \hline 1 & 7 & 8 & 6 \end{array} \qquad \text{(d)}\ \begin{array}{ccc} 6 & 9 & \\ \cancel{7} & (\cancel{1})\cancel{0} & (1)5 \\ -5 & 2 & 8 \\ \hline 1 & 7 & 7 \end{array}$$

Notice in Example 24–29(d) that, since 8 is greater than 5, a borrow is indicated. However, the next significant digit is 0 and no borrow can take place (we can't borrow something from nothing). Therefore, we must borrow from the next significant digit (7), giving the second position a value of 10. Now we can borrow from that position to yield 15 in the least significant position.

24-9-2 Subtracting Octal Numbers

In Example 24–30, the number 267_8 is subtracted from 512_8. Notice that a borrow is necessary in the first position since 7 is greater than 2. The borrow creates a new number in the minuend, 12. (This number is read "one-two," *not* "twelve," because twelve is a decimal number. One-two in base eight means $8 + 2$ in base ten.)

EXAMPLE 24–30

Subtract 267_8 from 512_8:

SOLUTION

$$\begin{array}{rccc}
 & 4 & \nearrow 10 & \\
\text{(minuend)} & \cancel{5} & \cancel{1} \rightarrow & (1)2 \\
\underline{\text{(subtrahend)}} & 2 & 6 & 7 \\ \hline
\text{(difference)} & 2 & 2 & 3 = 223_8
\end{array}$$

One-two minus 7 equals 3. If you think in base 10, then $(8 + 2) - 7$ equals 3. In the next position, 1 was borrowed from the number in the minuend, making its value 0. A borrow is again required; it makes the value of the minuend 10 (or $8 + 0$ in base ten). Subtracting 6 from 10 yields a difference of 2. In the next position, 1 was borrowed from the minuend, making its value 4. The difference in this position is 2.

EXAMPLE 24–31

Solve the following subtraction problems in base eight:

$$\text{(a)}\ \begin{array}{r} 43 \\ (-)\underline{16} \end{array} \qquad \text{(b)}\ \begin{array}{r} 161 \\ (-)\underline{72} \end{array} \qquad \text{(c)}\ \begin{array}{r} 2172 \\ (-)\underline{1717} \end{array}$$

SOLUTIONS

$$\text{(a)}\ \begin{array}{rc}
3 & \\
\cancel{4} & (1)3 \\
(-)1 & 6 \\ \hline
2 & 5 = 25_8
\end{array}
\qquad
\text{(b)}\ \begin{array}{rcc}
0 & (1)5 & \\
\cancel{1} & \cancel{6} & (1)1 \\
(-) & 7 & 2 \\ \hline
 & 6 & 7 = 67_8
\end{array}$$

$$\text{(c)}\ \begin{array}{rccc}
1 & & 6 & \\
\cancel{2} & (1)1 & \cancel{7} & (1)2 \\
(-)1 & 7 & 1 & 7 \\ \hline
 & 2 & 5 & 3 = 253_8
\end{array}$$

PRACTICE PROBLEMS 24–9

Subtract the following octal numbers:

1. $\begin{array}{r} 73 \\ (-)\underline{56} \end{array}$
2. $\begin{array}{r} 376 \\ (-)\underline{277} \end{array}$
3. $\begin{array}{r} 673 \\ (-)\underline{176} \end{array}$
4. $\begin{array}{r} 2133 \\ (-)\underline{1574} \end{array}$
5. $\begin{array}{r} 6014 \\ (-)\underline{3247} \end{array}$

24–10 SUBTRACTING HEXADECIMAL NUMBERS

Applying the same rules as in base eight, let's subtract the hexadecimal number 85E from the number C37.

EXAMPLE 24–32

$C37 - 85E = (?)_{16}$

SOLUTION

$$\begin{array}{rccc} & B & (1)2 & \\ \text{(minuend)} & \cancel{C} & \cancel{3} \longrightarrow & (1)7 \\ \text{(subtrahend)} & 8 & 5 & E \\ \hline \text{(difference)} & 3 & D & 9 = 3D9_{16} \end{array}$$

Notice that a borrow is necessary in the first position since E is greater than 7. One-seven (*not* seventeen) minus E equals 9 or, in base 10, $(16 + 7) - 14$ equals 9. (Remember, the borrow has a weight of 16.) In the next position, a borrow is required, so 5 is subtracted from 12 (one-two), which equals D. (In base ten, it would be $16 + 2 - 5 = 13$.) Finally, in the next position, subtracting 8 from B (remember that we borrowed 1, making C a B) yields 3 or, in base ten, $11 - 8 = 3$.

EXAMPLE 24–33

Subtract the following hexadecimal numbers:

$$\text{(a)}\ \begin{array}{r} A29 \\ \underline{-7BF} \end{array} \qquad \text{(b)}\ \begin{array}{r} 3A12 \\ \underline{-29A0} \end{array} \qquad \text{(c)}\ \begin{array}{r} F1B2 \\ \underline{-ABC7} \end{array}$$

SOLUTIONS

$$\text{(a)}\ \begin{array}{ccc} 9 & (1)1 & \\ \cancel{A}\nearrow & \cancel{2} \rightarrow & (1)9 \\ 7 & B & F \\ \hline 2 & 6 & A = 26A_{16} \end{array}$$

$$\text{(b)}\ \begin{array}{cccc} & 9 & & \\ 3 & \cancel{A} \rightarrow & (1)1 & 2 \\ 2 & 9 & A & 0 \\ \hline 1 & 0 & 7 & 2 = 1072_{16} \end{array}$$

$$\text{(c)}\ \begin{array}{cccc} E & (1)0 & (1)A & \\ \cancel{F}\nearrow & \cancel{1}\nearrow & \cancel{B} \rightarrow & (1)2 \\ A & B & C & 7 \\ \hline 4 & 5 & E & B = 45EB_{16} \end{array}$$

PRACTICE PROBLEMS 24–10

Subtract the following hexadecimal numbers:

1. A17
(−)26B

2. B27
(−)6A4

3. 7A29
(−)2BC9

4. 4A9B
(−)100F

5. FADE
(−)2C3F

24–11 SUBTRACTING BINARY NUMBERS

The process of subtraction in base two is similar to that in the other bases, as was discussed earlier in this chapter.

EXAMPLE 24–34

Subtract 0101 from 1110:

SOLUTION

$$\begin{array}{rcccl}
\text{borrow} & & & 0 & \\
\text{(minuend)} & 1 & 1 & \cancel{1}\rightarrow & (1)0 \\
\text{(subtrahend)} & 0 & 1 & 0 & 1 \\ \hline
\text{(difference)} & 1 & 0 & 0 & 1 = 1001_2
\end{array}$$

EXAMPLE 24–35

Subtract 01101 from 10011:

SOLUTION

$$\begin{array}{rccccl}
\text{borrow} & 0 & 1 & & & \\
\text{(minuend)} & 1\rightarrow & (\cancel{1})\cancel{0}\rightarrow & (1)0 & 1 & 1 \\
\text{(subtrahend)} & 0 & 1 & 1 & 0 & 1 \\ \hline
 & 0 & 0 & 1 & 1 & 0 = 00110_2
\end{array}$$

In Example 24–34, a borrow is necessary in the least significant position since the minuend is 0. The borrow has a weight of 2 because we are working in base two. Note that in Example 24–35 a borrow is required in the 2^2 position. However, since the 2^3 position contains a 0, the borrow must come from the 2^4 position to the 2^3 position. A borrow is now possible from the 2^3 position, and the minuend in the 2^2 position is 10 (one-zero), resulting in a difference of 1. The new minuend in the next position (2^3) is now 1, and the difference is 0. Since our original borrow came from the next position (2^4), the minuend is 0 and the difference is 0.

PRACTICE PROBLEMS 24–11

Subtract the following binary numbers:

1. $\begin{array}{r} 1010 \\ (-)\underline{0101} \end{array}$
2. $\begin{array}{r} 10110 \\ (-)\underline{01101} \end{array}$
3. $\begin{array}{r} 110010 \\ (-)\underline{100111} \end{array}$
4. $\begin{array}{r} 100111 \\ (-)\underline{001101} \end{array}$
5. $\begin{array}{r} 1100110 \\ (-)\underline{1011001} \end{array}$

24–12 COMPLEMENT METHOD OF SUBTRACTION

In the modern computer, subtraction is usually performed by addition of the complement of the subtrahend. This is done to simplify the design of the arithmetic section of the computer. To help us understand the complement method of subtraction in binary, we will first use the complement method of subtraction in decimal.

To subtract numbers in decimal, we must first find the nine's complement of the subtrahend. The nine's complement of a digit is the difference between 9 and that digit and can be found by a simple inspection procedure. Let's find the nine's complement of 235:

$$\begin{array}{r l} 999 & \\ -235 & \\ \hline 764 & \text{(nine's complement of 235)} \end{array}$$

The nine's complement may be converted to the ten's complement by adding 1. Find the ten's complement of 5789:

$$\begin{array}{r l} 9999 & \\ -5789 & \\ \hline 4210 & \text{(nine's complement)} \\ +\quad 1 & \\ \hline 4211 & \text{(ten's complement)} \end{array}$$

EXAMPLE 24–36

Subtract 676 from 835 using the complement method.

SOLUTION Find the ten's complement of the subtrahend; add this number to the minuend and drop the carry.

$$\begin{array}{r l} 999 & \\ -676 & \\ \hline 323 & \text{(nine's complement)} \\ +\ 1 & \\ \hline 324 & \text{(ten's complement)} \end{array}$$

$$\begin{array}{r l} 835 & \text{(minuend)} \\ 324 & \text{(ten's complement of 676)} \\ \hline \cancel{1}159 & \text{(difference = 159)} \end{array}$$

EXAMPLE 24–37

Subtract 4678 from 6392 using the complement method.

SOLUTION

$$\begin{array}{r} 9999 \\ -4678 \\ \hline 5321 \\ +\quad 1 \\ \hline 5322 \end{array}$$

$$\begin{array}{r l} 6392 & \text{(minuend)} \\ +\ 5322 & \text{(ten's complement of 4678)} \\ \hline \cancel{1}1714 & \text{(difference = 1714)} \end{array}$$

24–12–1 One's Complement

The one's complement of a binary number is similar to the nine's complement of a decimal number. The binary number to be complemented is subtracted from a binary number made up of all 1's. In the following example the one's complement of 10110010 is found.

EXAMPLE 24–38

$$\begin{array}{r r r r r r r r r l} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \\ - & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & \\ \hline & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & \text{(one's complement of 10110010)} \end{array}$$

Careful inspection reveals that the one's complement of a binary number may be produced by changing all the ones in the number to zeros and changing all of the zeros to ones.

1 0 1 1 0 0 1 0 (invert this number to get the one's complement)
0 1 0 0 1 1 0 1

The computer is able to perform this operation easily by means of a circuit called an *inverter*. Notice that carries or borrows are not involved in this operation.

To subtract by using the one's complement method, the one's complement of the subtrahend is added to the minuend. The carry out of the highest bit position is then added to the *lowest bit position* (LBP). This is called *end-around carry*.

EXAMPLE 24–39

Find the difference between 10110011 and 01101101:

SOLUTION

$$\begin{array}{rr} \text{(minuend)} & 10110011 \\ \text{(subtrahend)} & \underline{01101101} \end{array} \qquad \begin{array}{rl} 1 & \\ 10110011 & \\ \underline{10010010} & \text{(one's complement)} \\ 101000101 & \\ \underline{\longrightarrow 1} & \text{(end-around carry)} \\ 01000110 & \text{(difference)} \end{array}$$

EXAMPLE 24–40

Find the difference between 11011000 and 10110011:

SOLUTION

$$\begin{array}{rr} \text{(minuend)} & 11011000 \\ \text{(subtrahend)} & \underline{10110011} \end{array} \qquad \begin{array}{rl} 1 & \\ 11011000 & \\ \underline{01001100} & \text{(one's complement)} \\ 100100100 & \\ \underline{\longrightarrow 1} & \text{(end-around carry)} \\ 00100101 & \text{(difference)} \end{array}$$

24-12-2 Two's Complement

KEY POINT Computers subtract by using the two's complement method.

The two's complement of a number may be found by adding 1 to the one's complement. Let's find the two's complement of 10011101. First, we change all 0s to 1s and all 1s to 0s (one's complement). Then we add 1 as follows:

$$\begin{array}{lrl} 10011101 & 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0 & \text{(one's complement)} \\ (+) & \underline{\qquad\qquad\qquad 1} & \\ & 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1 & \text{(two's complement)} \end{array}$$

To subtract, we obtain the two's complement of the subtrahend and then add to the minuend. A quick way to find the two's complement of a number is to copy all bits starting at the LSB and continuing through the first 1 bit. Then complement the remaining bits. This is the two's complement of the original binary number. Consider the number 10010100. To find the two's complement, copy all the bits starting at the LSB and continuing through the first 1 bit. This gives us the three least significant bits (100). Next, complement the remaining bits. The answer is 01101100.

EXAMPLE 24–41

Let's subtract 01001010 from 01100111:

SOLUTION

(minuend) 01100111 01100111

(subtrahend) 01001010 (+) 10110110 (two's complement)

1 00011101 (difference)

(this carry is ignored)

The carry resulting from the most significant bit position is ignored. There is no end-around carry as there is with the one's complement method. The computer obtains the one's complement by using an inverter. The two's complement is then obtained by starting the addition process with a carry, as shown in Example 24–42.

EXAMPLE 24–42

Subtract 00110110 from 01011011.

SOLUTION

(minuend) 01011011 01011011

(subtrahend) 00110110 11001001 (one's complement)

+ 1 (add 1 to get the two's complement)

1 00100101 (difference)

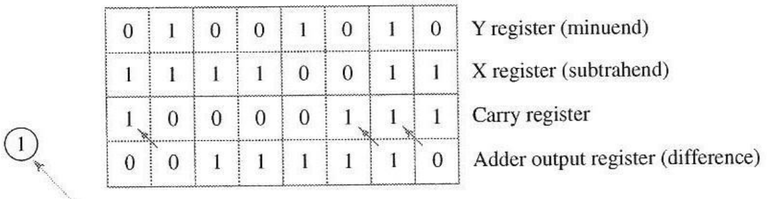
(carry is dropped)

The number of bit positions in the subtrahend must agree with the number of bit positions in the minuend. This is usually accomplished by providing a fixed number of storage cells in the computer. In the preceding examples, eight bit positions were used; therefore, eight storage cells were used to store the numbers. Eight cells were also used to store the difference, causing the carry out of the most significant bit position to be dropped because there was no place to store it. Each of the above eight-cell combinations is called a *register*. Example 24–43 illustrates the concept of registers with fixed numbers of bit positions.

EXAMPLE 24–43

Subtract 00001100 from 01001010 (Figure 24–5).

SOLUTION The minuend is placed in the Y register; the one's complement of the subtrahend is placed in the X register. The LSB is in the rightmost bit position. A 1 is placed in the LSB



0	1	0	0	1	0	1	0	Y register (minuend)
1	1	1	1	0	0	1	1	X register (subtrahend)
1	0	0	0	0	1	1	1	Carry register
0	0	1	1	1	1	1	0	Adder output register (difference)

(Carry out is dropped because there is nowhere to store it)

FIGURE 24–5

position of the carry register. All the other positions of the carry register store a zero at the start. If a carry is produced with the addition of a column, a 1 is stored in the *next* bit position of the carry register. Since the output register is limited to eight bit positions, the carry out of the MSB position is dropped.

PRACTICE PROBLEMS 24-12

For the following decimal numbers, find (a) the nine's complement and (b) the ten's complement:

1. 17
2. 36
3. 88
4. 276
5. 8976

For the following binary numbers, find (a) the one's complement and (b) the two's complement:

6. 1101
7. 1001
8. 100110
9. 110001
10. 11000011

Subtract the following binary numbers by using (a) the one's complement method and (b) the two's complement method:

11. $\begin{array}{r} 11001 \\ \underline{(-)01101} \end{array}$

12. $\begin{array}{r} 101101 \\ \underline{(-)001011} \end{array}$

13. $\begin{array}{r} 1100011 \\ \underline{(-)0011001} \end{array}$

14. $\begin{array}{r} 10110010 \\ \underline{(-)00011101} \end{array}$

15. $\begin{array}{r} 11100100 \\ \underline{(-)00011011} \end{array}$

24-13 SIXTEEN'S AND EIGHT'S COMPLEMENT

When large signed binary numbers are added or subtracted, it is convenient to use the sixteen's complement method. The sixteen's complement is obtained by taking the fifteen's complement and adding one. The fifteen's complement is obtained by substituting each digit with the difference between its value and 15. For example, the sixteen's complement of 3B8 is

$$\begin{array}{rrl} \text{FFF} & \text{C47} & \text{(fifteen's complement)} \\ \underline{(-)\text{3B8}} & \underline{1} & \text{(plus 1)} \\ \text{C47} & \text{C48} & \text{(sixteen's complement)} \end{array}$$

In many systems, octal is used to represent large binary numbers. Therefore, it is convenient to use the eight's complement method for performing signed binary number additions and subtractions. The seven's complement is first obtained by substituting each octal digit with the difference between its value and 7. Then 1 is added to produce the eight's complement. For example, the eight's complement of 136_8 is

$$\begin{array}{rrl} 777 & 641 & \text{(seven's complement)} \\ \underline{(-)136} & \underline{1} & \text{(plus 1)} \\ 641 & 642 & \text{(eight's complement)} \end{array}$$

The following subtraction is performed by using the two's complement method:

$$\begin{array}{rcrl} 0011\ 1100\ 1011 & \longrightarrow & 0011\ \ 1100\ \ 1011 & \text{(minuend)} \\ (-)0010\ 0110\ \underline{0111} & \underline{\longrightarrow} & \underline{1101\ \ 1001\ \ 1001} & \text{(two's complement)} \\ \text{(carry is dropped)} & \longrightarrow & \cancel{1}\ \ 0001\ \ 0110\ \ 0100 & \text{(difference)} \end{array}$$

The same problem may be solved by using the sixteen's complement method:

$$\begin{array}{rrrl} \text{3CB} & & \text{3CB} & \text{(minuend)} \\ \underline{(-)267} & & \underline{\text{D99}} & \text{(subtrahend in sixteen's complement form)} \\ & \cancel{1} & 164 & \text{(difference)} \end{array}$$

(carry is dropped)

If the above binary numbers are expressed in octal, the eight's complement method may be used:

$$\begin{array}{rrrl} 1713_8 & & 1713 & \text{(minuend)} \\ \underline{(-)1147_8} & & \underline{6631} & \text{(subtrahend in eight's complement form)} \\ & \cancel{1} & 0544 & \text{(difference)} \end{array}$$

(carry is dropped)

PRACTICE PROBLEMS 24–13

For the following octal numbers, find (a) the seven's complement and (b) the eight's complement:

1. 6
2. 12
3. 56
4. 173
5. 726

For the following hexadecimal numbers, find (a) the fifteen's complement and (b) the sixteen's complement.

6. A
7. 2C
8. 87
9. 4A3
10. F209

Perform the following subtractions: (a) in binary by using the two's complement method, (b) in octal by using the eight's complement method, and (c) in hexadecimal by using the sixteen's complement method.

11. $\begin{array}{r} 11010011 \\ \underline{(-)00111101} \end{array}$

12. $\begin{array}{r} 10011101 \\ \underline{(-)00110011} \end{array}$

13. $\begin{array}{r} 10111100 \\ \underline{(-)01001110} \end{array}$

14. $\begin{array}{r} 11110000 \\ \underline{(-)00111100} \end{array}$

15. $\begin{array}{r} 11001111 \\ \underline{(-)10000011} \end{array}$

SELF-TEST 24–9 THROUGH 24–13

Work the following problems by using (a) the straight subtraction method and (b) the complement method:

1. $\begin{array}{r} 11101011_2 \\ \underline{(-)01001101_2} \end{array}$

2. $\begin{array}{r} 11000110_2 \\ \underline{(-)10011011_2} \end{array}$

3. $\begin{array}{r} 73_8 \\ \underline{(-)46_8} \end{array}$

4. $\begin{array}{r} 603_8 \\ \underline{(-)364_8} \end{array}$

5. $\begin{array}{r} \text{F6}_{16} \\ \underline{(-)\text{3A}_{16}} \end{array}$

6. $\begin{array}{r} \text{B03}_{16} \\ \underline{(-)\text{8AE}_{16}} \end{array}$

CHAPTER 24 AT A GLANCE

PAGE	KEY POINT OR RULE	EXAMPLE
492	*Rule 24–1:* In converting from binary to decimal, find the value or weight of the MSB. Work down to the LSB, adding the weight of that position if a 1 is present or a 0 if a 0 is present.	$101_2 = 1 \times 2^2$ $+ 0 \times 2^1$ $+ 1 \times 2^0 = 5_{10}$
496	*Rule 24–2:* In converting from octal to decimal, find the weight of the digit in each position. Add the values of the digits in each position to determine the decimal equivalent.	$473_8 = 4 \times 8^2 +$ $7 \times 8^1 +$ $3 \times 8^0 = 315_{10}$
499	*Rule 24–3:* In converting from hexadecimal to decimal, find the weight of the digit in each position. Add the values of the digits in each position to determine the decimal equivalent.	$2BC_{16} = 2 \times 16^2$ $+ 11 \times 16^1$ $+ 12 \times 16^0 = 700_{10}$
500	*Key Point:* To convert from binary to octal, separate the bits into 3-bit groups, starting with the LSB and moving left to the MSB.	$10110110_2 = 266_8$
501	*Key Point:* To convert from binary to hexadecimal, separate the bits into 4-bit groups, starting with the LSB and moving left to the MSB.	$10011011_2 = 9B_{16}$
514	*Key Point:* Computers subtract by using the two's complement method. The two's complement of the subtrahend is added to the minuend. The carry out of the MSB is ignored.	$01100111 - 01001010$ Solution 01100111 $\underline{(+)10110110}$ (two's complement) $\cancel{1}$ 00011101 (difference)

END OF CHAPTER PROBLEMS 24–1

Convert the following binary numbers to decimal numbers:

1. 111_2
2. 101_2
3. 1010_2
4. 1101_2
5. 1101_2
6. 1000_2
7. 10111_2
8. 11001_2
9. 100110_2
10. 111001_2
11. 111000_2
12. 101010_2
13. 1100101_2
14. 1001101_2
15. 1110111_2
16. 1111111_2
17. 11110000_2
18. 10101010_2
19. 11000011_2
20. 10011001_2

Convert the following decimal numbers to binary numbers:

21. 5
22. 5
23. 12
24. 14

25. 21
26. 31
27. 45
28. 55
29. 59
30. 60
31. 68
32. 84
33. 96
34. 117
35. 135
36. 180
37. 210
38. 254

END OF CHAPTER PROBLEMS 24–2

Convert the following numbers from octal to decimal:

1. 14_8
2. 25_8
3. 77_8
4. 100_8
5. 276_8
6. 676_8
7. 4176_8
8. 6237_8
9. $11{,}714_8$
10. $24{,}576_8$

Convert the following numbers from decimal to octal:

11. 20_{10}
12. 46_{10}
13. 80_{10}
14. 103_{10}
15. 360_{10}
16. 617_{10}
17. 1417_{10}
18. 5916_{10}
19. 3789_{10}
20. 6063_{10}

END OF CHAPTER PROBLEMS 24–3

Convert the following numbers from hexadecimal to decimal:

1. $1A_{16}$
2. $B1_{16}$
3. $4C_{16}$
4. $1AC_{16}$
5. 200_{16}
6. 500_{16}
7. $11AA_{16}$
8. $FADE_{16}$
9. $A02B_{16}$
10. $84AB_{16}$

Convert the following numbers from decimal to hexadecimal:

11. 22_{10}
12. 50_{10}
13. 97_{10}
14. 127_{10}
15. 512_{10}
16. 873_{10}
17. 2700_{10}
18. 5606_{10}
19. 6075_{10}
20. 8088_{10}

END OF CHAPTER PROBLEMS 24–4

Convert the following binary numbers to (a) octal numbers and then (b) hexadecimal numbers.

1. 11001010_2
2. 10110110_2
3. 10011011_2
4. 11100011_2
5. 10000001_2
6. 10101111_2
7. 110011001001_2
8. 110011001100_2
9. 101011000011_2
10. 110100011010_2

Convert the following numbers to binary numbers.

11. 17_8
12. 25_8
13. 44_8
14. 61_8
15. 200_8
16. 400_8
17. 560_8
18. 777_8
19. 1000_8
20. 1102_8
21. C_{16}
22. E_{16}
23. 24_{16}
24. 40_{16}
25. $4F_{16}$
26. $C4_{16}$
27. $E3_{16}$
28. $B7_{16}$
29. $19A_{16}$
30. $2EF_{16}$

END OF CHAPTER PROBLEMS 24–5

1. $75_8 = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
2. $112_8 = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
3. $140_8 = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
4. $165_8 = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
5. $2A_{16} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10}$
6. $6D_{16} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10}$
7. $1A7_{16} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10}$
8. $3EC_{16} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10}$
9. $100110_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
10. $101101_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
11. $111000101_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
12. $101111011011_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{10} = \underline{\hspace{3cm}}_{16}$
13. $10_{10} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{16}$
14. $200_{10} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{16}$
15. $290_{10} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{16}$
16. $433_{10} = \underline{\hspace{3cm}}_2 = \underline{\hspace{3cm}}_8 = \underline{\hspace{3cm}}_{16}$

END OF CHAPTER PROBLEMS 24–6

Add the following octal numbers:

1. $\begin{array}{r}123\\ \underline{234}\end{array}$
2. $\begin{array}{r}451\\ \underline{116}\end{array}$
3. $\begin{array}{r}456\\ \underline{317}\end{array}$
4. $\begin{array}{r}517\\ \underline{126}\end{array}$
5. $\begin{array}{r}667\\ \underline{107}\end{array}$
6. $\begin{array}{r}273\\ \underline{706}\end{array}$
7. $\begin{array}{r}604\\ \underline{617}\end{array}$
8. $\begin{array}{r}376\\ \underline{412}\end{array}$
9. $\begin{array}{r}726\\ \underline{161}\end{array}$
10. $\begin{array}{r}571\\ \underline{707}\end{array}$

END OF CHAPTER PROBLEMS 24–7

Add the following hexadecimal numbers:

1. $\begin{array}{r}\mathrm{4A}\\ \underline{\mathrm{A3}}\end{array}$
2. $\begin{array}{r}\mathrm{D2}\\ \underline{\mathrm{1C}}\end{array}$

3. A07
29E

4. B38
398

5. 4AB
8AB

6. C37
8B9

7. ABF
CDE

8. FED
ABC

9. F0C6
E9A6

10. BA09
F6E4

END OF CHAPTER PROBLEMS 24–8

Add the following binary numbers:

1. 1100
1001

2. 1011
1111

3. 11100
10111

4. 10011
11101

5. 10011
10110

6. 10111
11101

7. 110011
101111

8. 111001
011101

9. 101110
111011

10. 111101
101111

END OF CHAPTER PROBLEMS 24–9

Subtract the following octal numbers:

1. 73
(−)26

2. 64
(−)40

3. 616
(−)554

4. 624
(−)267

5. 540
(−)273

6. 405
(−)267

7. 2004
(−)1445

8. 3001
(−)2653

9. 6334
(−)4617

10. 5503
(−)3573

END OF CHAPTER PROBLEMS 24–10

Subtract the following hexadecimal numbers:

1. A6
(−)8C

2. CA
(−)A9

3. E06
(−)A25

4. C0A
(−)2A9

5. 9A0
(−)3AF

6. 8B4
(−)4E8

7. $\begin{array}{r}\text{C00}\\(-)\underline{\text{103}}\end{array}$

8. $\begin{array}{r}\text{CA0}\\(-)\underline{\text{6A6}}\end{array}$

9. $\begin{array}{r}\text{B029}\\(-)\underline{\text{A04D}}\end{array}$

10. $\begin{array}{r}\text{E24A}\\(-)\underline{\text{174C}}\end{array}$

END OF CHAPTER PROBLEMS 24–11

Subtract the following binary numbers:

1. $\begin{array}{r}1011\\(-)\underline{0110}\end{array}$

2. $\begin{array}{r}1101\\(-)\underline{0011}\end{array}$

3. $\begin{array}{r}1101\\(-)\underline{1011}\end{array}$

4. $\begin{array}{r}1101\\(-)\underline{0110}\end{array}$

5. $\begin{array}{r}11001\\(-)\underline{10111}\end{array}$

6. $\begin{array}{r}111011\\(-)\underline{010011}\end{array}$

7. $\begin{array}{r}10010\\(-)\underline{01111}\end{array}$

8. $\begin{array}{r}110110\\(-)\underline{100111}\end{array}$

9. $\begin{array}{r}110011\\(-)\underline{011101}\end{array}$

10. $\begin{array}{r}111001\\(-)\underline{011001}\end{array}$

END OF CHAPTER PROBLEMS 24–12

Find the two's complement of:

1. 1010_2
2. 10110_2
3. 101101_2
4. 11001110_2

Use the one's complement method to find:

5. $\begin{array}{r}11001_2\\(-)\underline{01100_2}\end{array}$

6. $\begin{array}{r}11100_2\\(-)\underline{11011_2}\end{array}$

7. $\begin{array}{r}101101_2\\(-)\underline{001010_2}\end{array}$

8. $\begin{array}{r}1101001_2\\(-)\underline{0100110_2}\end{array}$

Use the two's complement method to find:

9. $\begin{array}{r}10110_2\\(-)\underline{01011_2}\end{array}$

10. $\begin{array}{r}10011_2\\(-)\underline{01000_2}\end{array}$

11. $\begin{array}{r}1011001_2\\(-)\underline{0010110_2}\end{array}$

12. $\begin{array}{r}11100101_2\\(-)\underline{10110010_2}\end{array}$

END OF CHAPTER PROBLEMS 24–13

Find the eight's complement of:

1. 64_8
2. 436_8
3. 635_8
4. 3732_8

Find the sixteen's complement of:

5. A3_{16}
6. 4C3_{16}
7. C1E_{16}
8. 3E04_{16}

Use the eight's complement method to find:

9. $\begin{array}{r}63_8\\(-)\underline{25_8}\end{array}$

10. $\begin{array}{r}503_8\\(-)\underline{236_8}\end{array}$

11. $\begin{array}{r} 626_8 \\ (-)\underline{377_8} \end{array}$

12. $\begin{array}{r} 6026_8 \\ (-)\underline{3662_8} \end{array}$

Use the sixteen's complement method to find:

13. $\begin{array}{r} \mathrm{A2}_{16} \\ (-)\underline{\mathrm{8C}_{16}} \end{array}$

14. $\begin{array}{r} \mathrm{E4D}_{16} \\ (-)\underline{\mathrm{AAB}_{16}} \end{array}$

15. $\begin{array}{r} \mathrm{6A2}_{16} \\ (-)\underline{\mathrm{4FF}_{16}} \end{array}$

16. $\begin{array}{r} \mathrm{F40B}_{16} \\ (-)\underline{\mathrm{6BDD}_{16}} \end{array}$

CHAPTER 25

Boolean Algebra

CHAPTER OBJECTIVES

In this chapter you will learn how to:

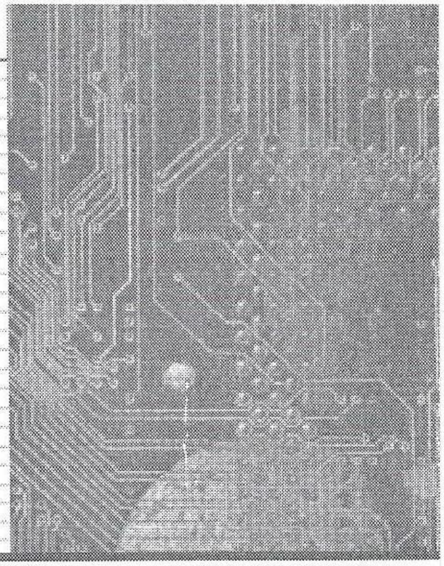
1. Identify the three basic logic operations.
2. Draw the logic symbols and construct truth tables for the three basic logic operations.
3. Write the Boolean expression for various logic circuits.
4. Use Boolean postulates and theorems to simplify logic expressions.

MATH IN ACTION

"I think the numbers on the display are wrong," said Pat. "Is there an easy way to test it?" Sal looked at the circuit and said, "I see it's a seven-segment display and this chip is the decoder/driver. A binary number is applied to these pins, and the chip determines which segments of the seven-segment display must light up to show the correct number in decimal form. You could test it by applying a binary number to the input of the decoder/driver, and making sure the right decimal number shows up on the display." "Why do they use binary numbers?" asked Pat. "It's the number system used by computers and microcontrollers," answered Sal. "Binary is the natural choice for computers because they use transistors that can be quickly switched between their ON and OFF states and these two states can represent the two symbols or numbers used in the binary number system. A typical computer may represent a binary 1 by switching on a transistor to provide a 5-volt signal, and when the transistor is in its OFF state the signal will be 0 volts." "So I can apply 5 volts or 0 volts to the inputs to simulate a binary number; but how do I know what decimal number should appear on the display?" asked Pat. "You need to convert the binary number to a decimal number," explained Sal. "Suppose you put in the binary number 1001. Just like decimal numbers, the actual value depends on where the bit, the binary digit, is in the number. In decimal numbers we have a ones column, a tens column, a hundreds column, and so on. With binary numbers, it's the same idea, but instead we have a ones column, a twos column, a fours column, and, in this case, an eights column. Notice that there's a 1 in the eights column. That means it has a value of eight, just like a 1 in the hundreds column of a decimal number has the value of one hundred." "I see," said Pat. "There's also a 1 in the ones column, so that should have a value of one. So the decimal value of the number will be nine." "That's right," said Sal. "And knowing how to do conversions between binary and decimal numbers will come in handy at other times too."

INTRODUCTION

In today's world, technicians must have some background in digital logic circuits. In Chapter 24 we introduced the various kinds of number systems used in computers. In this chapter we will examine the basic building blocks of any logic system, the AND,



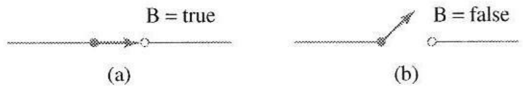


FIGURE 25–1 SPST switch: (a) closed; (b) open.

OR, and NOT functions. We will learn how to relate between logic diagrams and logic expressions. We will simplify logic circuits and construct truth tables to verify the accuracy of our simplifications.

25–1 LOGIC GATES

25–1–1 Logic Variables

Boolean algebra is used in manipulating logic variables. A variable is either completely true or completely false; partly true or partly false values are not allowed. When a variable is not true, by implication it must be false. Conversely, if the variable is not false, it must be true. Because of this characteristic, Boolean algebra is ideally suited to variables that have two states or values, such as YES and NO, or for a number system that has two values, 1 and 0 (for example, the binary number system). For example, B (a variable) could represent the presence of Bob. B has two values: if Bob is present, B equals "true"; if Bob is absent, B equals "false." Note that Bob is not the variable; B is the variable that represents the presence of Bob.

A switch is ideally suited to represent the value of any two-state variable because it can only be "off" or "on." Consider the SPST switch in Figure 25–1. When the switch is in the closed position, it indicates that Bob is present (B = true). When it is in the open position, it then represents that Bob is absent (B = false).

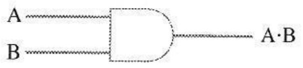
A closed switch could also represent values such as true, yes, one (1), HIGH (H), go, and so on; an open switch could represent values such as false, no, zero (0), LOW (L), no go, and so on.

In digital circuits the TRUE state of a logic variable is usually indicated by the presence of 5 volts. The FALSE state is usually indicated by zero volts. There are a variety of terms that are used, instead of TRUE and FALSE, depending on the application. Table 25–1 shows examples of the synonyms for TRUE and FALSE used in industry.

Table 25–1 Synonyms for TRUE and FALSE.

TRUE	T	YES	ON	CLOSED	HIGH	1	5 V
FALSE	F	NO	OFF	OPEN	LOW	0	0 V

FIGURE 25–2 Two-input AND gate: (a) logic symbol; (b) truth table.



(a)

Condition	A	B	A·B
1	F	F	F
2	F	T	F
3	T	F	F
4	T	T	T

(b)

25-1-2 Logic Operations

There are only three basic logic operations:

1. The *conjunction* (logical product), commonly called AND, symbolized by (·). For example: A · B.
2. The *disjunction* (logical sum), commonly called OR, symbolized by (+). For example: A + B.
3. The *negation*, commonly called NOT, symbolized by ($\overline{\ \ }$). For example: $\overline{A}$.

These operations are performed by logic circuits. All functions within a computer can be performed by combinations of these three basic logic operations.

25-1-3 The AND Function

KEY POINT *For the AND function, the output is true only when ALL inputs are true. Otherwise, the output is false.*

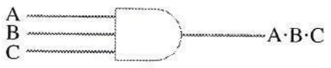
Figure 25–2 shows the logic symbol and truth table for a two-input AND gate. A and B are the input variables. The function (A · B) is true when A is true AND B is true. That is, if both inputs are true, the output is true. If either input, or both, is false, the output is false. To show all the conditions that may exist at the input and output of a logic gate, truth tables are used. Since there are two inputs and each input has two possible states (true and false), the number of possible conditions at the input would be 2 raised to the second power (2^2), or 4. Note in the truth table that condition 4 is the only one in which all the inputs are true so that the AND function is produced and a true output appears. If A is true AND B is true, the output will be true.

AND gates may have two or more inputs. Figure 25–3 shows a three-input AND gate and its truth table. The function (A · B · C) is produced when A is true AND B is true AND C is true. Ones (1) and zeros (0) are used for the values of the variables (true = 1 and false = 0). Because three variables are used, eight possible conditions exist ($2^3 = 8$). Condition 8 is the only one that will produce a true output (1) because all the input variables are true. Remember, in an AND gate, *all* the inputs must be true in order to produce a true output.

The (·) symbol used in the expression A · B · C is the AND operator and indicates logical multiplication. For example, condition 8 in Figure 25–3 can be interpreted as $1 \times 1 \times 1 = 1$, whereas condition 7 can be interpreted as $1 \times 1 \times 0 = 0$.

Using the same approach, we find that all other conditions (1 through 6) also produce a zero. It is important to remember that in the binary system a 2 cannot exist, only 1s and 0s. As with

FIGURE 25–3 Three-input AND gate: (a) logic symbol; (b) truth table.



(a)

Condition	A	B	C	A·B·C
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	0
5	1	0	0	0
6	1	0	1	0
7	1	1	0	0
8	1	1	1	1

(b)

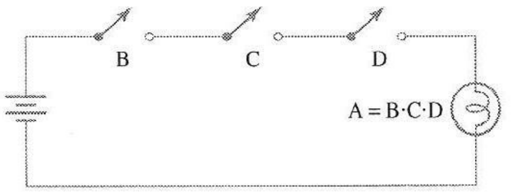


FIGURE 25–4 Switches connected in series to produce the AND function.

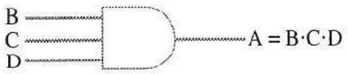


FIGURE 25–5 Logic symbol for the AND function BCD.

ordinary algebra, the operator symbol (·) can be omitted; thus, $A \cdot B \cdot C = ABC$ and is read "A AND B AND C."

The AND function can be illustrated by the following analogy. The members of group A are Bob, Charley, and Dick. Note that the names in this group are combined by the conjunction "and." That means group A equals the presence of Bob AND Charley AND Dick. This may be symbolized as

$$A = BCD$$

A is true (group A is present) when B is true (Bob is present), AND C is true (Charley is present), AND D is true (Dick is present). A is not true if any one or more of the members are absent.

The circuit in Figure 25–4 can be used to produce the AND function of the above example. The light indicates that group A is present only when all members of the group are present. (All switches are closed.) A logic circuit producing this AND function is symbolized in Figure 25–5.

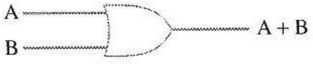
25–1–4 The OR Function

KEY POINT For the OR function, the output is true when any one of the inputs is true. Otherwise, the output is false.

Figure 25–6 shows the logic symbol and truth table for a two-input OR gate. The function $(A + B)$ is produced when A is true OR B is true. That is, if either input is true, the output is true. If both inputs are false, the output is false. Because there are two input variables, a total of four conditions is possible ($2^2 = 4$). Note in the truth table that conditions 2, 3, and 4 produce a true output because at least one of the input variables is true. Condition 1 produces a false output because neither of the input variables is true.

OR gates may have two or more inputs. Figure 25–7 shows a three-input OR gate and its truth table. Ones (1) and zeros (0) are used for the values of the variables. Because three variables are used, eight possible conditions occur ($2^3 = 8$). Conditions 2 through 8 produce a true (1) output because at least one of the input variables is true (1). Condition 1 is the only condition that does not produce a true output.

The + symbol used in the expression $A + B + C$ is the OR operator and indicates logical addition. In logical addition $1 + 0 = 1$ and $1 + 1 = 1$. One OR one does not equal two! Condition 8 in Figure 25–7 may be interpreted as $1 + 1 + 1 = 1$, whereas condition 5 can be interpreted as $1 + 0 + 0 = 1$. Condition 1 produces a zero output because $0 + 0 + 0 = 0$.



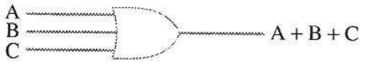
(a)

Condition	A	B	A + B
1	F	F	F
2	F	T	T
3	T	F	T
4	T	T	T

(b)

FIGURE 25–6 Two-input OR gate: (a) logic symbol; (b) truth table.

FIGURE 25–7
Three-input OR gate:
(a) logic symbol;
(b) truth table.



(a)

Condition	A	B	C	A + B + C
1	0	0	0	0
2	0	0	1	1
3	0	1	0	1
4	0	1	1	1
5	1	0	0	1
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

(b)

The OR function is illustrated by the following analogy. The members of group A are Bob, Charley, and Dick. A representative of this group could be Bob OR Charley OR Dick, or any combination of them. This expression may be symbolized as

$$R = B + C + D$$

where R is a representative of group A. R is true (a representative of group A is present) when B is true (Bob is present) OR C is true (Charley is present) OR D is true (Dick is present). Only one of the members needs to be present for group A to be represented. However, group A is also represented when more than one member is present. Group A will not be represented (false) when all members are absent.

The circuit in Figure 25–8 can be used to produce the OR function of the above example. The light (R) indicates that a representative of group A is present when one OR more members of the group are present. (At least one switch is closed.) A logic circuit producing this OR function is symbolized in Figure 25–9.

25–1–5 The NOT Function

KEY POINT For the NOT function, the output is the complement of the input.

The concept of the NOT function can be illustrated by the circuit in Figure 25–10. When the switch is closed, the indicator lamp lights (true). Opening the switch will break the circuit and the lamp will go out (false). Only two conditions may exist. Either the lamp is on (true) or it is off (false). No other condition may exist. The conditions are *complements* of each other.

A logic circuit producing the NOT function is called an *inverter;* its symbol appears in Figure 25–11. An inverter converts the state or value of a variable to its complement. Thus, if variable A appears at the input, $\overline{A}$ (not A) is produced at the output; and, conversely, when $\overline{A}$ appears at the input, A is produced at the output. When performing the NOT function, a 1 will be changed to a 0, and vice versa, as shown in the truth table.

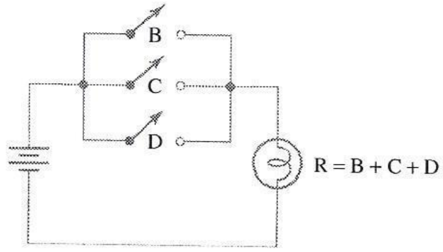


FIGURE 25–8 Switches connected in parallel to produce the OR function.

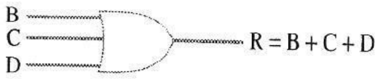


FIGURE 25–9 Logic symbol for the OR function B + C + D.

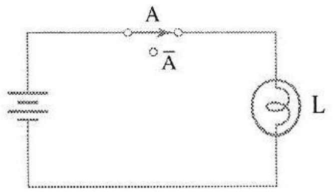


FIGURE 25–10 Representation of the NOT function.

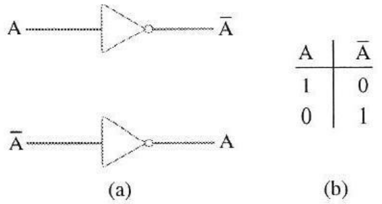


FIGURE 25–11 Inverter: (a) logic symbol; (b) truth table.

25–1–6 Boolean Expressions

The three basic logic functions discussed, AND, OR, and NOT, either individually or in various combinations, are the basic building blocks for all computer logic circuits. A few of these combinations will be illustrated by our group A analogy consisting of the presence of Bob (B), Charley (C), and Dick (D). Suppose we wish to describe a situation in which the group (A) is represented by the presence of Bob (B) and Charley (C), but not Dick (D). This situation could be expressed as $X = BC\overline{D}$, which states that Bob and Charley are present but Dick is NOT present; it is illustrated in Figure 25–12. Note the use of the inverter.

How would you describe a condition in which the group (A) is represented by at least two members—in other words, a majority? This situation could be expressed in Boolean algebra as $X = BC + BD + CD$ and is symbolized in Figure 25–13. The output is true if B AND C is true (Bob AND Charley are present), OR B AND D is true (Bob AND Dick are present), OR C AND D is true (Charley AND Dick are present).

How could the group be represented by the presence of Charley and the absence of both Bob and Dick? One possible method would be $\overline{B}C\overline{D}$, as shown in Figure 25–14.

Suppose we wish to express the situation in which the entire group is NOT present. In other words, one or more members are absent. The Boolean expression would be $\overline{BCD}$ and is illustrated in Figure 25–15. Note that the entire group is affected by the inverter.

Finally, how would we express a situation in which the group is represented by either Bob and Charley or by Dick alone? One possible method is shown in Figure 25–16.

Logic diagrams are drawn to symbolize logic expressions. In the following problems, logic diagrams are to be interpreted or logic diagrams are to be drawn to symbolize logic expressions.

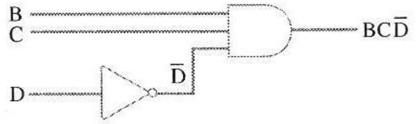


FIGURE 25–12 Logic circuit used to represent the expression $A = BC\overline{D}$.

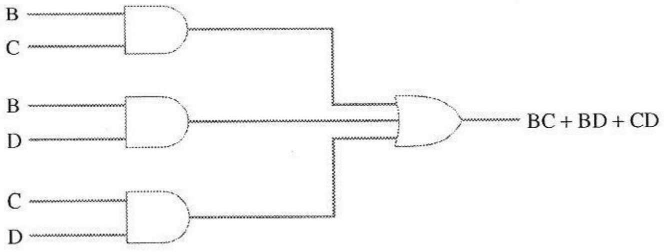


FIGURE 25–13 Logic circuit used to represent the expression $BC + BD + CD$.

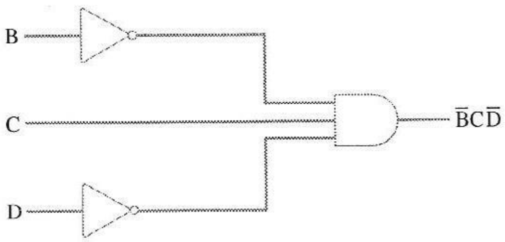


FIGURE 25–14 Logic circuit used to represent the expression $\overline{B}C\overline{D}$.

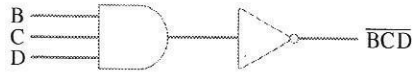


FIGURE 25–15 Logic circuit used to represent the expression $\overline{BCD}$.

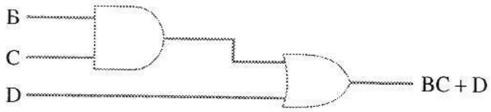


FIGURE 25–16 Logic circuit used to represent the expression BC + D.

EXAMPLE 25–1

Draw the logic diagrams that represent the following Boolean expressions:

1. $B + \overline{C}$
2. $\overline{A}B$
3. $AB + \overline{C}$
4. $A(\overline{B} + C)$
5. $\overline{B + \overline{C} + D}$

SOLUTIONS

1. In problem 1, Figure 25–17, the OR function is indicated; therefore, an OR gate is drawn. To realize the input $\overline{C}$, an inverter is used.
2. In problem 2, Figure 25–17, the AND function is indicated; therefore, an AND gate is drawn. To realize the input $\overline{A}$, an inverter is used.
3. In problem 3, Figure 25–17, both AND and OR functions are indicated. A and B are the inputs to the AND gate. The output of the AND gate (AB) and $\overline{C}$ are the inputs to the OR gate. An inverter is used to produce the input variable $\overline{C}$.
4. In problem 4, Figure 25–17, parentheses are used to indicate logical multiplication as in any algebraic expression. Therefore, a two-input AND gate is indicated. The inputs are A and $\overline{B}$ OR C. The input $\overline{B}$ OR C results from a two-input OR gate whose inputs are $\overline{B}$ and C.
5. In problem 5, Figure 25–17, the entire output function is negated. The bar across the entire expression tells us this. Therefore, an inverter must be connected to the output of the OR gate. At the output of the OR gate is the expression $B + \overline{C} + D$. Because there are three variables, the gate is a three-input OR gate.

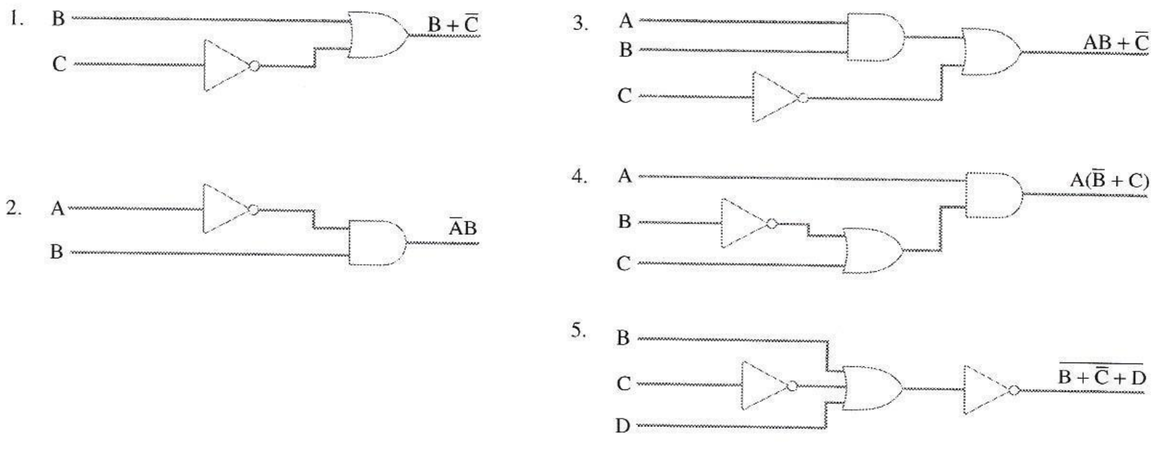


FIGURE 25–17 Answers to Example 25–1.

EXAMPLE 25–2

Write the Boolean expression represented by the logic circuits in Figure 25–18.

SOLUTIONS

1. $F = \overline{AB}$. In problem 1 the AND function is indicated, followed by an inverter. Because the inverter is on the output side of the gate, the entire expression is complemented.

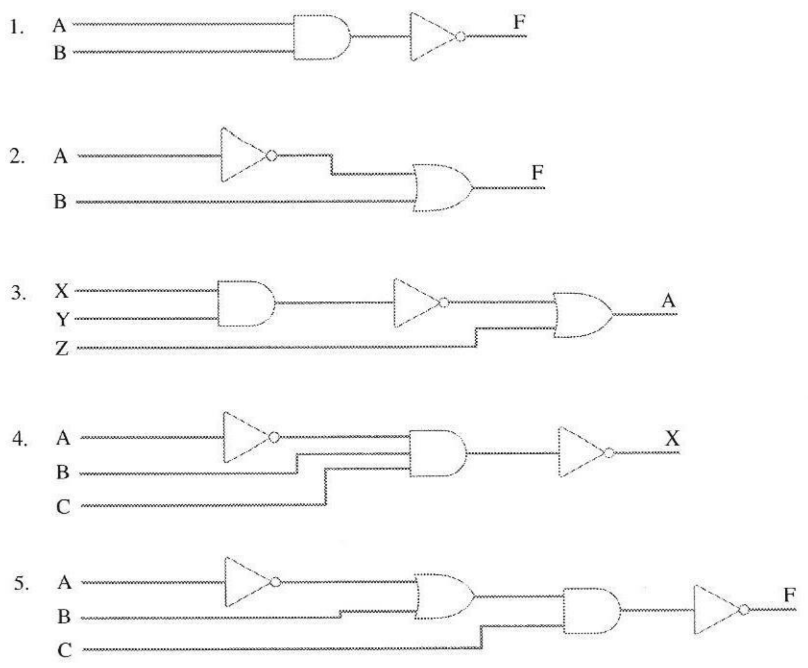


FIGURE 25–18 Logic circuits for Example 25–2.

2. F = $\overline{A} + B$. In problem 2 the inverter complements one input to the OR gate and results in the output $\overline{A} + B$.
3. A = $\overline{XY} + Z$. The inverter complements the output of the AND gate in problem 3 to give the expression $\overline{XY}$. This expression is one input to the OR gate. The other input is Z. The output of the OR gate then is $\overline{XY} + Z$.
4. X = $\overline{\overline{A}BC}$. In problem 4 the three inputs to the AND gate are $\overline{A}$, B, and C. The output is $\overline{A}BC$. This output is complemented by the inverter and results in $\overline{\overline{A}BC}$ at its output.
5. F = $\overline{C(\overline{A} + B)}$. In problem 5 the output of the OR gate $(\overline{A} + B)$ is one input to the AND gate. C is the other input. The output of the AND gate is $C(\overline{A} + B)$. This output is complemented and results in $\overline{C(\overline{A} + B)}$ at the output of the inverter.

PRACTICE PROBLEMS 25–1

In problems 1 through 5, draw the logic diagrams that represent the Boolean expressions. In problems 6 through 10, write the Boolean expression for the logic circuits in Figure 25–20.

1. $A\overline{B}$
2. $\overline{A} + B$
3. $\overline{F + G}$
4. $\overline{C}(B + \overline{D})$
5. $A\overline{B} + C$

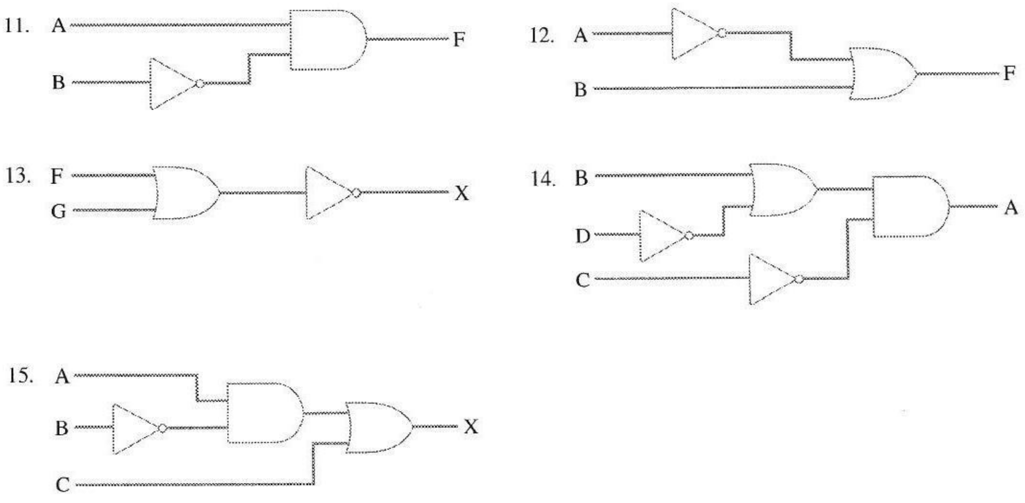
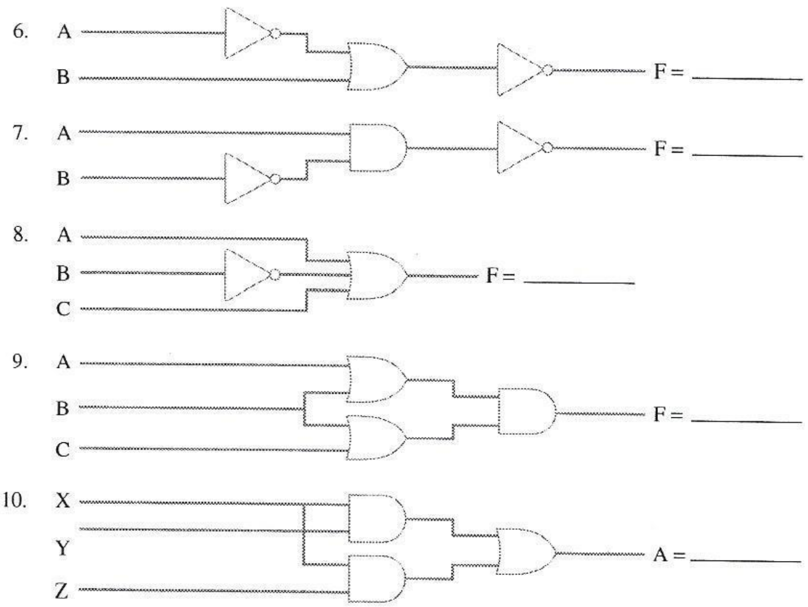


FIGURE 25–19

FIGURE 25–20 Logic circuits for Practice Problems 25–1, problems 6 through 10.

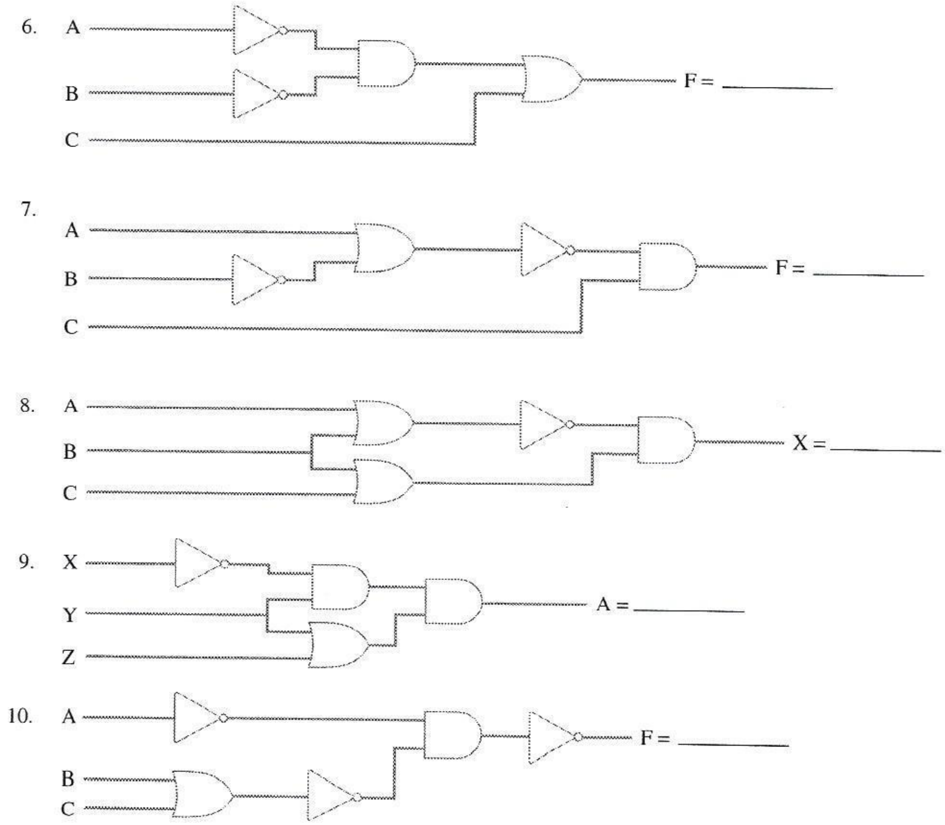


SELF-TEST 25–1

In problems 1 through 5, draw the logic diagrams that represent the following Boolean expressions. In problems 6–10, write the Boolean expressions for the logic circuits in Figure 25–21.

1. $AB + AC$
2. $(X + Z)(X + Y)$
3. $A\overline{B} + \overline{CD}$
4. $\overline{A(B + C)}$
5. $\overline{B\overline{C} + D}$

FIGURE 25–21 Logic circuits for Self-Test 25–1, problems 6 through 10.



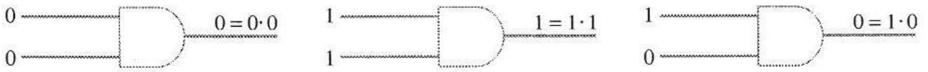


FIGURE 25–22 Logic symbolization of postulates 2a, 3a, and 4a.

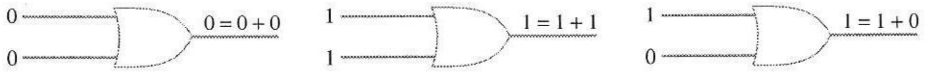


FIGURE 25–23 Logic symbolization of postulates 2b, 3b, and 4b.

FIGURE 25–24 Logic symbolization of postulates 5a and 5b.

25–2 BOOLEAN POSTULATES AND THEOREMS

The following postulates, laws, and theorems are important in the simplification and manipulation of logic expressions. The student is encouraged to become familiar with these because they will be used throughout this chapter. Note that each postulate, law, or theorem is described in two parts. These are *duals* of each other; that is, the dual of the OR operator is the AND, while the dual of a given variable is its complement.

25–2–1 Postulates

Postulates are self-evident truths. Consider postulates 1a and 1b. A variable is either true (1) or false (0). Postulates 2a, 3a, and 4a represent the conjunctive (AND) form and define the function of the AND operator, as shown in Figure 25–22. Postulates 2b, 3b, and 4b represent the disjunctive (OR) form and define the function of the OR operator, as shown in Figure 25–23. Postulates 5a and 5b define the function of the NOT operator, as illustrated in Figure 25–24.

POSTULATES	
1a. $A = 1$ (if $A \neq 0$)	1b. $A = 0$ (if $A \neq 1$)
2a. $0 \cdot 0 = 0$	2b. $0 + 0 = 0$
3a. $1 \cdot 1 = 1$	3b. $1 + 1 = 1$
4a. $1 \cdot 0 = 0$	4b. $1 + 0 = 1$
5a. $\overline{1} = 0$	5b. $\overline{0} = 1$

25–2–2 Algebraic Properties

The following are properties of ordinary algebra that also apply to Boolean algebra. Remember, Boolean expressions contain variables having only two possible values.

ALGEBRAIC PROPERTIES	
Commutative	
6a. $AB = BA$	6b. $A + B = B + A$
Associative	
7a. $A(BC) = AB(C)$	7b. $A + (B + C) = (A + B) + C$
Distributive	
8a. $A(B + C) = AB + AC$	8b. $A + BC = (A + B)(A + C)$

FIGURE 25–25 Logic symbols used to demonstrate the commutative property.

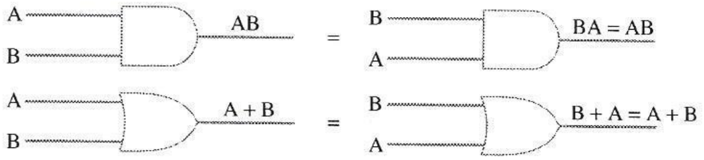
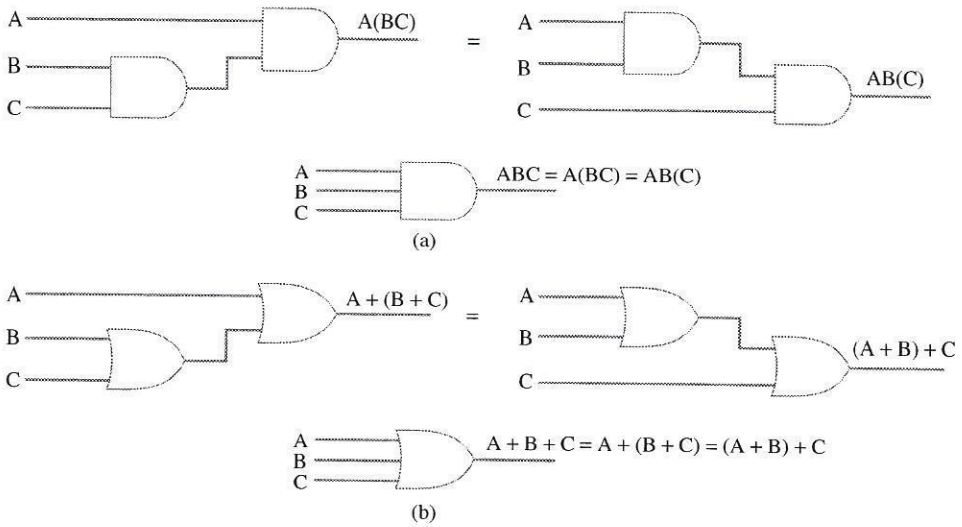


FIGURE 25–26 Logic symbols and circuits used to demonstrate the associative property: (a) logic circuit to demonstrate AND; (b) logic circuit to demonstrate OR.



The *commutative* property simply means that the circuit is not affected by the order or sequence of the variables. This is illustrated in Figure 25–25.

The *associative* property pertains to the parentheses. It shows that a sequence exclusively of AND functions (7a) or a sequence exclusively of OR functions (7b) is not affected by the placement of the parentheses, as indicated in Figure 25–26.

The *distributive* property for 8a and 8b may be proved by performing the algebraic multiplication or by factoring. Figure 25–27 shows that the application of the distributive property produces two forms of an expression and, thus, two circuits may be realized for each.

EXAMPLE 25–3

The distributive property (8a) states that A(B + C) = AB + AC. Prove that they are equal by constructing a truth table.

FIGURE 25–27 Logic symbols and circuits used to demonstrate the distributive property: (a) logic circuit to demonstrate A(B + C) = AB + AC; (b) logic circuit to demonstrate A + BC = (A + B)(A + C).

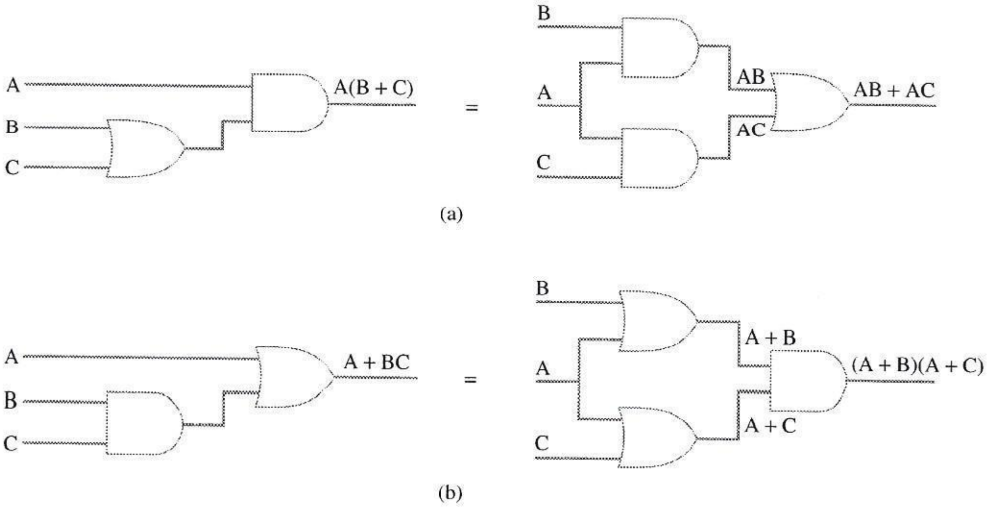
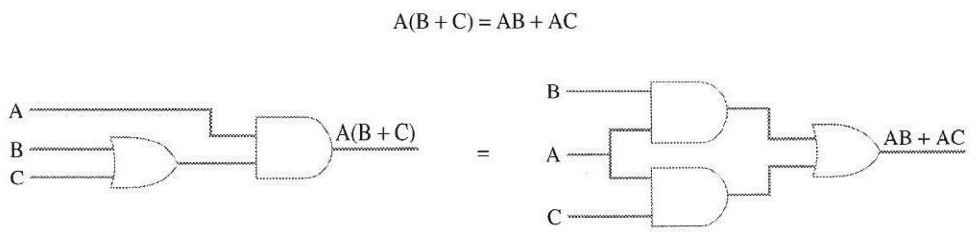


FIGURE 25–28 Logic diagrams and truth table for A(B + C) = AB + AC.



A	B	C	B + C	AB	AC	A(B + C)	AB + AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Equal

SOLUTION The logic diagrams and truth table are shown in Figure 25–28. First, the three variables are listed. Because there are three variables, eight conditions are possible. Next we list each intermediate step. B + C is the result of ORing the input variables B and C. The columns AB and AC are the result of ANDing the input variables A and B and the input variables A and C. Finally, there are columns showing the expressions A(B + C) and AB + AC.

EXAMPLE 25–4

The distributive property (8b) states that A + BC = (A + B)(A + C). Prove that they are equal by constructing a truth table.

SOLUTION The logic diagrams and truth table are shown in Figure 25–29. Notice again how each part of each expression is listed in the truth table.

The process of using truth tables to prove the equality of two expressions is called *proof by perfect induction*. The student is encouraged to use truth tables to evaluate logic expressions or to prove the results of logic operations. Truth tables can also be used to define the functions of various logic circuits.

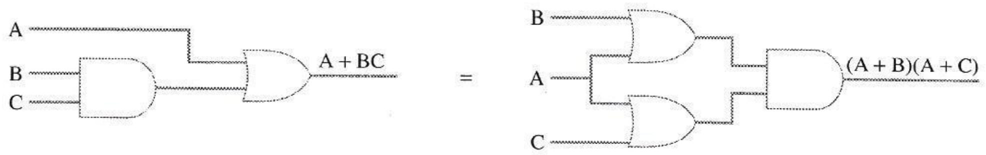
25–2–3 Theorems

The following theorems define the application of the operators to variables:

THEOREMS	
9a. $A \cdot 0 = 0$	9b. $A + 0 = A$
10a. $A \cdot 1 = A$	10b. $A + 1 = 1$
11a. $A \cdot A = A$	11b. $A + A = A$
12a. $A \cdot \overline{A} = 0$	12b. $A + \overline{A} = 1$
13a. $\overline{\overline{A}} = A$	13b. $A = \overline{\overline{A}}$

FIGURE 25–29 Logic diagrams and truth table for A + BC = (A + B)(A + C).

(A + B)(A + C) = A + BC



A	B	C	BC	A + B	A + C	A + BC	(A + B)(A + C)
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Equal

Theorems 9a, 10a, 11a, and 12a pertain to the AND function. These are symbolized in Figure 25–30. For theorem 9a, one of the variables is always a zero. Therefore, the output will be a zero regardless of the value of A. Theorem 10a tells us the output will be determined by the input variable A. If A = 1, then $1 \cdot 1 = 1$. If A = 0, then $1 \cdot 0 = 0$. For theorem 11a, if the input variable A = 1, the output will be $1 \cdot 1 = 1$. If the input variable A = 0, the output will be $0 \cdot 0 = 0$.

For theorem 12a, the output will always be 0, because, when the input variable A = 1, the other input variable will be 0 ($\overline{A}$), causing the output to be $1 \cdot 0 = 0$. The same output will be produced when the values of the input variables are reversed. Theorem 12a is called a *self-contradiction.*

Theorem 13a expresses double negation. An expression that has been inverted twice equals its original value, as shown in Figure 25–31.

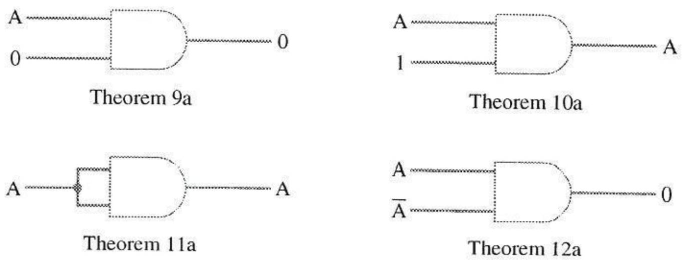
Theorems 9b, 10b, 11b, and 12b pertain to the OR function and are illustrated in Figure 25–32. For the OR function, one or more of the input variables must be true in order that a true output be present. For theorems 9b and 11b, the output is determined by the input variable A. For 10b and 12b, either A or $\overline{A}$ must always equal 1.

Consider the logic diagram in Figure 25–33. The Boolean expression at its output is (AB)(BC). By applying various postulates, laws, and theorems, we can simplify the expression to ABC. Here's how we could do it.

1. (AB)(BC)	Original expression
2. ABBC	7a
3. ABC	11a

Step 1 shows the original expression. In step 2 we remove the parentheses. Our justification for this step is the associative property (7a). The simplified expression in step 3 results from applying theorem 11a. Any variable AND that same variable equals that variable ($B \cdot B = B$). The

FIGURE 25–30 Logic symbolization of the AND function.



Theorem 9a

Theorem 10a

Theorem 11a

Theorem 12a

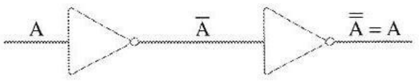
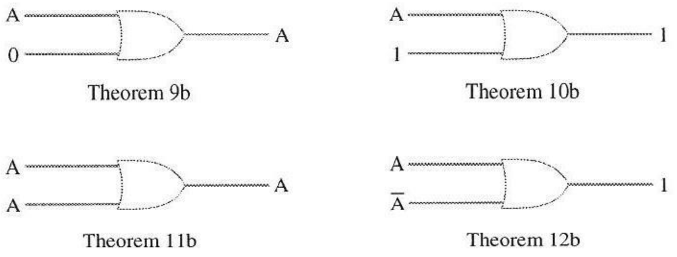


FIGURE 25–31 Logic symbolization of double negation.



Theorem 9b
Theorem 10b
Theorem 11b
Theorem 12b

FIGURE 25–32 Logic symbolization of the OR function.

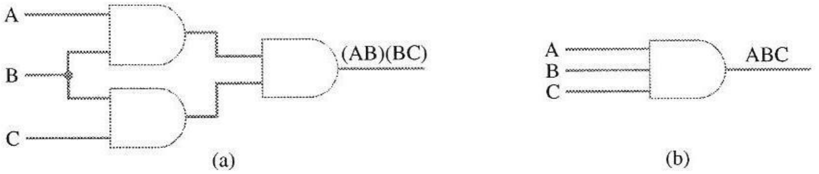
simplified expression results in 3 two-input AND gates being replaced by 1 three-input AND gate, a substantial savings of both space and money to the designer.

Let's prove that the expressions are equal by using a truth table as in Figure 25–33(c). Notice that there are eight conditions. There are three input variables (A, B, and C) and each variable has two possible states ($2^3 = 8$). When we construct a truth table, it is important that we keep the order of the original statement and not skip any steps along the way. Thus, we list the variables first, then we list the function of the first two AND gates (AB and BC), and then we write the original expression and the simplified expression. The truth table shows that $(AB)(BC) = ABC$.

Let's see if it is possible to simplify the logic diagram in Figure 25–34. The step-by-step simplification and the justification for each step are shown by the following:

1.	$A + B + AC + AB$	Original expression
2.	$A + AB + B + AC$	6b
3.	$A(1 + B) + B + AC$	8a
4.	$A(1) + B + AC$	10b
5.	$A + B + AC$	10a
6.	$A + AC + B$	6b
7.	$A(1 + C) + B$	8a
8.	$A(1) + B$	10b
9.	$A + B$	10a

When many steps are involved in the simplification, there is usually more than one way to arrive at the final answer. The student may find alternative methods to the above simplification, but in each case the final simplification should be $A + B$. The original logic diagram, simplified logic diagram, and truth table are found in Figure 25–34.



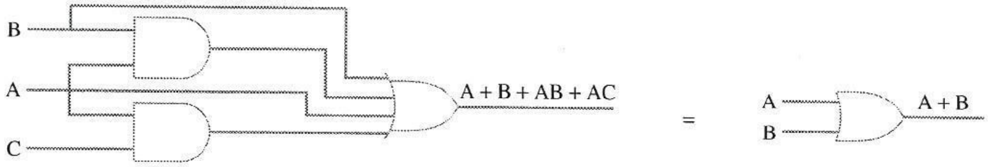
A	B	C	AB	BC	(AB)(BC)	ABC
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

(c)

Equal

FIGURE 25–33 (a) Original circuit; (b) simplified circuit; (c) truth table.

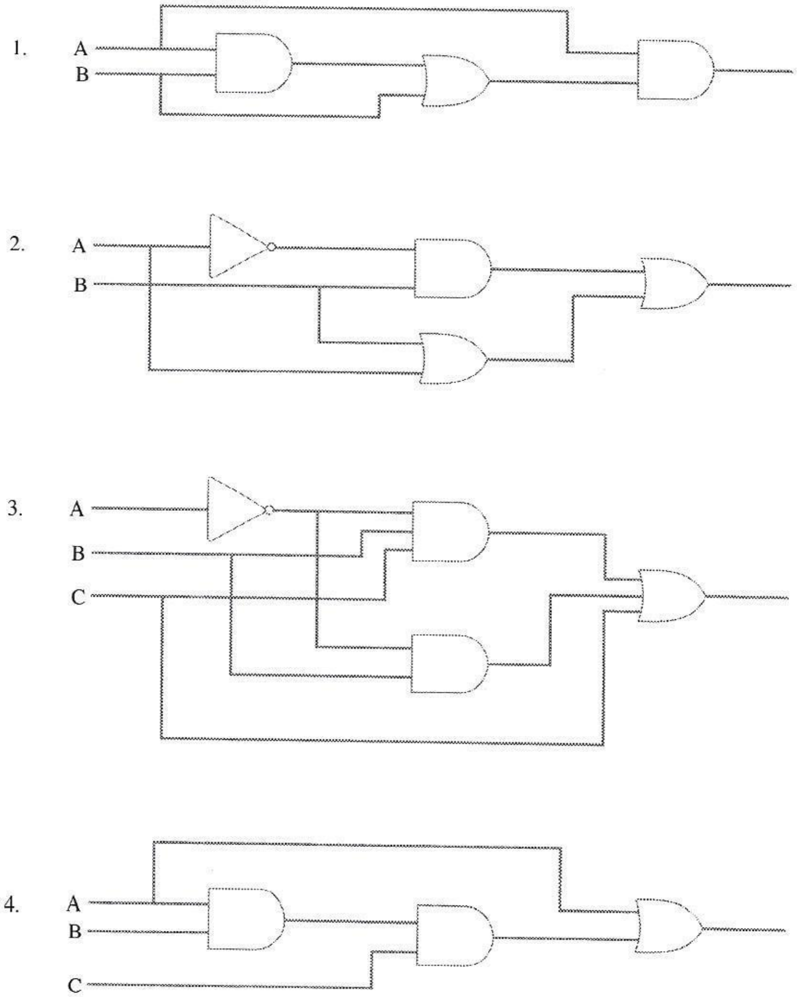
FIGURE 25–34 Logic symbols and truth table for A + B + AB + AC = A + B.



A	B	C	AC	AB	A + B + AC + AB	A + B
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	1	1
1	1	1	1	1	1	1

Equal

FIGURE 25–35 Logic circuits for Practice Problems 25–2.



PRACTICE PROBLEMS 25–2

For each logic diagram in Figure 25–35, (a) write the Boolean expression, (b) simplify the Boolean expression and justify each step, (c) draw the simplified logic diagram, and (d) construct a truth table to prove that the two expressions are equal.

25–3 DE MORGAN'S THEOREM AND THE ABSORPTION THEOREM

25–3–1 De Morgan's Theorem

De Morgan's theorem provides us with an invaluable tool to use in simplifying Boolean expressions. By applying De Morgan's theorem, we can find the equivalent of a Boolean expression.

DE MORGAN'S THEOREM

14a. $\bar{A}\bar{B}\bar{C} = \overline{A + B + C}$ 14b. $\bar{A} + \bar{B} + \bar{C} = \overline{ABC}$

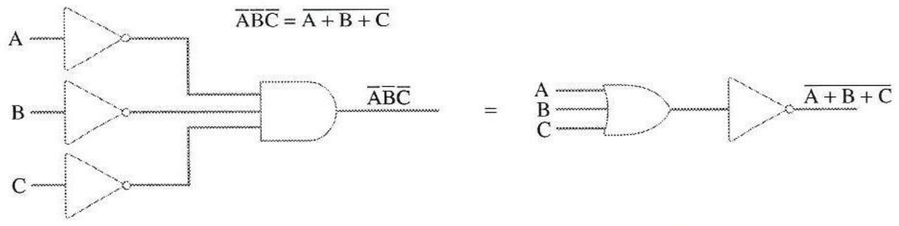
Three steps are required in De Morganizing a Boolean expression: First, we replace all the OR operator symbols (+) with AND operator symbols (•) and all the AND operator symbols with OR operator symbols; next, we replace all variables with their complements; finally, we complement the entire expression.

To De Morganize the expression $\bar{A}\,\bar{B}\,\bar{C}$, in theorem 14a, we use the following steps:

- Step 1. Replace the AND operators with the OR operators: $\bar{A} + \bar{B} + \bar{C}$
- Step 2. Complement each variable: $\bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}} = A + B + C$
- Step 3. Complement the entire expression: $\overline{A + B + C}$

Therefore, $\bar{A}\bar{B}\bar{C} = \overline{A + B + C}$

The logic diagrams and the truth table verifying that they are equal are found in Figure 25–36.

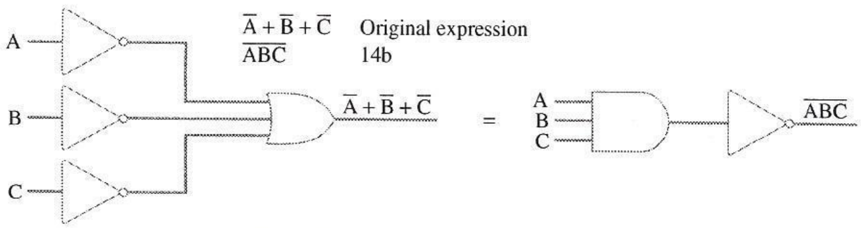


A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A}\bar{B}\bar{C}$	A + B + C	$\overline{A+B+C}$
0	0	0	1	1	1	1	0	1
0	0	1	1	1	0	0	1	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	0	0	1	0
1	0	0	0	1	1	0	1	0
1	0	1	0	1	0	0	1	0
1	1	0	0	0	1	0	1	0
1	1	1	0	0	0	0	1	0

Equal

FIGURE 25–36 Logic diagrams and truth table for $\bar{A}\bar{B}\bar{C} = \overline{A + B + C}$.

FIGURE 25–37 Logic diagrams and truth table for $\overline{A} + \overline{B} + \overline{C} = \overline{ABC}$.


A
B
C
$\overline{A}+\overline{B}+\overline{C}$ Original expression
$\overline{ABC}$ 14b
$\overline{A}+\overline{B}+\overline{C}$
=
A
B
C
$\overline{ABC}$

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A}+\overline{B}+\overline{C}$	ABC	$\overline{ABC}$
0	0	0	1	1	1	1	0	1
0	0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0	1
0	1	1	1	0	0	1	0	1
1	0	0	0	1	1	1	0	1
1	0	1	0	1	0	1	0	1
1	1	0	0	0	1	1	0	1
1	1	1	0	0	0	0	1	0

Equal

To De Morganize the expression $\overline{A} + \overline{B} + \overline{C}$, in theorem 14b, we again use the following three steps:

- Step 1. Replace the OR operators with the AND operators: $\overline{A}\cdot\overline{B}\cdot\overline{C}$
- Step 2. Complement each variable: $\overline{\overline{A}}\cdot\overline{\overline{B}}\cdot\overline{\overline{C}} = A\cdot B\cdot C$
- Step 3. Complement the entire expression: $\overline{A\cdot B\cdot C}$ or $\overline{ABC}$

The logic diagrams and the truth table verifying that they are equal are found in Figure 25–37.

EXAMPLE 25–5

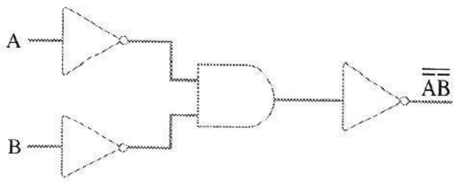
Consider the logic diagram in Figure 25–38. Simplify the expression using De Morgan's theorem, and construct a truth table verifying that the two outputs are equal.

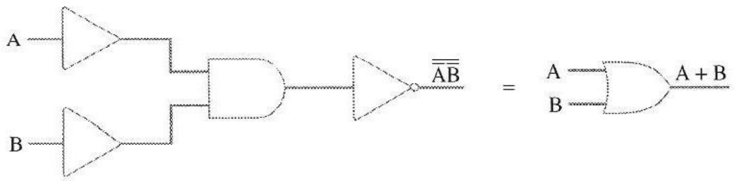
SOLUTION The Boolean expression at the output is $\overline{\overline{A}\,\overline{B}}$. Applying De Morgan's theorem we get:

- Step 1. Replace the AND operator with the OR operator: $\overline{\overline{A} + \overline{B}}$
- Step 2. Complement each variable: $\overline{\overline{\overline{A}} + \overline{\overline{B}}} = \overline{A + B}$
- Step 3. Complement the entire expression: $\overline{\overline{A + B}} = A + B$

$\overline{\overline{A}\,\overline{B}}$ and A + B produce the same logical output.
The logic diagrams and truth table are shown in Figure 25–39. The equivalent logic circuit is much easier to understand and much more simple to construct.

FIGURE 25–38 Logic diagram for the expression $\overline{\overline{A}\,\overline{B}}$.





A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{\overline{A}\,\overline{B}}$	A + B
0	0	1	1	1	0	0
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	0	1	1

Equal

FIGURE 25–39 Logic diagrams and truth table for $\overline{\overline{A}\,\overline{B}}$ = A + B.

EXAMPLE 25–6

Consider the logic diagram in Figure 25–40. Simplify the expression using De Morgan's theorem, and construct a truth table verifying that the two outputs are equal.

SOLUTION The Boolean expression at the output is $\overline{\overline{A} + \overline{B}}$. Applying De Morgan's theorem we get:

- Step 1. Replace the OR operator with the AND operator: $\overline{\overline{A} \cdot \overline{B}}$
- Step 2. Complement each variable: $\overline{\overline{\overline{A}} \cdot \overline{\overline{B}}} = \overline{AB}$
- Step 3. Complement the entire expression: $\overline{\overline{AB}} = AB$

$$\overline{\overline{A} + \overline{B}} = AB$$

The logic diagrams and truth table are shown in Figure 25–41.

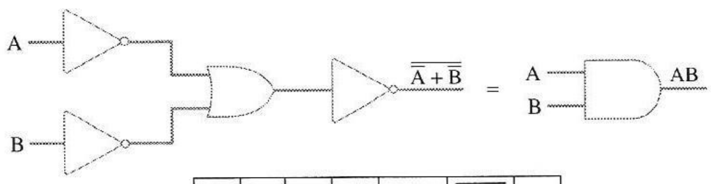
The three-step solution process can be simplified. Consider the expression $\overline{\overline{A} + \overline{B}}$ that we simplified earlier. To simplify, we just break the bar and change the operator.

$$\overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = AB$$

This is an example of theorem 14a. When breaking the bar results in a double negation, we apply theorem 13a ($\overline{\overline{A}} = A$).

EXAMPLE 25–7

Let's simplify the Boolean expression $\overline{\overline{A} + B + \overline{C}}$.

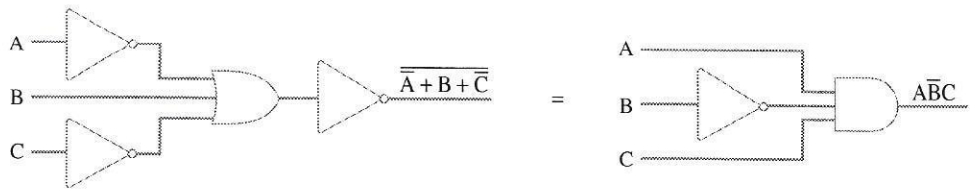


A	B	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$	$\overline{\overline{A}+\overline{B}}$	AB
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	0	1	1

Equal

FIGURE 25–40 Logic diagrams and truth table for $\overline{\overline{A} + \overline{B}}$ = AB.

FIGURE 25–41 Logic diagrams and truth table for $\overline{\overline{A} + B + \overline{C}} = A\overline{B}C$.



A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A}+B+\overline{C}$	$\overline{\overline{A}+B+\overline{C}}$	$A\overline{B}C$
0	0	0	1	1	1	1	0	0
0	0	1	1	1	0	1	0	0
0	1	0	1	0	1	1	0	0
0	1	1	1	0	0	1	0	0
1	0	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	1	0	0
1	1	1	0	0	0	1	0	0

Equal

SOLUTION

$\overline{\overline{A} + B + \overline{C}}$ Original expression

$\overline{\overline{A}} \cdot \overline{B} \cdot \overline{\overline{C}}$ 14b

$A\overline{B}C$ 13a

Notice we broke the bar between each variable and changed each OR operator to AND. The logic diagrams and truth table are shown in Figure 25–41.

Let's simplify the expression $\overline{\overline{A} + \overline{B + C}}$. Again we use De Morgan's theorem since simplification requires removing the signs of negation (the bars). Let's start by breaking the bar between $\overline{A}$ and $\overline{B + C}$ and changing the operator.

$\overline{\overline{A} + \overline{B + C}}$ Original expression

$\overline{\overline{A}} \cdot \overline{\overline{B + C}}$ 14a

$A \cdot (B + C)$ 13a

Or

$$A(B + C)$$

Notice the parentheses (sign of grouping) around B + C. We must treat $\overline{B + C}$ in the original expression as a single term (in algebra the bar is a sign of grouping). Therefore, when we De Morganize, we must include the sign of grouping around B + C to avoid an invalid expression. Without the sign of grouping, we get $A \cdot B + C$ or $AB + C$, which is not the same as $A(B + C)$. We could have included the sign of grouping in the original expression:

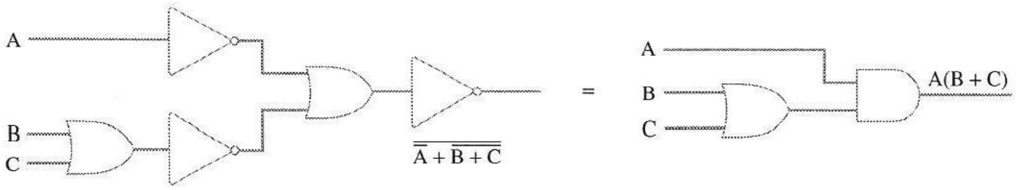
$$\overline{\overline{A} + \overline{B + C}} = \overline{\overline{A} + \overline{(B + C)}}$$

It isn't necessary though because the bar ($\overline{B + C}$) tells us to treat B + C as a single term or variable.

$$A(B + C)$$

We could have De Morganized $\overline{B + C}$ first:

$\overline{\overline{A} + \overline{B + C}}$ Original expression $\overline{\overline{A} + \overline{B}\,\overline{C}}$ 14a



A	B	C	$\overline{A}$	B + C	$\overline{B+C}$	$\overline{A}+\overline{B+C}$	$\overline{\overline{A}+\overline{B+C}}$	A(B + C)
0	0	0	1	0	1	1	0	0
0	0	1	1	1	0	1	0	0
0	1	0	1	1	0	1	0	0
0	1	1	1	1	0	1	0	0
1	0	0	0	0	1	1	0	0
1	0	1	0	1	0	0	1	1
1	1	0	0	1	0	0	1	1
1	1	1	0	1	0	0	1	1

Equal

FIGURE 25–42 Logic diagrams and truth table for $\overline{\overline{A}+\overline{B+C}}$ = A(B + C).

Notice this did not affect the bar that extends over the entire expression since we were De Morganizing just $\overline{B+C}$. Next

$\overline{\overline{A}}\cdot\overline{\overline{\overline{B}\,\overline{C}}}$ 14a (again) $A\cdot\overline{\overline{\overline{B}\,\overline{C}}}$ 13a

$A\cdot(\overline{\overline{B}}+\overline{\overline{C}})$ 14b $A(B+C)$ 13a

We arrive at the same answer by a much more difficult route. Notice the parentheses when theorem 14b was used. Only the expression BC was De Morganized. Therefore, we include the sign of grouping to indicate that B + C is a single variable (in this case one input to an AND gate). Figure 25–42 includes the logic diagrams and truth table.

25-3-2 Absorption Theorem

Sometimes variables appear in a certain pattern that can obviously be simplified. The *absorption* theorem shows how substitution may be used on some commonly derived patterns.

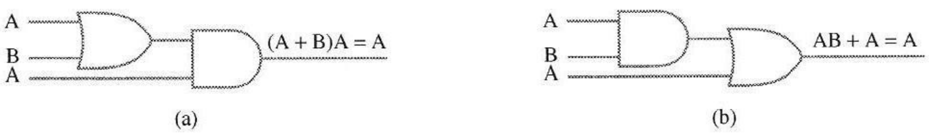
ABSORPTION THEOREM

15a. $A(A+B)=A$ 15b. $A+AB=A$
16a. $A(\overline{A}+B)=AB$ 16b. $A+\overline{A}B=A+B$

Theorems 15a and 15b show that the value of B is redundant. A true output occurs only when A is true, as shown in Figure 25–43.

Theorem 16a states that only when A AND B are true is the statement true, as shown in Figure 25–44. Recall that A and $\overline{A}$ cannot be true at the same time.

Theorem 16b is the dual of theorem 16a and is shown in Figure 25–45. For the expression to be true, either A must be true or B must be true. $\overline{A}$ is redundant.



(a) (b)

FIGURE 25–43 Logic symbolization of (a) theorem 15a and (b) theorem 15b.

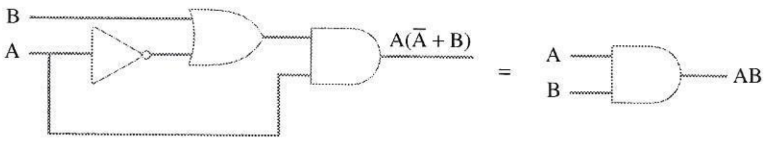


FIGURE 25–44 Logic symbolization of theorem 16a.

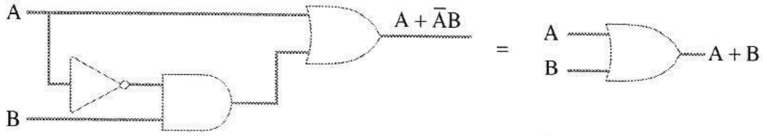
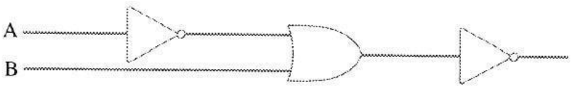


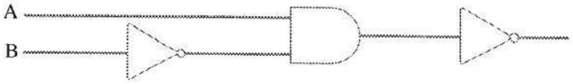
FIGURE 25–45 Logic symbolization of theorem 16b.

FIGURE 25–46 Practice Problems 25–3.

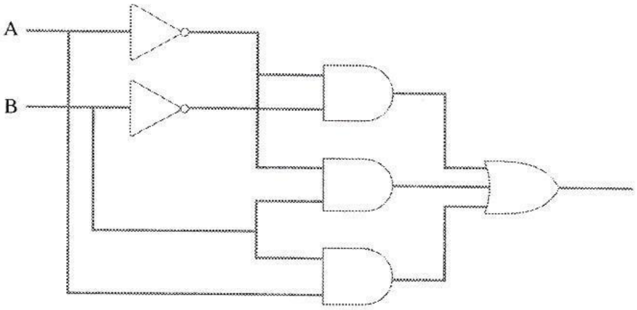
1.



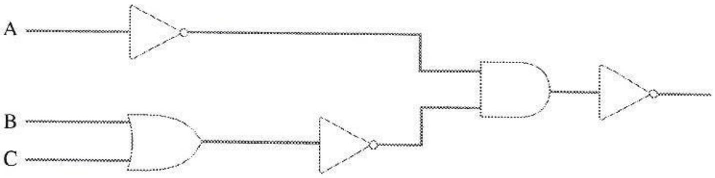
2.



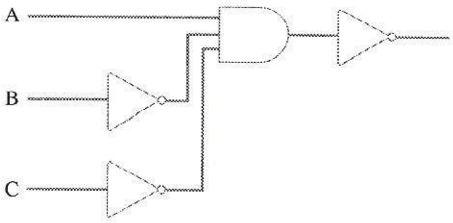
3.



4.



5.



PRACTICE PROBLEMS 25–3

For each logic diagram in Figure 25–46, (a) write the Boolean expression, (b) simplify the Boolean expression and justify each step, (c) draw the simplified logic diagram, and (d) construct a truth table to prove that the two expressions are equal.

25–4 APPLICATION PROBLEMS

Computers use thousands of logic circuits to perform their many functions. Each of these logic circuits can be separated into the basic logic circuits that have been presented in this chapter. We can write a Boolean equation for each of these circuits that will be a complete description of the logic functions that it performs. By combining all the Boolean equations for all the logic circuits in a computer, it is possible to completely describe the logical functions of a computer using Boolean equations. Emulators are software programs containing thousands of Boolean equations that describe complex logic circuits. Intel sells emulators for their microprocessors so that their customers can simulate the operations of Intel microprocessors on a computer. This type of simulation allows computer controlled testing in a fraction of the time it would take for bench testing.

Every logic circuit, no matter how complex, can be separated into simpler logic circuits with a single output. Every logic circuit in this chapter has a single output and we can write a Boolean equation for each of these circuits that takes the form:

Boolean expression using input varibles = one logical output

In the following application problems we must analyze the English words to determine the inputs, the logical operators, and the desired output. The logical operators are described using Boolean expressions. A Boolean equation has the logical output on one side of the equation and Boolean expressions on the other side of the equation.

The sequence of reasoning we can follow to solve these problems is to determine:

1. the logical output desired—this will become the output of the equivalent logic circuit
2. the input conditions, or signals—we can select alphabetic characters to represent these logic variables
3. the logical operations (AND, OR, NOT) that must be performed on the input variables in order to obtain the desired output
4. the Boolean equation

EXAMPLE 25–8

In a home, when a light switch is flipped to the ON position, a lamp will come on if its branch circuit breaker and the main circuit breaker are in the ON position. Write the Boolean equation that represents the condition for the lamp to glow.

SOLUTION First, we can identify that the logical output is that the lamp is glowing and we can specify the output variable to be:

$$L = \text{lamp is glowing}$$

Second, we can specify symbols to represent the input conditions necessary for the lamp to turn on.

M = main circuit breaker is on
B = branch circuit breaker is on
S = light switch is on

Third, we can determine, by analyzing the problem statement, that we must use the AND function for all three inputs. Fourth, we write the Boolean equation: $M \cdot B \cdot S = L$

EXAMPLE 25–9

The interior lights in an automobile are turned on when a door is opened or when the key is in the ignition and the interior light switch is put in the ON position. Write the Boolean equation that represents the conditions for the interior lights to be on.

SOLUTION First, we can identify that the logical output is for the lights to be on. We can specify the output variable to be:

$$L = \text{lights ON}$$

Second, we can specify symbols to represent the input conditions necessary for the interior lights to glow.

D = door is opened
K = key in ignition
S = switch in ON position

Third, we must determine the logical operators by analyzing the problem statement. There are two conditions that can cause the light to come on. Either the door is opened *or* the key is in the ignition *and* the switch is on. By analyzing the use of the words *or* and *and* in the sentence, we can determine that both an AND and an OR operator are needed.

Fourth, we write the Boolean equation: $D + (K \cdot S) = L$.

PRACTICE PROBLEMS 25–4

1. A wall switch and an infrared (IR) detector that can sense the presence of a live body control the lights in a classroom. The wall switch has three positions—ON, AUTO, and OFF. In the ON position the lights are turned on, in the AUTO position the lights are turned on if the infrared detector senses the presence of a body, and in the OFF position the lights are always off. Write the Boolean equation that represents the conditions for the classroom lights to be on.
2. A single remote control can control both a TV and a VCR because of dual function keys. For example, if the TV button is pressed and then the ON button, the TV will turn on; if the VCR button is pressed and then the ON button, the VCR will turn on. Write the Boolean equation that represents the conditions to control the TV and VCR.
3. A calculator has several dual function keys. The key labeled sin is used to calculate the sine of an angle. The same key can also calculate the inverse sine if the SHIFT key is pressed first. Write the Boolean equation that represents the conditions to perform each calculation.
4. The buttons on a computer mouse perform different functions, depending on the location of the mouse pointer. For example, when the mouse pointer is placed on the printer icon and the left mouse button is pressed, the printer will print a copy of the displayed document; if the mouse pointer is placed on the page icon and the left mouse button is pressed, the computer will display the document in the same format as it will be printed. Write the Boolean expression that represents the conditions to perform each function.

SELF-TEST 25–2 THROUGH 25–4

Problems **1** through **5**: for the logic diagrams in Figure 25–47, (a) write the Boolean expression at the output, (b) simplify and justify each step, (c) draw the logic diagram of the simplified expression, and (d) construct a truth table to prove that the expressions are equal.

For the following Boolean expressions, (a) draw the logic diagram, (b) simplify and justify each step, (c) draw the logic diagram of the simplified expression, and (d) construct a truth table to prove that the expressions are equal.

6. $C(\overline{B + D})$
7. $\overline{A\overline{B} + \overline{C}}$
8. $\overline{\overline{A} + \overline{B}} + C$
9. $(AB + BC)(AC + BC)$
10. $\overline{\overline{\overline{A}B + A\overline{C}} + \overline{BC}}$

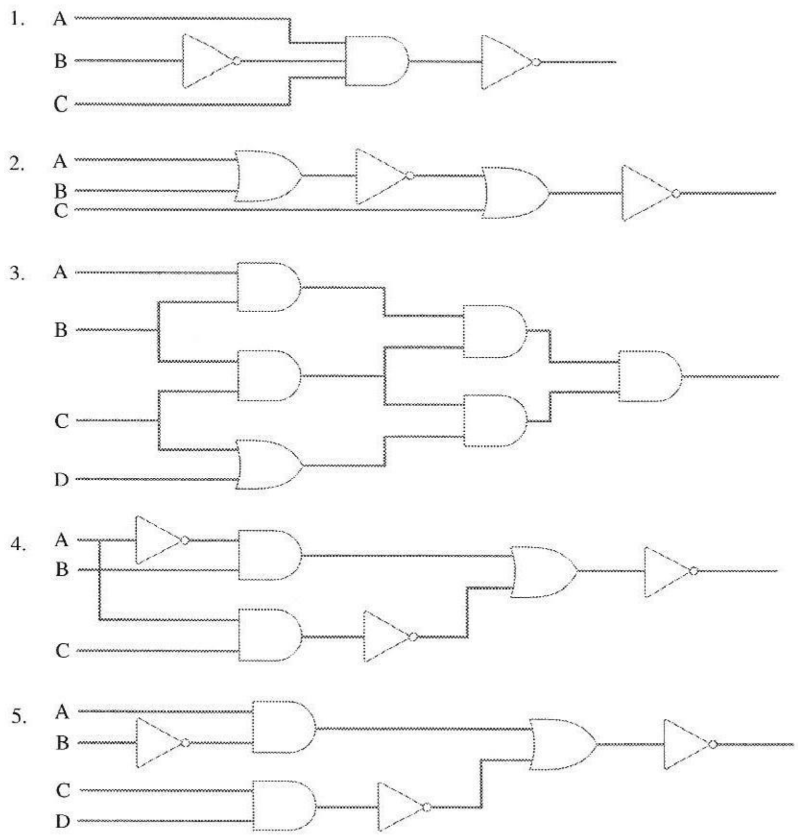
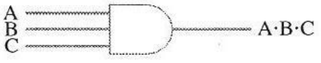


FIGURE 25–47 Problems 1 through 5, Self-test 25–2.

CHAPTER 25 AT A GLANCE

PAGE — **LOGIC FUNCTIONS, POSTULATES, AND THEOREMS** — **LOGIC SYMBOLS AND CIRCUITS**

526 For the AND function, the output is true only when all inputs are true. Otherwise, the output is false.



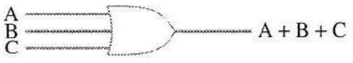
(a)

Condition	A	B	C	A·B·C
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	0
5	1	0	0	0
6	1	0	1	0
7	1	1	0	0
8	1	1	1	1

(b)

FIGURE 25–48 Three-input AND gate: (a) logic symbol; (b) truth table.

527 For the OR function, the output is true when any one of the inputs is true. Otherwise, the output is false.



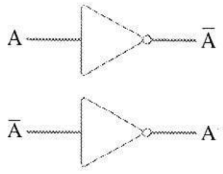
(a)

Condition	A	B	C	A + B + C
1	0	0	0	0
2	0	0	1	1
3	0	1	0	1
4	0	1	1	1
5	1	0	0	1
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

(b)

FIGURE 25–49 Three-input OR gate: (a) logic symbol; (b) truth table.

528 For the NOT function, the output is the complement of the input.



(a)

A	$\overline{A}$
1	0
0	1

(b)

FIGURE 25–50 Inverter: (a) logic symbol; (b) truth table.

533 Postulates, laws, and theorems are used in the simplification and manipulation of logic expressions.

END OF CHAPTER PROBLEMS 25–1

Draw the logic diagrams that represent the following Boolean expressions:

1. $\overline{A + \overline{B}}$
2. $\overline{AB} + A\overline{C}$
3. $ABC + D$
4. $\overline{X}(\overline{Y + Z})$
5. $\overline{A + B\overline{C}}$
6. $\overline{A}B + \overline{AC}$
7. $A\overline{B} + \overline{A}B$
8. $B(\overline{\overline{C} + D})$
9. $\overline{AB} + C + D$
10. $\overline{\overline{B}CD + E}$
11. $AB + A\overline{C} + B\overline{C}$
12. $(B + C)(\overline{C + D})$
13. $\overline{A\overline{B} + BC}$
14. $\overline{\overline{B}(A + \overline{C})}$
15. $\overline{A\overline{B}C} + D$
16. $\overline{A} + \overline{B + C}$
17. $(\overline{X + Y})(\overline{Y + Z})$
18. $\overline{\overline{AC} + \overline{D}}$
19. $\overline{\overline{AB} + C\overline{D}}$
20. $\overline{\overline{A}B + \overline{C}}$

Write the Boolean expression represented by the logic diagrams in Figures 25–51 through 25–53.

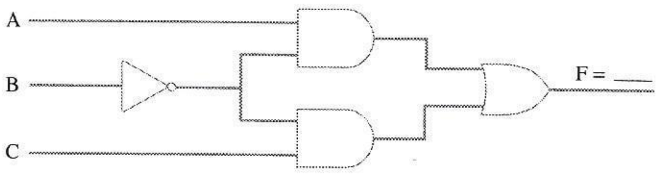
END OF CHAPTER PROBLEMS 25–2

For each logic diagram in Figure 25–54, (a) write the Boolean expression, (b) simplify the Boolean expression and justify each step, (c) draw the simplified logic diagram, and (d) construct a truth table to prove that the two expressions are equal.

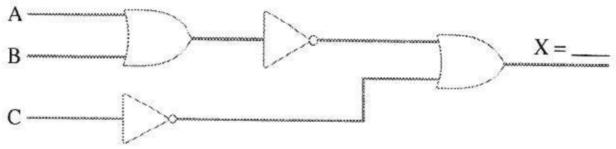
1. See Figure 25–54, circuit 1.
2. See Figure 25–54, circuit 2.
3. See Figure 25–54, circuit 3.
4. See Figure 25–54, circuit 4.
5. See Figure 25–54, circuit 5.
6. See Figure 25–54, circuit 6.

FIGURE 25–51 Logic diagrams for End of Chapter Problems 25–1, problems 21 and 22.

21.



22.



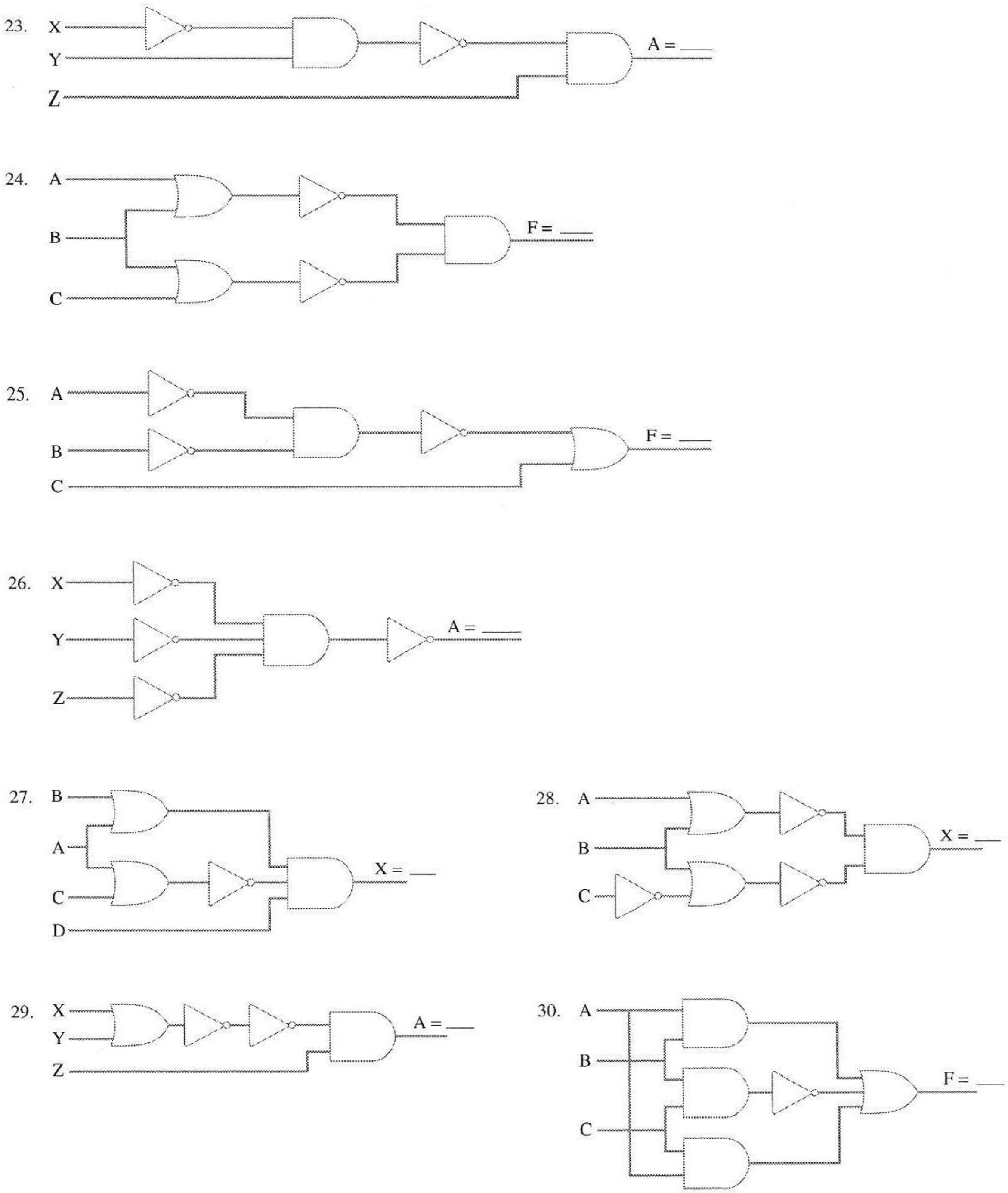


FIGURE 25–52 Logic diagrams for End of Chapter Problems 25–1, problems 23 through 30.

END OF CHAPTER PROBLEMS 25–3

In each problem, (a) simplify the output expression and justify each step, (b) draw the simplified logic diagram, and (c) construct a truth table to prove that the expressions are equal.

1. See Figure 25–51, circuit for Problem 22.
2. See Figure 25–52, circuit for Problem 25.
3. See Figure 25–52, circuit for Problem 27.
4. See Figure 25–52, circuit for Problem 30.

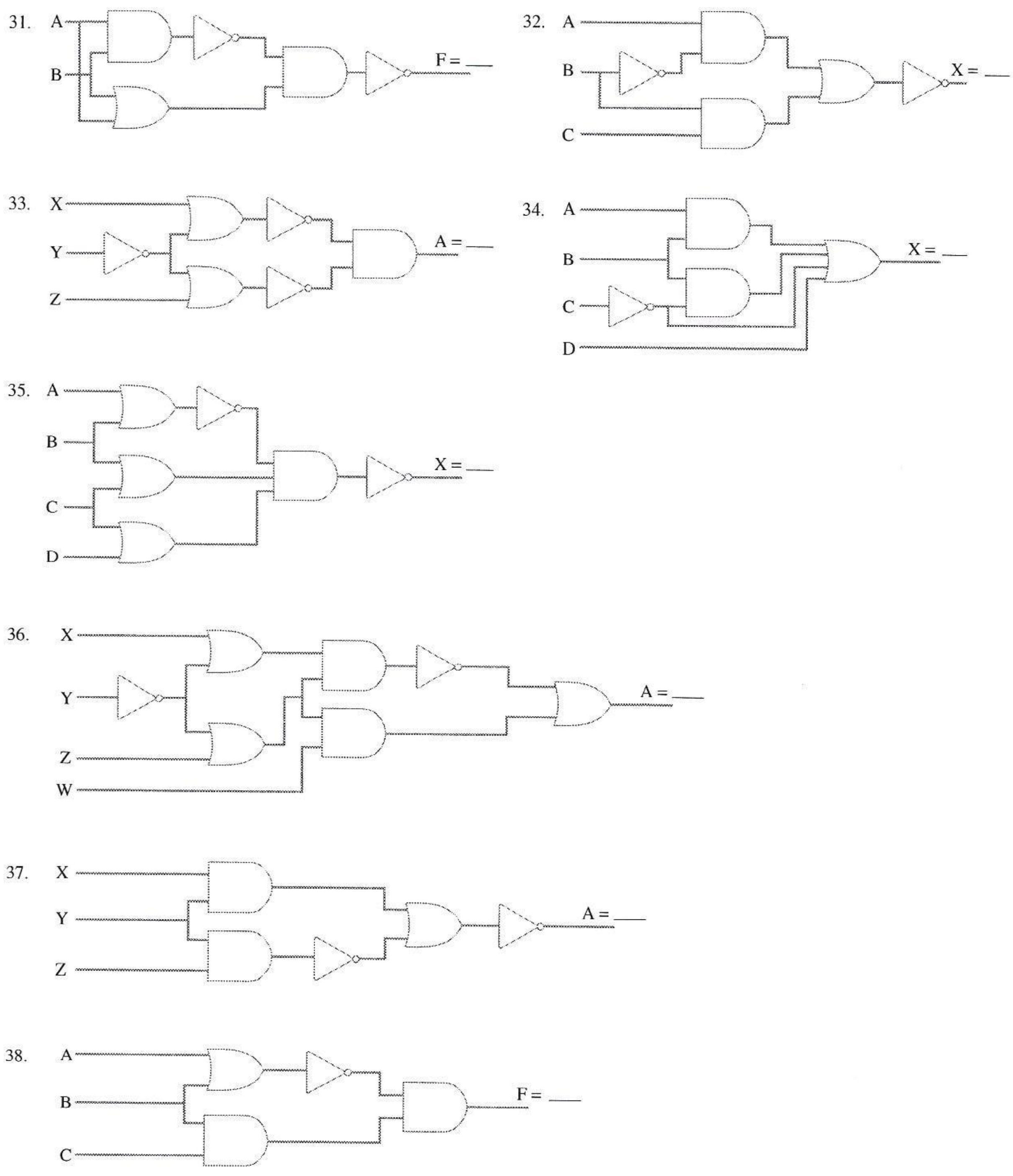
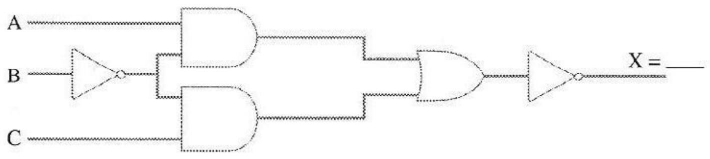


FIGURE 25–53 Logic diagrams for End of Chapter Problems 25–1, problems 31 through 40.

5. See Figure 25–53, circuit for Problem 32.
7. $\overline{A + B\overline{C}}$
9. $\overline{(B + C)}(C + D)$
11. $\overline{\overline{AB} + C\overline{D}}$

6. See Figure 25–53, circuit for Problem 40.
8. $\overline{AB} + \overline{A}C$
10. $\overline{\overline{B}(\overline{A} + \overline{C})}$
12. $\overline{A\overline{B}} + C$

39.



40.

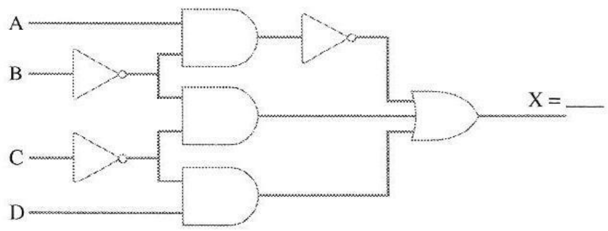
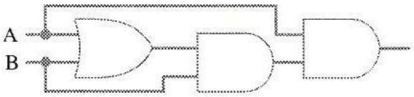
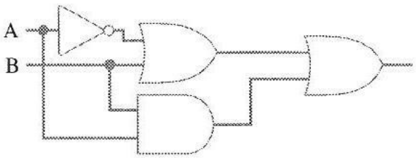


FIGURE 25–53
(*cont.*)

1.



2.



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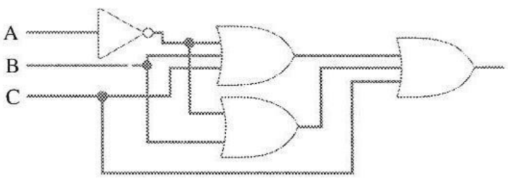


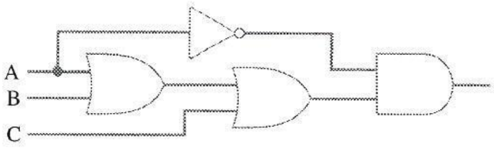
FIGURE 25–54

END OF CHAPTER PROBLEMS 25–4

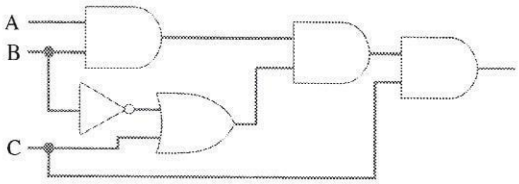
1. In some automobiles, in order to start the engine, the transmission must be in PARK and the ignition key must be turned to ON. Write the Boolean equation that represents the conditions for the engine to start.
2. To start a truck, the brake must be ON, the transmission must be in neutral, and the ignition key must be turned to ON. Write the Boolean equation that represents the conditions for the engine to start.
3. A computer needs three things to start up correctly. It needs power, the ON button must be pressed, and it needs operating system software. Write the Boolean equation that represents the conditions for the computer to start up.
4. A computer needs three things to start up correctly. It must be plugged into a power source, it must be turned on, and it must have operating system software. However, if there is a data disk in the floppy drive it will not start up. Write the Boolean equation that represents the conditions for the computer to start up correctly.

FIGURE 25–54 *(cont.)*

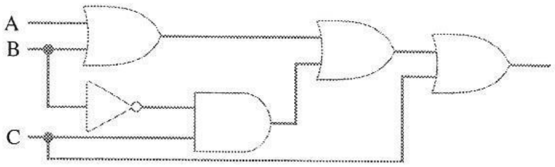
4.



5.



6.

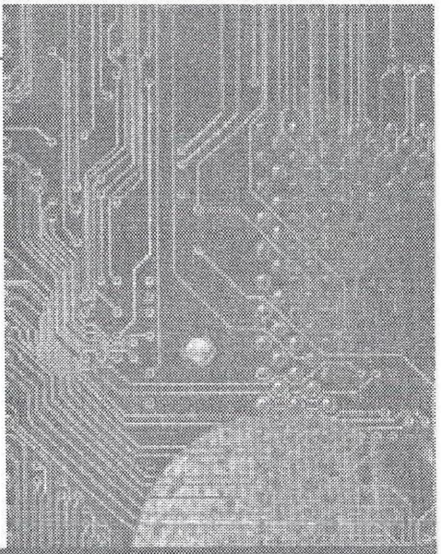


5. When using a word processor, you can scroll down a document by using the down arrow key, the page down key, or by placing the mouse pointer in the scroll bar and clicking the left mouse button. Write the Boolean equation that represents the condition to scroll down a document.

6. A "warm boot" can be initiated in some computers by pressing the "restart" button or by simultaneously pressing three keys—the Ctrl, Alt, and Del keys. Write the Boolean equation that represents the conditions to initiate a warm boot.

SECTION SEVEN

Introduction to Statistics in Electronics



26 Introduction to Statistics

Statistics is that part of math that deals with the collection, analysis, and interpretation of numerical data. In the electronics and computer industry, statistics are used to predict:

- The reliability of spacecraft electronic equipment that must operate for years with no maintenance
- The failure rates of products during the manufacturing process
- The average time it should take to repair equipment

Spacecraft computers are very reliable and expensive. Personal computers are not very reliable (they may have to be rebooted often), but they are inexpensive. Chapter 26 provides some of the fundamental concepts of statistics that must be understood in order to advance to complicated problems such as calculating the reliability of a computer system.

CHAPTER 26

Introduction to Statistics

CHAPTER OBJECTIVES

In this chapter you will learn how to:

1. Use frequency distribution to clarify or group data.
2. Construct a histogram.
3. Calculate the mean, median, and mode of a set of data.
4. Compute the standard deviation for a set of data.
5. Determine whether measured data fall within acceptable limits.

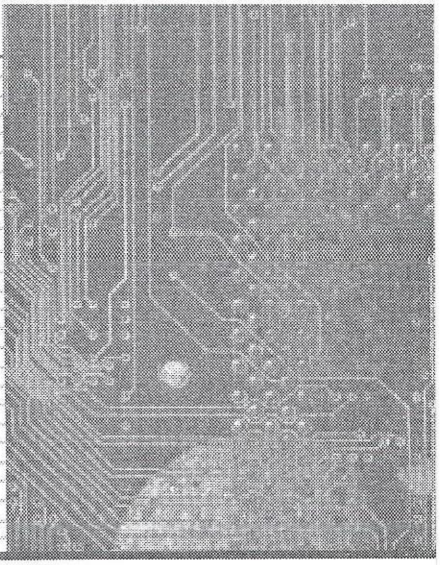
MATH IN ACTION

"Why are we analyzing just these fifty PC repair reports when we've had over 1,000 calls for service on PCs over the past three years?" asked Sandy. Sandy and Gail were assigned to determine the mean time to repair (MTTR) for computers in their company. Their department was considering purchasing a blanket service contract for all their PCs and wanted to compare that cost with an estimate of the cost to repair the PCs in the IT department, which is what they have done for the past three years. Gail responded, "Engineering randomly selected this sample of failure reports and determined that it is statistically large enough to give us reliable data." "So let me make sure I understand what we're looking for," said Sandy. "We are going to check these reports to determine how many minutes it took our IT department to fix each of these computers?" "Right," said Gail. "We'll add up the total number of repair minutes for all the PCs in our sample and then divide by the number of PCs in the sample to find the average repair time. Then we can multiply the average repair time by the average repair labor rate to determine the average cost to repair a single PC." "Then we'll multiply the average cost by the number of PCs we have," Sandy offered. "Yes," said Gail, "but we'll also have to do more statistical analysis to find the failure rate of the computers because the total expected annual repair cost will be the product of the average repair cost times the number of PCs times the PCs' failure rate." "I see," said Sandy. "By applying basic statistics, we can get a pretty good idea of how much it will cost us to do our own PC repairs and compare that with the cost of a service contract."

INTRODUCTION

Statistics are numbers that are used to analyze and interpret data in order to make decisions. Statistics is the relationship of how many to how much. How many scores were there? How often did each score appear? For example, statistics are kept on players and teams in sports. As the season progresses, teams and players are evaluated: batting and pitching averages, and team win-loss records in baseball; passing and rushing averages, and team win-loss records in football are a few of the statistics most often found in the sports section of the newspaper.

Statistical quality control and statistical process control are integral parts of all manufacturing processes. Components and assemblies that make up a complete unit—such



as a television, VCR, or automobile—must be built to meet certain standards. To be competitive, units must be of high quality. At the same time, to stay in business, a company product must be profitable. In short, to be competitive a company must make a quality product in as short a time as possible. Only with strict quality control can a company maintain a quality product that is competitive.

Schools, hospitals, and other organizations use statistical data in many different ways: Determining students' grades, predicting students' chances for succeeding in a particular course of study, and predicting the success of certain operations or medicines are a few examples.

In statistics, the entire lot or group to be tested is called a *population*. When testing an entire population would be too costly, both in terms of time and money, a *sample* is taken. From the sample, conclusions are made concerning the entire population. To be a fair gauge of the entire population, the sample taken must be a *random* sample. That is, each item in the population must have an equal chance of being selected.

Suppose a company manufactures 1000 widgets per shift. Customers for these widgets realize that there will be variations in the weight and size of the widgets (there are *always* variations), so some variations are allowed. Causes of variations in manufactured goods could be temperature variations, humidity, wear on the machines making the goods, and operator error. (And, of course, whenever you measure *anything* there is always the nagging question of how accurate the measuring instrument is.)

Suppose that the customer agrees that each widget is to weigh 10 ounces ± 0.1 ounce. That means any widget that weighs between 9.90 oz and 10.1 oz is a good widget. The cost of weighing each individual widget per shift would make the price too great, so the customer accepts random sampling each shift as a way of determining the weight of the entire population. Random sampling not only tells the customer about the weight of the entire population, but it also provides the company with valuable information about its quality control. From such data, trouble areas can be spotted and corrected. For example, the analyzed data from the samples might show that one shift is turning out a higher percentage of defective parts than another shift. Such data might also alert maintenance that a particular machine is beginning to show signs of wear and needs to be overhauled or replaced. The early detection of faults, for whatever reason, is one of the most important functions of statistical process control.

One of the jobs of the technician is the analysis and interpretation of data: A circuit is built and tested; the results are recorded and compared to expected results. A resistor is measured. Was the measured value within the acceptable range? A sample of components is removed from the assembly line and tested. The test results are used to determine the quality of the entire lot.

Over the last decade, it has become more and more important that the technician be aware of data collection and the statistical methods used in the analyses and interpretation of these data.

This chapter introduces the student to some of the basic tools used in statistics and presents examples of how these tools can be used. In order to obtain valid statistical data it is often necessary to use hundreds, or even thousands, of data samples. We use artificially low numbers of data samples in this chapter's problems to make the solutions less time consuming for students. However, the mathematical procedures are the same regardless of the number of data samples.

26–1 FREQUENCY DISTRIBUTION

In data collection, we will first arrange the data in a table. Such a table is called a *frequency distribution* table. A frequency distribution table contains the information *how much* and *how many*. The *how much* could be the magnitude of an item or score such as the measured value of a resistor or the score on a test. The *how many* is simply a tally of how many times that value appears.

To construct a frequency distribution table we determine the number of items. If there are fewer than 12 *different* items, construct a table, listing each item. Then tally the number of times each item appears.

EXAMPLE 26–1

In an electronics math quiz, the following scores were recorded: 75, 80, 60, 95, 75, 70, 90, 75, 85, 80, 70, 65, 80, 75, 70, 75, 90, 80, 65, 75. Construct a frequency distribution table:

FIGURE 26–1 Frequency distribution table for Example 26–1.

<table>
<tr><th><i>Scores</i></th><th><i>60</i></th><th><i>65</i></th><th><i>70</i></th><th><i>75</i></th><th><i>80</i></th><th><i>85</i></th><th><i>90</i></th><th><i>95</i></th></tr>
<tr><td rowspan="2">Tally</td><td>|</td><td>||</td><td>|||</td><td>卌|</td><td>||||</td><td>|</td><td>||</td><td>|</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>6</td><td>4</td><td>1</td><td>2</td><td>1</td></tr>
</table>

KEY POINT
A frequency distribution table tells us how many items are in a study and how often each item appears.

SOLUTION There are eight different scores, so we can construct a table with eight columns, one for each score. In the table, we need three rows: one for the scores, and two for the tally. Enter the scores in the eight columns. Tally the number of times each score appears. In the first row of the tally, we put a mark for each score as we read it. When all scores have been read, add up the tally and place the results in the second row. Check your tally to make sure you have 20 scores recorded. The frequency distribution table for Example 26–1 appears in Figure 26–1.

26–1–1 Histogram

A frequency distribution can be represented graphically in a bar graph called a *histogram*. In the histogram, the items or scores are plotted along the horizontal axis in equal intervals. The number of times each score appears, the frequency, is plotted along the vertical axis.

A histogram of the frequency distribution in Example 26–1 is shown in Figure 26–2. In a histogram, the midpoint of each interval is the data value. The widths of the intervals are the

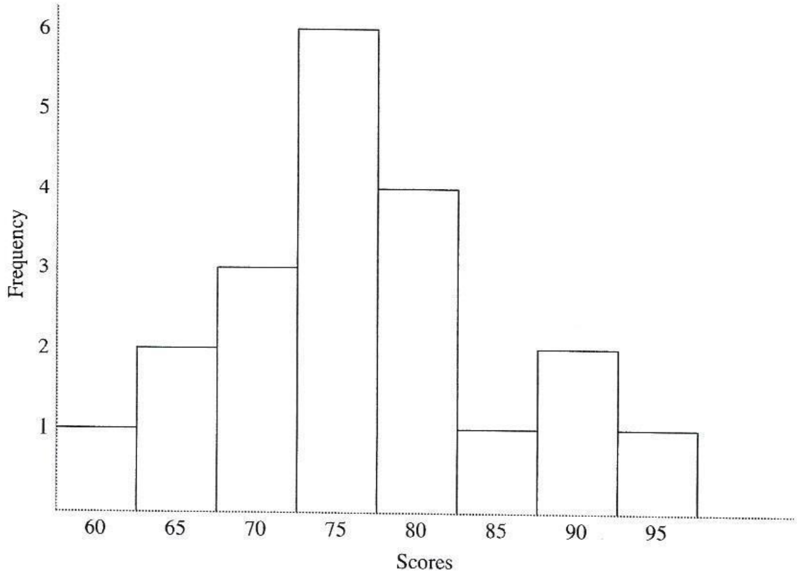


FIGURE 26–2 Histogram for Example 26–1.

<table>
<tr><th>Defective parts/100</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td rowspan="2">Tally</td><td>||||</td><td>||||</td><td>𝍸|</td><td>|||</td><td>||</td><td>|</td></tr>
<tr><td>4</td><td>4</td><td>6</td><td>3</td><td>2</td><td>1</td></tr>
</table>

FIGURE 26–3 Frequency distribution table for Example 26–2.

same and are usually determined by the distance between adjacent midpoints. In Example 26–1, the interval width is 5 because the distance between scores is 5.

EXAMPLE 26–2

In a production run, the number of defective parts per 100 was tallied over a 20-day period. The results were: day 1—3, day 2—6, day 3—4, day 4—3, day 5—2, day 6—1, day 7—5, day 8—1, day 9—3, day 10—4, day 11—2, day 12—1, day 13—3, day 14—3, day 15—4, day 16—3, day 17—2, day 18—2, day 19—1, day 20—5. Construct a frequency distribution table and a histogram.

SOLUTION There are six different values (defective parts per 100), so the frequency distribution table will have six columns. Tally how many times each number of defective parts per 100 appears. The frequency distribution table for Example 26–2 is shown in Figure 26–3. Check your tally (it should be 20) to make sure you didn't lose or add a day. The histogram for Example 26–2 is shown in Figure 26–4.

KEY POINT In a histogram, the frequency is plotted along the *y*-axis and the values are plotted along the *x*-axis.

If there are *more* than 12 different items in a data set, the table and histogram become too unwieldy. When this happens, we divide the items into a convenient number of intervals or groups. In the table, make a column for each interval, just like we did for each score. Then tally the number of data in each interval as before. In such a table: (1) intervals cannot overlap; (2) data cannot appear in more than one interval; (3) the smallest data value fits in the first interval; (4) the largest data value fits in the last interval; (5) the width of all intervals is the same.

Example 26–3 shows how to construct a table for data that are separated into intervals.

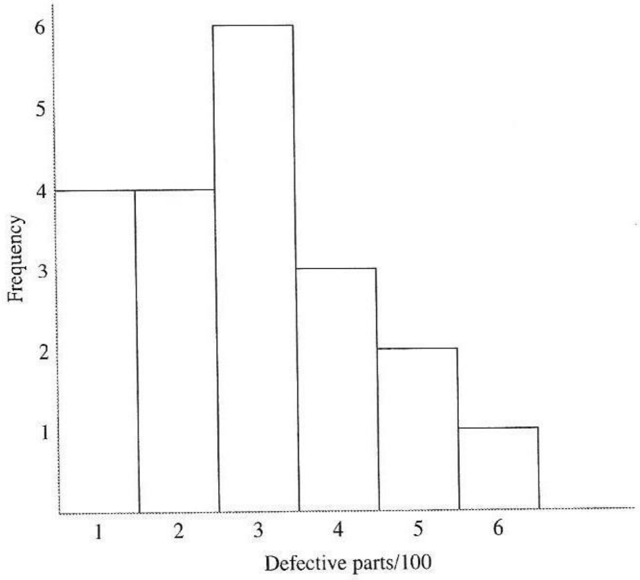


FIGURE 26–4 Histogram for Example 26–2.

FIGURE 26–5 Frequency distribution table for Example 26–3.

<table>
<tr><th>Score intervals</th><th>58–62</th><th>63–67</th><th>68–72</th><th>73–77</th><th>78–82</th><th>83–87</th><th>88–92</th></tr>
<tr><td rowspan="2">Tally</td><td>|</td><td>|</td><td>||</td><td>卌</td><td>||||</td><td>||||</td><td>||</td></tr>
<tr><td>1</td><td>1</td><td>2</td><td>5</td><td>4</td><td>4</td><td>2</td></tr>
</table>

EXAMPLE 26–3

In a second quiz, the following scores were recorded: 82, 84, 76, 70, 75, 86, 58, 82, 74, 78, 86, 88, 68, 92, 84, 82, 75, 77, 66. Construct a frequency distribution table and a histogram. Determine the range and interval midpoints.

SOLUTION There are more than 12 *different* scores, so we will divide the scores into intervals. To determine how many intervals to use, we will first determine the *range*. The range is the difference between the maximum and minimum scores. An examination of the scores shows that the highest score was 92 and the lowest score was 58.

$$\text{range} = 92 - 58 = 34$$

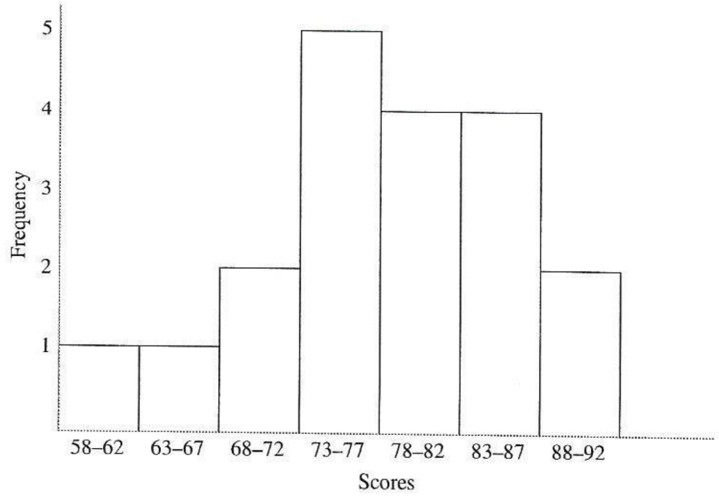
Choosing the number of intervals is arbitrary. We try to choose a number of intervals that provides a reasonable picture of the population distribution. With a range of 34 we could conveniently choose seven groups with an interval of 5. That is, each interval can consist of five different scores.

$$34 \div 7 = 4.86 \simeq 5$$

We always round up to the next highest whole number to ensure that all data fall within the selected intervals. The actual data spread will always be one greater than the range because the range is not *inclusive*. That is, the range includes all data from the lowest number to *but not including* the highest number. In our example, the actual number of data is from 58 *through* 92, a total of 35. Therefore 7 intervals of 5 gives us an exact fit. In our table, we need seven columns—one for each interval. Tally the number of data in each interval. The frequency distribution is shown in Figure 26–5. Notice that all the scores from 58 through 62, five scores (58, 59, 60, 61, and 62), are tallied in the first interval. There is only one score (58) in this interval, so our tally is one. In the interval 63 through 67, there is only one score (66). In the interval 68 through 72, there are two scores (68 and 70). In the interval 73 through 77, there are five scores (74, 75 twice, 76, and 77). And finally in the highest interval, 88 through 92, there are two scores (88 and 92).

The histogram for Example 26–3, using intervals of 5, is shown in Figure 26–6.

FIGURE 26–6 Histogram for Example 26–3.



Often, the number of intervals selected leaves us with more possible data values than we have. This is often referred to as an *overlap*. For example, suppose the highest score had been 90. Then the range would have been

$$\text{range} = 90 - 58 = 32$$

Again using seven intervals of five scores each, the highest interval, 88 through 92, will be wider than necessary creating an overlap of 2. Ninety-two must be the upper boundary though, because all intervals must be of the same width.

Sometimes the overlap is dealt with at the lower end. If we had chosen to do so, our boundaries would have been from 56 through 90. Sometimes the overlap is split. In this example we could have split the overlap and started the table at 57 and ended at 91. As a rule of thumb, if the overlap is less than half the interval width, we deal with the overlap at one end or the other.

KEY POINT The range of a set of data is the difference between the largest and smallest numbers.

KEY POINT Select a number of intervals that will most nearly describe the spread of the population.

KEY POINT When using intervals in a frequency distribution table: (1) intervals cannot overlap; (2) data cannot appear in more than one interval; (3) the smallest data value fits in the first interval; (4) the largest data value fits in the last interval; (5) the width of all intervals is the same.

The extremes of an interval are called the end points or boundaries. The data within an interval include all the values from the lower boundary through the upper boundary. Thus, in Example 26–3, in the interval 58 through 62, scores of 58, 59, 60, 61, and 62, a total of 5 is included. The lower boundary (end point) is 58 and the upper boundary (end point) is 62.

All scores from 63 through 67 (63, 64, 65, 66, and 67) will fall in the next group. The lower boundary is 63 and the upper boundary is 67. Finally, all values from 88 through 92 will be part of the highest interval. The lower boundary is 88 and the upper boundary is 92.

The midpoint of an interval can be found by dividing the sum of the interval boundaries by 2. Finding the midpoint of each interval will make it easier to calculate the average, which we will do in the next section.

EXAMPLE 26–4

Twenty high-gain amplifiers were constructed in the lab using components off the shelf. The design called for a voltage gain of 400 dB. The voltage gains were measured to determine the actual gain of the amplifiers. The results (in dB) were: 408, 412, 401, 380, 402, 390, 398, 405, 400, 396, 404, 401, 388, 399, 406, 402, 391, 404, 398, 403. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.

SOLUTION There are more than 12 different measurements of gain, so we will divide the gains into intervals. To determine the number of intervals to use, we will first find the range. The highest gain (in dB) is 412 and the lowest gain is 380.

$$\text{Range} = 412 - 380 = 32$$

Let's use 5 intervals: $32/5 = 6.4 \simeq 7$

With 5 intervals, there will be 7 measurements in each interval. The frequency distribution is shown in Figure 26–7. Notice that there is an overlap of 2 in the highest interval. We could have chosen to have the overlap in the lowest interval or we could have created an overlap of 1 on each end. Since the overlap was less than half the number of gains in an interval, only one end was used. We chose to use the upper end.

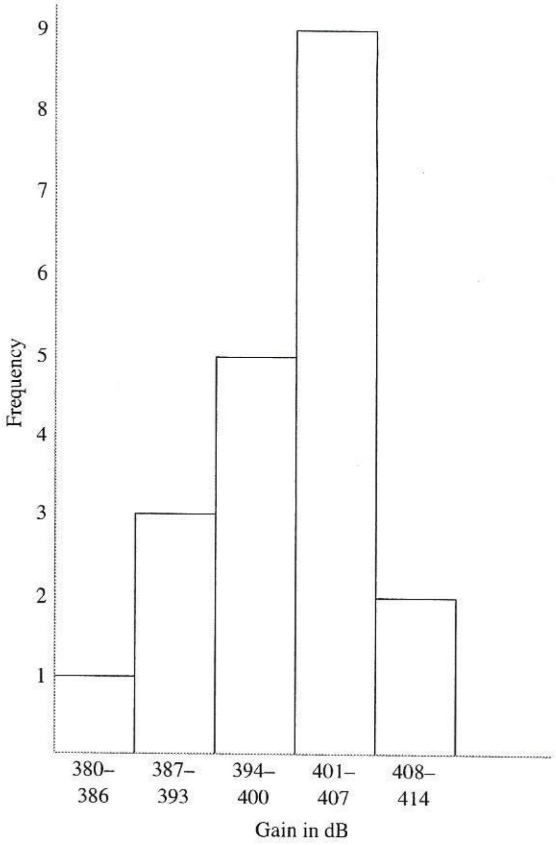
The interval midpoints are 382, 387, 392, 397, 402, 407, and 412. The histogram, using intervals of 5, is shown in Figure 26–8.

KEY POINT The midpoint of an interval is the number that is halfway between the upper and lower interval boundaries.

Gain in dB	*380–386*	*387–393*	*394–400*	*401–407*	*408–414*
Tally	|	|||	𝍸	𝍸||||	||
	1	3	5	9	2

FIGURE 26–7 Frequency distribution table for Example 26–4.

FIGURE 26–8



PRACTICE PROBLEMS 26–1

1. In an electronics math quiz, the following scores were recorded: 85, 60, 75, 90, 70, 80, 65, 75, 75, 85, 75, 90, 80, 85, 75, 80, 65, 75, 90, 80, 65, 95, 70, 80, 75. Construct a frequency distribution table. From the table, construct a histogram.

3. In a production run, the number of defective parts per 100 was tallied over a 20-day period. The results were: day 1—6, day 2—4, day 3—3, day 4—4, day 5—2, day 6—4, day 7—3, day 8—5, day 9—4, day 10—2, day 11—1, day 12—5, day 13—4, day 14—1, day 15—3, day 16—4, day 17—5, day 18—2, day 19—4, day 20—3. Construct a frequency distribution table and a histogram.

5. In a production run covering a 31-day period, the following numbers of items were produced each day: 25, 51, 72, 31, 40, 44, 37, 41, 48, 43, 57, 42, 31, 28, 46, 50, 63, 37, 78, 51, 62, 61, 51, 48, 41, 45, 34, 54, 32, 40, 79. Construct a frequency distribution table and histogram. Determine the range and interval midpoints.

2. From a bin of 100 1 kΩ ± 5% carbon resistors, 25 were selected to be measured in the laboratory. The results, in ohms, were: 990, 1000, 980, 1010, 1000, 1030, 1010, 1000, 990, 980, 990, 1020, 1010, 1000, 990, 1020, 990, 1010, 1000, 1010, 1020, 1010, 1020, 1010, 1040. Construct a frequency distribution table. From the table, construct a histogram.

4. In a mid-term exam, the following scores were recorded: 83, 82, 63, 91, 75, 68, 87, 80, 71, 72, 74, 85, 90, 62, 81, 77, 70, 86, 78, 73, 76, 84, 81, 74, 72. Construct a frequency distribution table and a histogram. Determine the range and the midpoints of the intervals.

6. Twenty-five high-gain amplifiers were constructed in the lab using components off the shelf. The design called for a voltage gain of 200 dB. The voltage gains were measured to determine the actual gain of the amplifiers. The results (in dB) were: 203, 201, 213, 193, 184, 200, 210, 196, 198, 191, 203, 206, 199, 204, 201, 202, 199, 197, 200, 208, 201, 198, 191, 206, 197. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.

26–2 MEASURES OF CENTRAL TENDENCY

From the frequency distribution table, more specific information can be obtained. In a data distribution we usually are interested in the center of the distribution. We are interested in the *average*. Three methods are used to measure averages. These methods are referred to as measures of central tendency. These measures of central tendency are the *mean*, the *median*, and the *mode*. In a normal distribution (discussed later in the chapter) the three measures occur at the same point.

26–2–1 The Mean

The mean is the sum of all the data divided by the number of data. In the English language the word "average" is used for the mathematical "mean." A student's grades are *averaged* to determine his or her grade for the semester. A hitter's at-bats versus hits are *averaged* to determine a batting average. A quarterback's completions versus number of passes thrown are *averaged* to determine a completion average. In each case we are calculating the mean.

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{\text{number of data}} \qquad (26\text{–}1)$$

where $x_1, x_2, x_3, \ldots, x_n$ are the data. Σx (summation x) means the sum of all the values of x, the data. For example, if five test scores are 60, 65, 70, 75, and 80, what would be the average score?

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{\text{number of data}}$$

$$= \frac{60 + 65 + 70 + 75 + 80}{5} = \frac{350}{5} = 70$$

KEY POINT The mean is equal to the sum of the data divided by the number of data.

26–2–2 The Median

The median is that value where half of the values are greater and half are less. That is, half the data fall below this number and half the data are above this number. If the number of data is odd, the median is the middle number. If the number of data is even, then the median is half the distance between the middle two numbers. For example, again using the test scores of 60, 65, 70, 75, and 80, the median would be 70, the middle score. If six test scores are 60, 65, 70, 75, 80, and 100, then the median would be half the distance between the two middle scores, or 72.5. This value can be found by adding the two middle scores and dividing by 2.

$$70 + 75 = 145;\ 145 \div 2 = 72.5$$

KEY POINT The median is that value where half of the values are greater and half are less.

26–2–3 The Mode

The *mode* is the data (the numbers) that occur most frequently. For example, if six test scores are 65, 70, 70, 70, 75, and 80, the mode would be 70 because 70 is the score that occurs most often.

If the data are grouped (in intervals), then the midpoint of the interval is used in computing the mean, median, and mode.

KEY POINT The mode is the measurement that appears most frequently in the data.

EXAMPLE 26–5

Using the data from Figure 26–9 (same as Figure 26–1), calculate the mean, median, and mode.

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{\text{number of data}}$$

The TI-36X SOLAR, CASIO fx-115W, and SHARP EL-506G are three calculators that include statistical functions. You may use any one of these calculators to find the mean and Σx, as well as other statistical information. See the user's manual for your calculator for the key combinations to use. In the TI-36X SOLAR, the key combination for this example is:

$\boxed{\text{3rd}}$ $\boxed{\text{STAT 1}}$ 60 $\boxed{\Sigma+}$ 65 $\boxed{\Sigma+}$ 65 $\boxed{\Sigma+}$ 70 $\boxed{\Sigma+}$

FIGURE 26–9 Frequency distribution table for Example 26–5.

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	𝍷	𝍷𝍷	𝍷𝍷𝍷	𝍸𝍷	𝍷𝍷𝍷𝍷	𝍷	𝍷𝍷	𝍷
	1	2	3	6	4	1	2	1

and so on until all 20 of the data have been entered. Notice that after $\boxed{\Sigma+}$ has been keyed in, the number displayed is the number of data entered, *not* the data themselves. After you key in the last data, 95, and press $\boxed{\Sigma+}$, the number 20 should be displayed because you have entered 20 data numbers. If the number 20 is *not* displayed, you made a mistake. After you have entered 95 $\boxed{\Sigma+}$ and the number 20 is displayed, both the mean and the total of the scores, Σx, are stored in the calculator. Display the mean by pressing $\boxed{\bar{x}}$. Display Σx by pressing the $\boxed{\Sigma x}$ key.

$$\text{mean} = \bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_{20}}{\text{number of data}} = \frac{1530}{20} = 76.5$$

For the Casio the key sequence is:

$\boxed{\text{MODE}}$ $\boxed{\text{MODE}}$ 1 to enter the Statistical Mode.

$\boxed{\text{SHIFT}}$ $\boxed{\text{Scl}}$ $\boxed{=}$ to clear memory.

To enter data:

60 $\boxed{\text{DT}}$ 65 $\boxed{\text{DT}}$ $\boxed{\text{DT}}$ 70 $\boxed{\text{DT}}$ $\boxed{\text{DT}}$ $\boxed{\text{DT}}$ 75 $\boxed{\text{DT}}$ ...

until all 20 values have been entered.

To display the mean: $\boxed{\text{SHIFT}}$ $\boxed{\bar{x}}$ $\boxed{=}$

If you do not use a statistical calculator, simply add all the data and divide by 20, the number of data.

$$\text{mean} = \frac{60 + (2)65 + (3)70 + (6)75 + (4)80 + 85 + (2)90 + 95}{20} = \frac{1530}{20} = 76.5$$

The median is the score where half the data are above and half the data are below. Because the number of data is even (20), the median would be the score between the tenth and eleventh data values. Since both scores are 75, the median is 75.

An examination of the data shows that a score of 75 appears most frequently, six times. Therefore, the mode is 75.

$$\text{mean} = 76.5 \qquad \text{median} = 75 \qquad \text{mode} = 75$$

EXAMPLE 26–6

Using the data from Figure 26–10 (same as Figure 26–3), calculate the mean, median, and mode.

FIGURE 26–10 Frequency distribution table for Example 26–6.

Defective parts/100	*1*	*2*	*3*	*4*	*5*	*6*
Tally	𝍷𝍷𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷	𝍷𝍷𝍷	𝍷𝍷	𝍷
	4	4	6	3	2	1

SOLUTION

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_{20}}{20}$$

$$\bar{x} = \frac{1(4) + 2(4) + 3(6) + 4(3) + 5(2) + 6(1)}{20} = \frac{58}{20} = 2.90$$

There are 20 data values, so the median falls between the tenth and eleventh values. The tenth and eleventh values are both 3, so the median is 3. Three appears the most times, so the mode is also 3.

$$\text{mean} = 2.9 \quad \text{median} = 3 \quad \text{mode} = 3$$

EXAMPLE 26–7

Using the data from Figure 26–11 (same as Figure 26–5), compute the mean, median, and mode.

Score intervals	*58–62*	*63–67*	*68–72*	*73–77*	*78–82*	*83–87*	*88–92*
Tally	|	|	||	𝍸	||||	||||	||
	1	1	2	5	4	4	2

FIGURE 26–11

SOLUTION The data are assumed to occur at the midpoint of each interval or at 60, 65, 70, 75, 80, 85, and 90. Selecting the midpoint of each interval to compute the mean, rather than using the raw scores, is the accepted method in statistics. This method introduces some error when the number of scores is small, as is the case here (see Example 26–3 for actual scores), but for larger numbers of data, which is the general case in statistics, the error would be negligible.

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_{19}}{19}$$

$$\text{mean} = \frac{60 + 65 + (2)70 + (5)75 + 4(80) + 4(85) + (2)90}{19} = \frac{1480}{19}$$

$$= 77.9 \text{ rounded to three places}$$

The median would occur at the tenth score. There are nine scores above and nine scores below the tenth score. The tenth score is in the interval 78 through 82. The midpoint of the interval 78 through 82 is 80. Therefore, the median is 80.

The mode occurs in the interval 73 through 77. The midpoint of this interval is 75. Therefore, the mode is 75.

$$\text{mean} = 77.9 \quad \text{median} = 80 \quad \text{mode} = 75$$

EXAMPLE 26–8

Using the data from Figure 26–12 (same as Figure 26–7), compute the mean, median, and mode.

Gain in dB	*380–386*	*387–393*	*394–400*	*401–407*	*408–414*
Tally	|	|||	𝍸	𝍸||||	||
	1	3	5	9	2

FIGURE 26–12
Frequency distribution table for Example 26–8.

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	𝍷	𝍷𝍷𝍷	𝍷𝍷	𝍸𝍷𝍷	𝍸	𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷
	1	3	2	7	5	3	3	1

FIGURE 26–13 Frequency distribution table for Practice Problems 26–2 #1.

Score intervals	*62–66*	*67–71*	*72–76*	*77–81*	*82–86*	*87–91*
Tally	𝍷𝍷	𝍷𝍷𝍷	𝍸𝍷𝍷	𝍸	𝍸	𝍷𝍷𝍷
	2	3	7	5	5	3

FIGURE 26–14 Frequency distribution table for Practice Problems 26–2 #2.

SOLUTION To compute the mean we again use the interval midpoints, which in this case are 383, 390, 397, 404, and 411.

$$\text{Mean} = \overline{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_{20}}{20} = \frac{7996}{20} = 400$$

The median would occur between the tenth and eleventh data values. Since both gain measurements fall in the interval 401 to 407, the median will be at the midpoint of that interval, which is 404. The mode also occurs in the interval 401–407, so the mode is also 404, the midpoint of that interval.

$$\text{mean} = 400 \quad \text{median} = 404 \quad \text{mode} = 404$$

PRACTICE PROBLEMS 26–2

1. Using the data in Figure 26–13 (same as Figure 26–9), determine the mean, median, and mode.
2. Using the data in Figure 26–14 (same as Figure 26–15), determine the mean, median, and mode.
3. Using the data in Figure 26–15 (same as Figure 26–11), determine the mean, median, and mode.
4. Using the data in Figure 26–16 (same as Figure 26–17), determine the mean, median, and mode.
5. Using the data in Figure 26–17 (same as Figure 26–13), determine the mean, median, and mode.
6. Using the data in Figure 26–18 (same as Figure 26–19), determine the mean, median, and mode.

FIGURE 26–15 Frequency distribution table for Practice Problems 26–2 #3.

Resistor values in ohms	*980*	*990*	*1000*	*1010*	*1020*	*1030*	*1040*
Tally	𝍷𝍷	𝍸	𝍸	𝍸𝍷𝍷	𝍷𝍷𝍷𝍷	𝍷	𝍷
	2	5	5	7	4	1	1

FIGURE 26–16 Frequency distribution table for Practice Problems 26–2 #4.

Score intervals	*25–31*	*32–38*	*39–45*	*46–52*	*53–59*	*60–66*	*67–73*	*74–80*
Tally	𝍷𝍷𝍷	𝍸	𝍸𝍷𝍷𝍷	𝍸𝍷𝍷	𝍷𝍷	𝍷𝍷𝍷	𝍷	𝍷𝍷
	3	5	8	7	2	3	1	2

Number of defective parts/100	*1*	*2*	*3*	*4*	*5*	*6*
Tally	||	|||	||||	卌||	|||	|
	2	3	4	7	3	1

FIGURE 26–17
Frequency distribution table for Practice Problems 26–2 #5.

Gain in dB	*184–188*	*189–193*	*194–198*	*199–203*	*204–208*	*209–213*
Tally	|	|||	卌	卌卌	||||	||
	1	3	5	10	4	2

FIGURE 26–18
Frequency distribution table for Practice Problems 26–2 #6.

SELF-TEST 26–1 THROUGH 26–2

1. In an electronics math quiz, the following scores were recorded: 85, 70, 70, 65, 60, 95, 70, 75, 80, 85, 90, 70, 60, 85, 75, 75, 70, 80, 85, 70. Construct a frequency distribution table and histogram. Compute the mean, median, and mode.
2. During a 25-day period in a manufacturing facility, the following numbers of items were produced: 123, 147, 159, 185, 114, 136, 144, 170, 152, 140, 138, 158, 125, 136, 166, 151, 156, 148, 142, 153, 177, 155, 133, 145, 160. Construct a frequency distribution table and histogram. Determine the range and the interval midpoints. Compute the mean, median, and mode.

26–3 STANDARD DEVIATION

All data sets will have some variability. How much the data values are different depends on many factors. The factors involved depend on what we're measuring. As mentioned before, in manufacturing processes, factors such as temperature, humidity, the precision of the manufacturing equipment, and human error account for these variations.

One measure of variability (*spread* and *dispersion* are also used to describe these differences) is the *range*, which we defined previously as the difference between the maximum and minimum values. Although the range gives us the extremes, it doesn't tell us anything about how the data are spread out between these extremes.

The most widely used measure of variability is the *standard deviation*. The standard deviation is the averaged distances of all data from the mean. The standard deviation gives statisticians a clearer picture of the spread because it relates the spread to the mean.

Statisticians use two different methods for calculating standard deviation. One is defined as the *population* standard deviation and the second is called the *sample* standard deviation. The most commonly used method is the sample standard deviation and it is the one we will use. We will use the lowercase Greek letter *sigma* (σ) to define the sample standard deviation, or simply the standard deviation. On the TI-36X calculator, that key is designated with the letters σ_{xn-1}.

The equation for determining the standard deviation is:

$$\sigma = \sigma_{xn-1} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}.$$

$(x - \bar{x})^2$ means that we are to subtract a value in the set from the mean and square it. $\dfrac{\Sigma(x - \bar{x})^2}{n}$ means that we are to find the mean of all these values. For example, suppose a set of numbers is 4, 4, 6, 7, and 8. The mean would be

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5.80$$

The standard deviation would be

$$\sigma = \sqrt{\left[\frac{(4 - 5.8)^2 + (4 - 5.8)^2 + (6 - 5.8)^2 + (7 - 5.8)^2 + (8 - 5.8)^2}{5 - 1}\right]}$$

$$\sigma = 1.79$$

An easier way is to use the statistical calculator. When you find the mean ($\bar{x}$), you can also find σ.

$\boxed{\text{3rd}}\ \boxed{\text{STAT 1}}\ 4\ \boxed{\Sigma+}\ 4\ \boxed{\Sigma+}\ 6\ \boxed{\Sigma+}\ 7\ \boxed{\Sigma+}\ 8\ \boxed{\Sigma+}$

$\boxed{\text{2nd}}\ \boxed{\bar{x}}$ mean = 5.80

$\boxed{\text{2nd}}\ \boxed{\sigma_{xn-1}}$ standard deviation = 1.79

EXAMPLE 26–9

Using the data in Figure 26–19, find the mean, median, mode, range, and standard deviation.

FIGURE 26–19
Frequency distribution table for Example 26–9.

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸	𝍸𝍷𝍷𝍷	𝍸	𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷
	2	4	5	8	5	3	3	1

SOLUTION You may find the mean and standard deviation, σ, using Equations 26–1 and 26–2, or you may use your statistical calculator. Follow the directions in the user's manual. Using the calculator:

$$\bar{x} = 75.8 \quad \sigma = 9.05$$

$$\text{median} = 75 \quad \text{mode} = 75 \quad \text{range} = 95 - 60 = 35$$

EXAMPLE 26–10

Using the data in Figure 26–20, find the mean, median, mode, range, and standard deviation. Again, use the interval midpoints to find the mean. The midpoints are 60, 65, 70, 75, 80, 85, and 90.

FIGURE 26–20
Frequency distribution table for Example 26–10.

Score intervals	*58–62*	*63–67*	*68–72*	*73–77*	*78–82*	*83–87*	*88–92*
Tally	𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷𝍷𝍷	𝍸	𝍷𝍷𝍷𝍷	𝍷𝍷	𝍷
	2	4	8	5	4	2	1

SOLUTION Using the calculator: $\bar{x} = 72.9$; $\sigma = 7.64$.

$$\text{median} = 70 \quad \text{mode} = 70 \quad \text{range} = 92 - 58 = 34$$

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	||	|||	𝍸	𝍸|	𝍸||||	𝍸	||||	||
	2	3	5	6	9	5	4	2

FIGURE 26–21 Frequency distribution table for Practice Problems 26–3 #1.

Items	*600*	*610*	*620*	*630*	*640*	*650*	*660*
Tally	|	|||	𝍸||	𝍸𝍸	𝍸|||	||||	||
	1	3	7	10	8	4	2

FIGURE 26–22 Frequency distribution table for Practice Problems 26–3 #2.

PRACTICE PROBLEMS 26–3

1. From the data set in Figure 26–21, find the mean, median, mode, range, and standard deviation.
2. From the data set in Figure 26–22, find the mean, median, mode, range, and standard deviation.
3. From the data set in Figure 26–23, find the mean, median, mode, range, and standard deviation.
4. From the data set in Figure 26–24, find the mean, median, mode, range, and standard deviation.
5. From the data set in Figure 26–25, find the mean, median, mode, range, and standard deviation.

Score intervals	*59–63*	*64–68*	*69–73*	*74–78*	*79–83*	*84–88*	*89–93*
Tally	|||	𝍸|	𝍸𝍸	𝍸	||||	||	|
	3	6	10	5	4	2	1

FIGURE 26–23 Frequency distribution table for Practice Problems 26–3 #3.

Score intervals	*60–64*	*65–69*	*70–74*	*75–79*	*80–84*	*85–89*	*90–94*	*95–99*
Tally	|	|||	𝍸||||	𝍸|	||||	|||	||	|
	1	3	9	6	4	3	2	1

FIGURE 26–24 Frequency distribution table for Practice Problems 26–3 #4.

Item intervals	*200–216*	*217–233*	*234–250*	*251–267*	*268–284*	*285–301*	*302–318*	*319–335*
Tally	|	|||	𝍸	𝍸𝍸|	𝍸||||	𝍸|	𝍸	|||
	1	3	5	11	9	6	5	3

FIGURE 26–25 Frequency distribution table for Practice Problems 26–3 #5.

In order to understand the significance of the standard deviation, let's discuss the *normal distribution* curve and then tie the two together.

KEY POINT

The *standard deviation* is the averaged distances of all data from the mean.

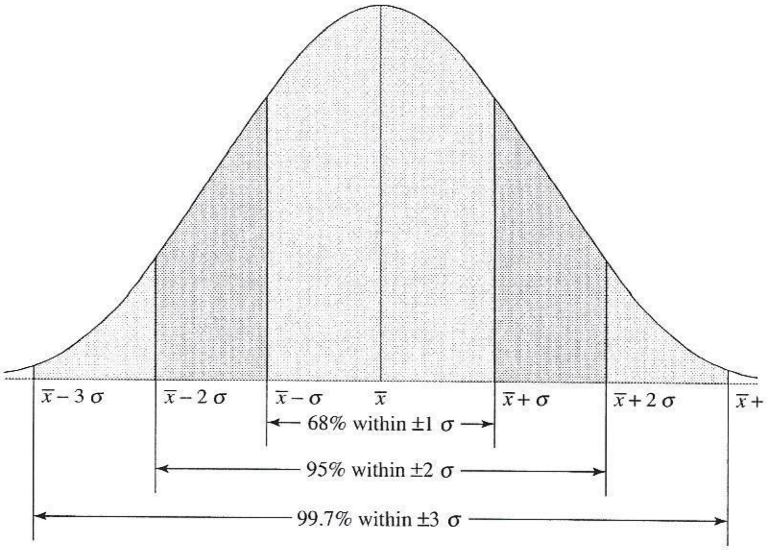


FIGURE 26–26 The normal curve.

26-4 THE NORMAL CURVE

When a population has a normal distribution, connecting all the midpoints in the histogram will result in a curve like the one in Figure 26–26. Such a curve is called the *normal curve*.

Characteristics of the normal curve are:

1. The curve is bell-shaped and has only one peak.
2. The shape on one side of the peak is the same as the shape on the other side.
3. There will be as many data points above the mean as there are below the mean.
4. The mean, median, and mode occur at the same point—the peak.
5. The mean defines the center of the curve. The standard deviation defines the spread.
6. 68% of the values fall within $\pm 1\ \sigma$, 95% of the values fall within $\pm 2\ \sigma$, and 99.7% of the values fall within $\pm 3\ \sigma$.

A small σ indicates a small spread from the mean. For example, suppose in a normal curve, the mean of a data set is 500 and σ equals 8. We know from the characteristics of the normal curve that 68% of the data values will fall between 492 and 508.

$$500 - \sigma = 500 - 8 = 492;\ 500 + \sigma = 500 + 8 = 508$$

Because the curve is symmetrical, 34% of the data fall in the group from 492 to 500 and 34% of the data fall in the group from 500 to 508.

95% of the data values will fall between $\pm 2\ \sigma$.

$$500 - 2\,\sigma = 500 - 16 = 484;\ 500 + 2\,\sigma = 500 + 16 = 516$$

Half of the 95% falls below the mean and half will be above the mean, or 47.5% of all the data lie between 484 and 500 and 47.5% of all the data lie between 500 and 516.

99.7% of the data values will fall between $\pm 3\ \sigma$.

$$500 - 3\,\sigma = 500 - 24 = 476;\ 500 + 3\,\sigma = 500 + 24 = 524$$

Half the 99.7% falls below the mean and half will be above the mean or 99.7% of all the data values fall between 476 and 524.

0.3% of all the data will be greater than $\pm 3\ \sigma$.

EXAMPLE 26–11

In a standardized test given over a period of years, 2000 students were tested. The test results yielded a normal curve. The mean was 75 and the standard deviation was 5. What was the distribution of grades?

SOLUTION If $\sigma = 5$, then:

$$75 - 1\,\sigma = 75 - 5 = 70$$

$$75 + 1\,\sigma = 75 + 5 = 80$$

68% of all the grades fall between 70 and 80.
34% (half of 68%) fall between 70 and 75 and 34% fall between 75 and 80.

$$2000 \times 0.34 = 680$$

680 of the students' scores were between 70 and 75.
680 of the students' scores were between 75 and 80.
95% of all scores fall $\pm 2\,\sigma$ from the mean.

$$75 - 2\,\sigma = 75 - 10 = 65$$

$$75 + 2\,\sigma = 75 + 10 = 85$$

47.5% of the scores (half of 95%) fall between 65 and 75.
47.5% are in the range from 75 to 85.

$$2000 \times 0.475 = 950$$

950 of the students' scores were between 65 and 75.
950 of the students' scores were between 75 and 85. $950 + 950 = 1900$.
The normal curve shows that almost all the scores (99.7%) are $\pm 3\,\sigma$ or less from the mean.

$$75 - 3\,\sigma = 75 - 15 = 60$$

$$75 + 3\,\sigma = 75 + 15 = 90$$

$$2000 \times 0.4985 = 997$$

997 of the students' scores were between 60 and 75.
997 of the students' scores were between 75 and 90.
Some scores were *greater than* 3 σ from the mean. In this case, 6 (0.3% of 2000). This area beyond 3 σ exists because in the normal curve, there are a few scores (or items) that are unusually large or small.

PRACTICE PROBLEMS 26–4

1. 600 amplifiers were produced in a given time period. The mean gain was 25 dB with a standard deviation of 2 dB. What would be a normal distribution?
2. 1500 scores were reported on a standardized test. The mean score was 70 with a standard deviation of 4. What would be the normal distribution?

In the examples we have used in this chapter, our curves have been *skewed*. That is, they are different from the normal curve. In our examples and practice problems we had more data points above the mean than below the mean, or vice versa. This is usually the case when dealing with small amounts of data. What this means is that the distribution of the data will be different from the normal, but all or almost all of the data will fall between $\pm 3\,\sigma$. A look at Example 26–5 shows that there were 20 scores. The mean was 76.5. Computing the standard deviation, we get

$\sigma = 8.90$. Twelve of the scores fall between the mean and $-3\ \sigma$. Eight of the scores lie between the mean and $+3\ \sigma$. Ideally, all scores will lie within $\pm 3\ \sigma$.

In manufacturing, if the limits of the tolerances of manufactured parts are set to $\pm 3\ \sigma$, then any item that lies beyond $\pm 3\ \sigma$ is a reject. If there are parts that exceed the established limits, then steps are taken immediately to determine and correct the cause.

The normal curve is the *ideal* curve. It is the curve that is expected as the result of thousands of tests or thousands of items produced. In reality, though, most curves will be skewed. In quality control, companies continually strive to eliminate any items, parts, or processes that fall beyond the acceptable limits. These limits are often set at $\pm 3\ \sigma$.

Further discussions of statistical methods are left to courses on statistics. Our purpose here has been to acquaint the student with some of the basic concepts he or she is most likely to encounter.

SELF-TEST 26–3 THROUGH 26–4

1. From the data in Figure 26–27, compute the mean, median, mode, range, and standard deviation.
3. In a standardized test given over a period of years, 3000 students were tested. The test results yielded a normal curve. The mean was 70 and the standard deviation was 6. What was the distribution of grades?
2. From the data in Figure 26–28, compute the mean, median, mode, range, and standard deviation.

FIGURE 26–27
Frequency distribution table for Self-Test 26–2 #1.

Scores	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	|		||	𝍸	𝍸|||	𝍸	||||	|||	||
	1	0	2	5	8	5	4	3	2

FIGURE 26–28
Frequency distribution table for Self-Test 26–2 #2.

Item intervals	*600–640*	*641–681*	*682–722*	*723–763*	*764–804*	*805–845*	*846–886*
Tally	|	||	𝍸𝍸	𝍸𝍸||	𝍸	||	|
	1	2	10	12	5	2	1

CHAPTER 26 AT A GLANCE

PAGE	KEY POINTS	EXAMPLE

Assume the following data: 3, 3, 4, 4, 4, 5, 7, 7, 9.

556 *Key Point:* A frequency distribution table tells us how many items are in a study and how often each item appears.

Data	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Tally	||	|||	|		||		|
	2	3	1	0	2	0	1

FIGURE 26–29

557	*Key Point:* In a histogram, the frequency is plotted along the y-axis and the values are plotted along the x-axis.	

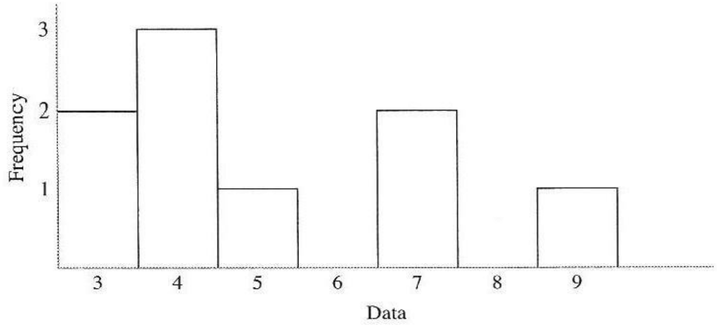


FIGURE 26–30

559	*Key Point:* The range of a set of data is the difference between the largest and smallest numbers.	range $= 9 - 3 = 6$
559	*Key Point:* Select a number of intervals that will most nearly describe the spread of the population.	
559	*Key Point:* When using intervals in a frequency distribution table: (1) intervals cannot overlap; (2) data cannot appear in more than one interval; (3) the smallest data value fits in the first interval; (4) the largest data value fits in the last interval; (5) the width of all intervals is the same.	
561	*Key Point:* The *mean* is equal to the sum of the data divided by the number of data.	$\overline{x} = \frac{\Sigma x}{n} = \frac{46}{9} = 5.11$
561	*Key Point:* The median is that value where half of the values are greater and half are less.	median $= 4$
561	*Key Point:* The *mode* is the measurement that appears most frequently in the data.	mode $= 4$
567	*Key Point:* The standard deviation is the averaged distances of all data from the mean.	$\sigma = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n - 1}} = 1.97$

END OF CHAPTER PROBLEMS 26–1

1. In an electronics math quiz, the following scores were recorded: 85, 65, 90, 75, 60, 70, 75, 65, 80, 75, 85, 70, 75, 80, 75. Construct a frequency distribution table. From the table, construct a histogram.
2. In an electronics math quiz, the following scores were recorded: 80, 70, 75, 60, 70, 85, 70, 65, 60, 70, 80, 60, 70, 65, 75. Construct a frequency distribution table. From the table, construct a histogram.
3. In an electronics math quiz the following scores were recorded: 80, 62, 77, 98, 78, 75, 70, 60, 90, 85, 75, 83, 78, 72, 65, 82, 70, 76, 65, 80, 94, 75, 66, 73, 85. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.
4. In an electronics math quiz the following scores were recorded: 83, 64, 75, 72, 78, 85, 57, 73, 70, 79, 80, 76, 95, 88, 72, 81, 75, 73, 85, 74, 76, 70, 82, 65, 78, 91, 82, 80, 75, 68. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.

5. Twenty high-gain amplifiers were constructed in the lab using components off the shelf. The design called for a voltage gain of 400 dB. The voltage gains were measured to determine the actual gain of the amplifiers. The results (in dB) were: 409, 397, 403, 418, 413, 382, 370, 407, 392, 400, 416, 402, 394, 392, 407, 405, 384, 432, 426, 404. Construct a frequency distribution table and a histogram. Determine the range and interval midpoints.

7. In a manufacturing company, the following items were produced over a 25-day period: 312, 276, 295, 300, 325, 301, 280, 277, 290, 315, 306, 279, 320, 302, 288, 270, 260, 293, 315, 294, 340, 292, 298, 330, 310. Construct a frequency distribution table and histogram. Determine the range and interval midpoints.

9. From a bin of 200 2-kΩ ±5% carbon resistors, 25 were selected to be measured in the laboratory. The results were: 1.98 kΩ, 2.00 kΩ, 1.96 kΩ, 2.02 kΩ, 2.00 kΩ, 1.93 kΩ, 2.03 kΩ, 2.00 kΩ, 1.98 kΩ, 1.96 kΩ, 1.90 kΩ, 2.05 kΩ, 2.02 kΩ, 2.00 kΩ, 1.98 kΩ, 2.04 kΩ, 1.98 kΩ, 2.04 kΩ, 2.00 kΩ, 2.03 kΩ, 2.09 kΩ, 2.04 kΩ, 2.06 kΩ, 2.05 kΩ, 2.08 kΩ. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.

11. In a production run, the number of defective parts per 100 were tallied over a 20-day period. The results were: day 1—3, day 2—4, day 3—3, day 4—2, day 5—6, day 6—4, day 7—3, day 8—5, day 9—2, day 10—1, day 11—3, day 12—5, day 13—4, day 14—3, day 15—1, day 16—3, day 17—5, day 18—2, day 19—4, day 20—3. Construct a frequency distribution table and a histogram.

6. Twenty high-gain amplifiers were constructed in the lab using components off the shelf. The design called for a voltage gain of 400 dB. The voltage gains were measured to determine the actual gain of the amplifiers. The results (in dB) were: 403, 402, 381, 417, 370, 397, 408, 388, 408, 415, 406, 398, 394, 411, 417, 405, 384, 434, 426, 404. Construct a frequency distribution table and histogram. Determine the range and interval midpoints.

8. In a manufacturing company, the following items were produced over a 25-day period: 125, 101, 113, 94, 78, 88, 94, 107, 112, 96, 102, 104, 96, 85, 89, 93, 98, 103, 119, 111, 102, 105, 98, 95, 104. Construct a frequency distribution table and histogram. Determine the range and interval midpoints.

10. From a bin of 100 47-kΩ ±5% carbon resistors, 25 were selected to be measured in the laboratory. The results were: 46.5 kΩ, 45.5 kΩ, 48.7 kΩ, 47.2 kΩ, 49.8 kΩ, 47.1 kΩ, 46.7 kΩ, 47.0 kΩ, 46.8 kΩ, 45.3 kΩ, 46.8 kΩ, 49.0 kΩ, 47.3 kΩ, 46.9 kΩ, 46.2 kΩ, 47.8 kΩ, 46.0 kΩ, 48.5 kΩ, 47.0 kΩ, 48.1 kΩ, 46.8 kΩ, 44.5 kΩ, 47.6 kΩ, 47.5 kΩ, 46.5 kΩ. Construct a frequency distribution table. From the table, construct a histogram. Determine the range and interval midpoints.

12. In a production run, the number of defective parts per 100 was tallied over a 30-day period. The results were: day 1—4, day 2—3, day 3—6, day 4—3, day 5—2, day 6—5, day 7—3, day 8—5, day 9—2, day 10—3, day 11—1, day 12—4, day 13—5, day 14—2, day 15—6, day 16—3, day 17—4, day 18—5, day 19—4, day 20—3, day 21—4, day 22—5, day 23—4, day 24—5, day 25—6, day 26—2, day 27—4, day 28—3, day 29—2, day 30—4. Construct a frequency distribution table and a histogram.

END OF CHAPTER PROBLEMS 26–2

1. Using the frequency distribution table developed for problem 1 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

3. Using the frequency distribution table developed for problem 3 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

5. Using the frequency distribution table developed for problem 5 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

7. Using the frequency distribution table developed for problem 7 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

9. Using the frequency distribution table developed for problem 9 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

2. Using the frequency distribution table developed for problem 2 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

4. Using the frequency distribution table developed for problem 4 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

6. Using the frequency distribution table developed for problem 6 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

8. Using the frequency distribution table developed for problem 8 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

10. Using the frequency distribution table developed for problem 10 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*
Tally	|	|||	𝍸	𝍸||	𝍸	||||	||	|	|
	1	3	5	7	5	4	2	1	1

FIGURE 26–31

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*
Tally	|	||	|||	𝍸	𝍸𝍸	𝍸|	||||	||	|
	1	2	3	5	10	6	4	2	1

FIGURE 26–32

11. Using the frequency distribution table developed for problem 11 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

12. Using the frequency distribution table developed for problem 12 of End of Chapter Problems 26–1, find the mean, median, and mode for the data set.

END OF CHAPTER PROBLEMS 26–3

1. From the data set in Figure 26–31, find the mean, median, mode, range, and standard deviation.
2. From the data set in Figure 26–32, find the mean, median, mode, range, and standard deviation.
3. From the data set in Figure 26–33, find the mean, median, mode, range, and standard deviation.
4. From the data set in Figure 26–34, find the mean, median, mode, range, and standard deviation.
5. From the data set in Figure 26–35, find the mean, median, mode, range, and standard deviation.
6. From the data set in Figure 26–36, find the mean, median, mode, range, and standard deviation.

Scores	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	||	|	||||	|||	𝍸|	𝍸𝍸	𝍸	||||	||
	2	1	4	3	6	10	5	4	2

FIGURE 26–33

Scores	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	|		|||	𝍸|	𝍸||	𝍸||||	𝍸	||	|
	1	0	3	6	7	9	5	2	1

FIGURE 26–34

Score intervals	*50–56*	*57–63*	*64–70*	*71–77*	*78–84*	*85–91*	*92–98*
Tally	||	𝍸	𝍸||	𝍸|||	𝍸|	||||	||
	2	5	7	8	6	4	2

FIGURE 26–35

Score intervals	*50–56*	*57–63*	*64–70*	*71–77*	*78–84*	*85–91*	*92–98*
Tally	|	||	𝍸	𝍸|||	𝍸𝍸	𝍸||	||||
	1	2	5	8	10	7	4

FIGURE 26–36

FIGURE 26–37

Score intervals	***50–54***	***55–59***	***60–64***	***65–69***	***70–74***	***75–79***	***80–84***	***85–89***	***90–94***
Tally	𝍷𝍷	𝍸	𝍸	𝍷𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷	𝍸𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷
	2	5	5	3	4	6	8	3	1

FIGURE 26–38

Score intervals	***50–54***	***55–59***	***60–64***	***65–69***	***70–74***	***75–79***	***80–84***	***85–89***	***90–94***
Tally	𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍸	𝍸	𝍷𝍷
	1	4	7	4	3	4	10	5	2

Item intervals	***300–340***	***341–381***	***382–422***	***423–463***	***464–504***	***505–545***	***546–586***	***587–627***
Tally	𝍷	𝍷𝍷𝍷	𝍸	𝍸𝍷𝍷𝍷	𝍸𝍸𝍷𝍷	𝍸𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷
	1	3	5	8	12	6	4	2

FIGURE 26–39

Item intervals	***710–760***	***761–811***	***812–862***	***863–913***	***914–964***	***965–1015***	***1016–1066***	***1067–1117***
Tally	𝍷𝍷	𝍷𝍷𝍷	𝍸𝍷𝍷	𝍸𝍸𝍷	𝍸𝍷𝍷	𝍸	𝍷𝍷𝍷	𝍷
	2	3	7	11	7	5	3	1

FIGURE 26–40

Item intervals	***500–520***	***521–541***	***542–562***	***563–583***	***584–604***	***605–625***	***626–646***
Tally	𝍷𝍷𝍷𝍷	𝍸𝍷	𝍸𝍸𝍷	𝍸𝍷𝍷	𝍸	𝍷𝍷𝍷	𝍷
	4	6	11	7	5	3	1

FIGURE 26–41

Item intervals	***140–170***	***171–201***	***202–232***	***233–263***	***264–294***	***295–325***	***326–356***	***357–387***	***388–418***
Tally	𝍷𝍷	𝍷𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷	𝍸𝍸	𝍸𝍷𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷	𝍷
	2	3	4	6	10	7	4	2	1

FIGURE 26–42

7. From the data set in Figure 26–37, find the mean, median, mode, range, and standard deviation.
9. From the data set in Figure 26–39, find the mean, median, mode, range, and standard deviation.
11. From the data set in Figure 26–41, find the mean, median, mode, range, and standard deviation.
8. From the data set in Figure 26–38, find the mean, median, mode, range, and standard deviation.
10. From the data set in Figure 26–40, find the mean, median, mode, range, and standard deviation.
12. From the data set in Figure 26–42, find the mean, median, mode, range, and standard deviation.

END OF CHAPTER PROBLEMS 26–4

1. 500 widgets were selected to be measured. The mean length was designed to be 40 mm. If the standard deviation was 1 mm, what would we expect a normal distribution to be?
3. The mean average of parts produced each day during the month of June was 600. If the standard deviation was 10, what would a normal distribution be for that 30-day period?
5. 5000 scores were reported on a standardized test. The mean score was 73, with a standard deviation of 5.6. What would be the normal distribution?
2. 200 widgets were selected to be measured. The mean length was designed to be 200 mm. If the standard deviation was 1.5 mm, what would we expect a normal distribution to be?
4. The mean average of parts produced each day during the month of June was 400. If the standard deviation was 7.5, what would a normal distribution be for that 30-day period?
6. 4000 scores were reported on a standardized test. The mean score was 77.5, with a standard deviation of 6.3. What would be the normal distribution?

APPENDIX C

Answers to Selected Problems

CHAPTER 1

PRACTICE PROBLEMS 1–1

1. (a) 9 ten-thousands or 90,000
2. (a) 0 (b) 1 (c) 6 (d) 8
3. $823 \times 1000 = 823{,}000$. The three has a value of 3000.

END OF CHAPTER PROBLEMS 1–1

1. (a) 0 (b) 2 (c) 8000 (d) 4
3. (a) 7 (b) 0 (c) 7000 (d) 4
5. (a) 3 (b) 9 (c) 800,000 (d) 4
7. 9000

PRACTICE PROBLEMS 1–2

1. (a) 0.04 (b) 0.023 (c) 0.00203
2. (a) $\frac{5}{100}$ (b) $\frac{73}{100{,}000}$ (c) $\frac{9}{100{,}000}$
3. (a) $\frac{7}{10} = 0.7$ (b) $\frac{37}{100} = 0.37$
   (c) $\frac{7}{10{,}000} = 0.0007$
   (d) $\frac{417}{100{,}000} = 0.00417$
   (e) $\frac{6}{1{,}000{,}000} = 0.000006$
4. The 2 appears in the thousandths position.
   The 6 appears in the hundred-thousandths position.
5. (a) 3 hundredths
   (b) 5 ten-thousandths
   (c) 73 hundred-thousandths
6. $\frac{18}{1000} = 0.018$. Record 0.018.
7. $\frac{29}{100{,}000}$ inch

END OF CHAPTER PROBLEMS 1–2

1. (a) 0.3 (b) 0.016 (c) 0.00278 (d) 0.1763 (e) 0.435 (f) 0.2060
3. (a) $\frac{7}{1000}$ (b) $\frac{432}{10{,}000}$ (c) $\frac{174}{1000}$ (d) $\frac{65}{1{,}000{,}000}$ (e) $\frac{16}{100{,}000}$ (f) $\frac{1234}{100{,}000}$
5. (a) $\frac{17}{1000} = 0.017$ (b) $\frac{4}{100} = 0.04$ (c) $\frac{460}{10{,}000} = 0.0460$
   (d) $\frac{27}{1{,}000{,}000} = 0.000027$ (e) $\frac{1780}{100{,}000} = 0.01780$ (f) $\frac{65}{1000} = 0.065$

7. (a) Thousandths (b) Millionths (c) Ten-thousandths
9. (a) Ten-thousandths (b) Millionths (c) Hundredths
11. (a) Six-thousandths (b) One hundred forty-seven thousandths
(c) Ninety-two hundred-thousandths (d) Seven-millionths
(e) Four hundred thirteen ten-thousandths (f) One hundred one ten-thousandths
13. 0.27
15. $\frac{43}{1000}$

PRACTICE PROBLEMS 1–3

1. (a) $24\frac{7}{1000}$ (b) $706\frac{24}{1000}$ (c) $4\frac{17}{100{,}000}$ (d) $7\frac{400}{1000}$ (e) $370\frac{6}{1000}$ (f) $47\frac{113}{1000}$
2. (a) 1076.07 (b) 8.0023 (c) 76.014 (d) 35.00270 (e) 713.007 (f) 8.0087
3. (a) $56.78 = 56\frac{78}{100}$ (b) $15.035 = 15\frac{35}{1000}$ (c) $106.0008 = 106\frac{8}{10{,}000}$
(d) $704.00214 = 704\frac{214}{100{,}000}$ (e) $4075.0014 = 4075\frac{214}{10{,}000}$ (f) $80.043 = 80\frac{43}{1000}$
4. $3 + 0.27 = 3.27$. You would record 3.27.

END OF CHAPTER PROBLEMS 1–3

1. (a) $7\frac{14}{100}$ (b) $50\frac{2}{100}$ (c) $710\frac{143}{1000}$ (d) $9\frac{99}{1000}$ (e) $73\frac{653}{1000}$ (f) $207\frac{7834}{10{,}000}$
(g) $28\frac{736}{100{,}000}$ (h) $8\frac{706}{10{,}000}$
3. (a) 5.68 (b) 25.007 (c) 7.0165 (d) 70.4 (e) 473.025 (f) 80.00743 (g) 2475.000035 (h) 307.00008
5. (a) $93.7 = 93\frac{7}{10}$ (b) $30.04 = 30\frac{4}{100}$ (c) $11.0001 = 11\frac{1}{10{,}000}$ (d) $905.052 = 905\frac{52}{1000}$
(e) $78.034 = 78\frac{34}{1000}$
7. (a) $273.00025 = 273\frac{25}{100{,}000}$ (b) $704.000704 = 704\frac{704}{1{,}000{,}000}$ (c) $2044.0504 = 2044\frac{504}{10{,}000}$
(d) $10{,}101.00089 = 10{,}101\frac{89}{100{,}000}$ (e) $90.0466 = 90\frac{466}{10{,}000}$ (f) $207.0000100 = 207\frac{100}{10{,}000{,}000}$
9. 4.079

PRACTICE PROBLEMS 1–4

	Tens	*Hundreds*	*Thousands*
1.	2710	2700	3000
2.	6530	6500	7000
3.	43,260	43,300	43,000
4.	76,550	76,500	77,000
5.	82,800	82,800	83,000
6.	26,760	26,800	27,000
7.	78,230	78,200	78,000
8.	19,000	19,000	19,000

9. Place in the bin marked 8300 ohms.

In problem 8, when we round to the nearest ten, we round up because the units digit is 9. Remember that we round up by adding 1 to the digit to the left (the tens position). Adding 1 to 9 in the tens position produces a 0 and a carry into the hundreds position. A carry into the hundreds position results in 0 and a carry into the thousands position, resulting in the answer 19,000.

END OF CHAPTER PROBLEMS 1–4

1. 20 3. 50 5. 60 7. 130 9. 870

	Tens	*Hundreds*
11.	270	300
13.	360	400
15.	1380	1400
17.	1410	1400
19.	8710	8700

	Tens	*Hundreds*	*Thousands*
21.	4820	4800	5000
23.	85,470	85,500	85,000
25.	78,670	78,700	79,000
27.	27,850	27,800	28,000
29.	35,490	35,500	35,000
31.	68,450	68,400	68,000
33.	73,650	73,700	74,000
35.	470,000		
37.	2500		

PRACTICE PROBLEMS 1–5

	Hundredth	*Tenth*	*One*	*Ten*
1.	73.65	73.6	74	70
2.	26.40	26.4	26	30
3.	17.04	17.0	17	20
4.	30.69	30.7	31	30
5.	50.47	50.5	50	50
6.	48.05	48.0	48	50
7.	33.78	33.8	34	30
8.	68.15	68.1	68	70

9. 28.3 volts
10. 1020, 6270, and 8760
11. 47.1 to the nearest tenth. 50 to the nearest ten.

END OF CHAPTER PROBLEMS 1–5

	Hundredths	*Tenths*	*Ones*	*Tens*
1.	163.78	163.8	164	160
3.	9.46	9.5	9	10
5.	88.89	88.9	89	90
7.	749.49	749.5	749	750
9.	39.28	39.3	39	40
11.	63.75	63.7	64	60
13.	478.67	478.7	479	480
15.	47.47	47.5	47	50
17.	16.55	16.5	17	20
19.	73.8			

PRACTICE PROBLEMS 1–6

1. (a) 4 (b) 3 (c) 5 (d) 5 (e) 3
2. (a) 4 (b) 2 (c) 7
3. 7 and 3

END OF CHAPTER PROBLEMS 1–6

1. (a) 5 (b) 4 (c) 4 (d) 4 (e) 3 (f) 4
3. (a) 8 (b) 2
5. (a) 6 (b) 3
7. (a) 8 (b) 7
9. 2 and 5

SELF-TEST 1–1 THROUGH 1–6

1. (a) 2 (b) 8 (c) 0 (d) 7 (e) 300 (f) 8
2. (a) 0.41 (b) 0.009 (c) 0.01783
3. (a) $\frac{3}{10} = 0.3$ (b) $\frac{85}{100} = 0.85$ (c) $\frac{18}{10{,}000} = 0.0018$
4. (a) Thirty-three hundredths
   (b) Four-thousandths
5. (a) $7\frac{46}{100}$ (b) $18\frac{6}{1000}$
6. (a) 76.14 (b) 6.023
7. (a) $3.07 = 3\frac{7}{100}$ (b) $28.063 = 28\frac{63}{1000}$

8.

	Ten	*Hundred*	*Thousand*
(a)	4770	4800	5000
(b)	9710	9700	10,000
(c)	15,790	15,800	16,000
(d)	37,050	37,000	37,000

9.

	Tenth	*Hundredth*	*Thousandth*
(a)	0.1	0.07	0.075
(b)	0.5	0.46	0.461
(c)	0.4	0.41	0.406
(d)	0.4	0.37	0.375

10.

	Ten	*Unit*	*Tenth*	*Hundredth*
(a)	20	17	17.5	17.49
(b)	20	23	23.5	23.46
(c)	40	37	36.5	36.55
(d)	20	21	20.7	20.71

PRACTICE PROBLEMS 1–7

1. 2
2. −20
3. 12
4. 12
5. −4
6. −19
7. 18
8. −5
9. 7
10. 16
11. −3
12. $5 - 3 - 4 + 2 = 0$ amps

END OF CHAPTER PROBLEMS 1–7

1. 5
3. 14
5. −6
7. 34
9. −28
11. 7
13. −14
15. 18
17. 21
19. 10
21. 3

PRACTICE PROBLEMS 1–8

1. −85
2. 224
3. −108
4. −168
5. −7
6. −7
7. −43
8. −7
9. 11
10. 12
11. 438
12. 26
13. 220
14. $E = IR = -3$ amps × 8 ohms $= -24$ volts

END OF CHAPTER PROBLEMS 1–8

1. −42
3. 54
5. −84
7. 225
9. 930
11. −7
13. −24
15. 12
17. −32
19. 378
21. 42
23. 760
25. −120 V

PRACTICE PROBLEMS 1–9

1. 26
2. −9
3. 10
4. −40
5. 3
6. −22
7. 19
8. 8
9. $E = I_1 \times R_1 + I_2 \times R_2 = 3 \times 8 + 7 \times 4 = 24 + 28 = 52$ V

END OF CHAPTER PROBLEMS 1–9

1. −17
3. 0
5. 17
7. 21
9. 18
11. 26
13. −21
15. 12
17. 15
19. −19
21. 21.156

PRACTICE PROBLEMS 1–10

1. −6
2. 0
3. 12
4. 21
5. −20
6. −8
7. −26
8. 102
9. 104
10. 202
11. $E = 18(12/24) + 22(21/42) = 18(0.5) + 22(0.5) = 9 + 11 = 20$

END OF CHAPTER PROBLEMS 1–10

1. 16
3. 7
5. −36
7. −83
9. 4
11. 51
13. 130
15. 45
17. −144
19. 20
21. 27.5

SELF-TEST 1–7 THROUGH 1–10

1. 5
2. −18
3. −48
4. 36
5. −12
6. −17
7. 16
8. −9
9. −57
10. −27
11. 13
12. 22
13. 10
14. 37
15. 986

CHAPTER 2

PRACTICE PROBLEMS 2–1

1. 10^2
2. 10^5
3. 10^1
4. 10^7
5. 10^3
6. 10^4
7. 10^6
8. 10^8

The answer to problem 3 could be written simply as 10 since $10^1 = 10$. In the future, when the number is raised to the first power, we will not write the exponent.

END OF CHAPTER PROBLEMS 2–1

1. 10^1
3. 10^3
5. 10^2

PRACTICE PROBLEMS 2–2

1. 100
2. 10
3. 1,000,000
4. 10,000,000
5. 1,000,000,000

END OF CHAPTER PROBLEMS 2–2

1. 1000
3. 1
5. 1,000,000

PRACTICE PROBLEMS 2–3

1. (a) 0.0001 (b) $\frac{1}{10{,}000}$
2. (a) 0.01 (b) $\frac{1}{100}$
3. (a) 0.1 (b) $\frac{1}{10}$
4. (a) 0.00000001 (b) $\frac{1}{100{,}000{,}000}$
5. (a) 0.001 (b) $\frac{1}{1000}$
6. (a) 0.0000001 (b) $\frac{1}{10{,}000{,}000}$
7. (a) 0.000001 (b) $\frac{1}{1{,}000{,}000}$
8. (a) 0.00001 (b) $\frac{1}{100{,}000}$

END OF CHAPTER PROBLEMS 2–3

1. $\frac{1}{100} = 0.01$
3. $\frac{1}{1000} = 0.001$
5. $\frac{1}{10} = 0.1$

PRACTICE PROBLEMS 2–4

1. 10^{-3}
2. 10^{-1}
3. 10^{-6}
4. 10^{-2}
5. 10^{-5}
6. 10^{-7}
7. 10^{-4}
8. 10^{-8}
9. 10^{-3}
10. 10^{-5}
11. 10^{-1}
12. 10^{-6}
13. 10^{-2}
14. 10^{-4}

END OF CHAPTER PROBLEMS 2–4

1. 10^{-4} 3. 10^{-6} 5. 10^{-4} 7. 10^{-3}

PRACTICE PROBLEMS 2–5

1. 10^{5}
2. 10^{10}
3. 10^{-4}
4. 10^{-8}
5. 10^{3}
6. 10^{2}
7. 10^{-1}
8. 10^{0}
9. 10^{3}
10. 10^{4}

END OF CHAPTER PROBLEMS 2–5

1. 10^{10}
3. 10^{6}
5. 10^{4}
7. 10^{-4}
9. 10^{3}
11. 10^{-10}
13. 10^{-12}
15. 10^{-9}

PRACTICE PROBLEMS 2–6

1. 10^{2}
2. 10^{3}
3. 10^{-3}
4. 10^{-5}
5. 10^{4}
6. 10^{8}
7. 10^{2}
8. 10^{-4}
9. 10^{2}
10. 10^{3}
11. 10^{-2}
12. 10^{-3}
13. 10^{-2}
14. 10^{4}
15. 10^{-6}

END OF CHAPTER PROBLEMS 2–6

1. 10^{4}
3. 10^{-5}
5. 10^{8}
7. 10^{12}
9. 10^{7}
11. 10^{-3}
13. 10^{-6}
15. 10^{-7}
17. 10^{3}
19. 10^{-4}

PRACTICE PROBLEMS 2–7

1. 10^{-5}
2. 10^{-10}
3. 10^{10}
4. 10^{-2}
5. 10^{-3}
6. 10^{6}
7. 10^{-7}
8. 10^{4}
9. 10^{6}
10. $10^{0} = 1$

END OF CHAPTER PROBLEMS 2–7

1. 10^{-7}
3. 10^{-7}
5. 10^{11}
7. 10^{10}
9. 10^{-12}
11. 10^{8}
13. 10^{13}
15. 10^{-4}
17. 10^{-36}

SELF-TEST 2–1 THROUGH 2–7

1. 10^{3}
2. 10^{6}
3. 10^{-3}
4. 10^{-7}
5. 10^{-4}
6. 10^{-5}
7. 1000
8. 100,000
9. 100

10. 1
11. 0.0001
12. 0.1
13. 0.00001
14. 0.00000001
15. $\frac{1}{100} = 0.01$
16. $\frac{1}{1000} = 0.001$
17. $\frac{1}{100{,}000} = 0.00001$
18. $\frac{1}{10} = 0.1$
19. 10^7
20. 10^{-1}
21. 10^4
22. 10^{-9}
23. 10^{-3}
24. 10^{10}
25. 10^{-9}
26. 10^3
27. 10^5
28. 10
29. 10^{-3}
30. 10^{-5}
31. 10^8
32. 10^{-3}
33. 10^{-3}

PRACTICE PROBLEMS 2–8

1. (a) 0.386×10^2, (b) 0.0386×10^3, (c) 0.00386×10^4
2. (a) 56.0×10^2, (b) 5.60×10^3, (c) 0.560×10^4
3. (a) 0.00905×10^{-2}, (b) 90.5×10^{-6}, (c) 9050×10^{-8}
4. (a) 0.0277×10^{-5}, (b) 2.77×10^{-7}, (c) 277×10^{-9}
5. (a) 0.180×10, (b) 0.00180×10^3, (c) 18.0×10^{-1}, (d) 1800×10^{-3}
6. (a) 0.000000833×10^6, (b) 0.000833×10^3, (c) 833×10^{-3}, (d) $833{,}000 \times 10^{-6}$

END OF CHAPTER PROBLEMS 2–8

1. (a) 4.75×10^2 (b) 0.0475×10^4 (c) 0.000475×10^6
3. (a) 0.659×10^2 (b) 0.00659×10^4 (c) 0.0000659×10^6
5. (a) 934×10^2 (b) 9.34×10^4 (c) 0.0934×10^6
7. (a) 47.8×10^2 (b) 0.478×10^4 (c) 0.00478×10^6
9. (a) 418×10^2 (b) 4.18×10^4 (c) 0.0418×10^6
11. (a) 0.465×10^{-2} (b) 46.5×10^{-4} (c) 4650×10^{-6}
13. (a) 0.000555×10^{-2} (b) 0.0555×10^{-4} (c) 5.55×10^{-6}
15. (a) 0.0673×10^{-2} (b) 6.73×10^{-4} (c) 673×10^{-6}
17. (a) $0.00000108 \times 10^{-2}$ (b) 0.000108×10^{-4} (c) 0.0108×10^{-6}
19. (a) 0.325×10^3 (b) $325{,}000 \times 10^{-3}$
21. (a) 0.722×10^3 (b) $722{,}000 \times 10^{-3}$
23. (a) 2.70×10^3 (b) $2{,}700{,}000 \times 10^{-3}$
25. (a) 0.180×10^3 (b) $180{,}000 \times 10^{-3}$
27. (a) 0.0000257×10^3 (b) 25.7×10^3 (c) 25700×10^{-6}
29. 0.680×10^3; 680×10^{-3}

PRACTICE PROBLEMS 2–9

1. 1.73×10^3
2. 2.67×10^4
3. 1.65×10^6
4. 4.27×10^1
5. 1.73×10^5
6. 4.74×10^4
7. 7.91×10^1
8. 6.78×10^5
9. 1.90×10^2
10. 6.75×10^3

END OF CHAPTER PROBLEMS 2–9

1. 2.76×10^4
3. 4.78×10
5. 1.77×10^6
7. 2.73×10^2
9. 1.73×10^5
11. 7.89×10
13. 1.67×10^6
15. 5.79×10^4
17. 8.10×10^4
19. 9.22×10^4

PRACTICE PROBLEMS 2–10

1. 1.67×10^{-2}
2. 6.41×10^{-1}
3. 3.99×10^{-4}
4. 6.07×10^{-3}
5. 1.09×10^{-1}
6. 7.06×10^{-6}
7. 1.76×10^{-5}
8. 9.10×10^{-3}
9. 7.65×10^{-2}
10. 5.67×10^{-4}

END OF CHAPTER PROBLEMS 2–10

1. 4.78×10^{-3}
3. 7.47×10^{-1}
5. 1.64×10^{-2}
7. 4.54×10^{-3}
9. 5.01×10^{-5}
11. 2.55×10^{-1}
13. 4.00×10^{-2}
15. 8.96×10^{-4}
17. 1.71×10^{-2}
19. 2.05×10^{-4}

PRACTICE PROBLEMS 2–11

1. 1.10×10^{3}
2. 1.01×10
3. 9.00×10^{2}
4. 3.20×10^{3}
5. 1.03
6. 1.00×10^{4}
7. 2.55×10^{-2}
8. 4.90×10^{2}
9. 1.31×10^{5}
10. 7.69×10^{-4}
11. $5830 + 0.245 = 5.83 \times 10^{3}$
12. $0.000054 \times 10^{2} - 37.8 \times 10^{-3} = -3.24 \times 10^{-2}$
13. 45.8×10^{2} ft $+ 7.96 \times 10^{2}$ ft $+ 2738$ ft $+ 357.5 \times 10$ ft $= 1.17 \times 10^{4}$ ft
14. $27{,}000\ \Omega + 7.5 \times 10^{3}\ \Omega + 3.3 \times 10^{4} + 10 \times 10^{3}\ \Omega = 7.75 \times 10^{4}\ \Omega$

END OF CHAPTER PROBLEMS 2–11

1. 1.10×10^{4}
3. 2.00×10^{-1}
5. 3.86×10^{3}
7. 4.82×10^{5}
9. 2.88×10^{-1}
11. 3.87×10^{-2}
13. 9.93×10^{4}
15. 8.00×10^{5}

PRACTICE PROBLEMS 2–12

1. $587 \approx 6 \times 10^{2}$ $43.8 \approx 4 \times 10$ $6 \times 10^{2} \times 4 \times 10 = 24 \times 10^{3} = 2.4 \times 10^{4}$
$587 \times 43.8 = 2.57 \times 10^{4}$
2. $713 \approx 7 \times 10^{2}$ $8.53 \approx 9$ $7 \times 10^{2} \times 9 = 63 \times 10^{2} = 6.3 \times 10^{3}$ $713 \times 8.53 = 6.08 \times 10^{3}$
3. $832.51 \approx 8 \times 10^{2}$ $47.956 \approx 5 \times 10$ $8 \times 10^{2} \times 5 \times 10 = 40 \times 10^{3} = 4.0 \times 10^{4}$
$832.51 \times 47.956 = 3.99 \times 10^{4}$
4. $397.93 \approx 4 \times 10^{2}$ $0.00052439 \approx 5 \times 10^{-4}$ $4 \times 10^{2} \times 5 \times 10^{-4} = 20 \times 10^{-2} =$
2.0×10^{-1} $397.63 \times 0.00052439 = 2.09 \times 10^{-1}$
5. $694 \approx 7 \times 10^{2}$ $137.9 \approx 1 \times 10^{2}$ $7 \times 10^{2} \div 1 \times 10^{2} = 7.0$ $694 \div 137.9 = 5.03$
6. $3870 \approx 4 \times 10^{3}$ $88.72 \approx 9 \times 10$ $4 \times 10^{3} \div 9 \times 10 \approx 0.5 \times 10^{2} = 5.0 \times 10$
$3870 \times 88.72 \approx 4.36 \times 10$
7. $55.374 \approx 6 \times 10$ $0.0356 \approx 4 \times 10^{-2}$ $6 \times 10 \div 4 \times 10^{-2} = 1.5 \times 10^{3}$
$55.374 \div 0.0356 = 1.56 \times 10^{3}$
8. $0.9327 \approx 9 \times 10^{-1}$ $9832.7 \approx 1 \times 10^{4}$ $9 \times 10^{-1} \div 1 \times 10^{4} = 9.0 \times 10^{-5}$ $0.9327 \div 9832.7 = 9.49 \times 10^{-5}$
9. To the nearest ten: \$30 × 40 hours = \$1200
To the nearest cent: \$33.25 × 37 = \$1230.25
10. To the nearest hundred: 1900 ft × 600 ft = 1,140,000 = 1.14×10^{6} sq ft
Actual area: 1940.7 ft × 620.4 ft = 1.20×10^{6} sq ft
11. $I = \frac{E}{R} = \frac{22\text{ V}}{47} \times 10^{3}\ \Omega \approx \frac{20\text{ V}}{50} \times 10^{3} = 0.4 \times 10^{-3} = 4 \times 10^{-4}\text{ A}$
Actual: $I = \frac{22\text{ V}}{47} \times 10^{3}\ \Omega = 4.68 \times 10^{-4}\text{ A}$

END OF CHAPTER PROBLEMS 2–12

Estimated answers are given, then answers rounded to three places are given.

1. $3.5 \times 10^{4}, 3.97 \times 10^{4}$
3. $3.6 \times 10^{-5}, 3.48 \times 10^{-5}$
5. $6.0 \times 10, 7.85 \times 10$
7. $3.3 \times 10^{7}, 3.18 \times 10^{7}$
9. $1.0 \times 10^{5}, 9.90 \times 10^{4}$
11. $2.0 \times 10, 2.08 \times 10$
13. $5.0 \times 10^{-2}, 4.68 \times 10^{-2}$
15. $3.0 \times 10^{2}, 2.71 \times 10^{2}$

PRACTICE PROBLEMS 2–13

1. 4.55
2. 8.42
3. 1.49×10
4. 9.14
5. 7.85
6. 1.42
7. 2.27
8. 1.82

END OF CHAPTER PROBLEMS 2–13

1. 1.09×10
3. 4.55
5. 1.48×10
7. 2.23×10
9. 1.07×10
11. 1.39
13. 1.74
15. 1.09
17. 2.53
19. $1316; $1292.50
21. 1.44×10^{6}; 1.45×10^{6}
23. 3.00×10^{-5}; 2.73×10^{-5}

PRACTICE PROBLEMS 2–14

1. 2.13×10^{-3}
2. 3.03×10^{-6}
3. 1.47×10^{7}
4. 2.70×10^{2}
5. 3.62×10^{-4}
6. 1.79×10^{-4}
7. 2.00×10^{-5}
8. 2.70×10^{3}
9. 2.50×10^{2}
10. 1.00×10^{-4}
11. 9.26×10^{-4}
12. 3.60×10^{-5}
13. 2.17×10^{-4}
14. 6.00×10^{-5}
15. 1.08×10^{4}
16. 3.07×10^{3}
17. 1.50×10^{-5}
18. 1.31×10^{2}
19. $G_T = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{470}\,\Omega + \dfrac{1}{560}\,\Omega = 2.13 \times 10^{-3} + 1.79 \times 10^{-3} = 3.91 \times 10^{-3}$ S. The siemen (S) is the unit of conductance.

END OF CHAPTER PROBLEMS 2–14

1. 1.06×10^{2}
3. 1.27×10^{-4}
5. 3.26×10^{-3}
7. 1.67×10^{-3}
9. 3.91×10^{-5}
11. 5.16×10^{-6}
13. 1.10×10^{-4}
15. 2.56×10^{3}
17. 8.46×10
19. 3.07×10^{4}
21. 2.44×10^{3}
23. 4.36×10^{-3}

SELF-TEST 2–8 THROUGH 2–14

1. 1.76×10^{3}
2. $6.75 \times 10^{0} = 6.75$
3. $1.99 \times 10^{0} = 1.99$
4. 4.35×10^{7}
5. 8.91×10^{9}
6. 5.26×10^{4}
7. 4.43×10^{-5}
8. 7.85×10^{-3}
9. 5.59×10^{5}
10. 5.18×10^{-2}
11. 2.08×10
12. 1.78×10^{-1}
13. 7.81×10^{-1}
14. 6.27×10^{-3}
15. $2.27 \times 10^{0} = 2.27$
16. 3.27×10^{-4}
17. 2.82×10^{4}
18. $1.89 \times 10^{0} = 1.89$
19. 2.56×10^{4}
20. 1.13×10^{3}

PRACTICE PROBLEMS 2–15

1. 10^{6}
2. 10^{8}
3. 10^{9}
4. 10^{-2}
5. 10^{-20}
6. 10^{-12}
7. 10^{6}
8. 10^{3}
9. 10^{4}
10. 10^{-15}
11. 10^{-2}
12. 10^{-12}
13. 10^{-8}
14. 10^{6}
15. 10^{9}

END OF CHAPTER PROBLEMS 2–15

1. 10^{6}
3. 10^{10}
5. 10^{-12}
7. 10^{12}
9. 10^{-8}

PRACTICE PROBLEMS 2–16

1. 10^{3}
2. 10^{3}
3. 10^{-3}
4. 10^{-4}
5. 10
6. 10^{-2}
7. 10
8. 10^{-2}
9. 10^{-5}
10. 10^{5}

END OF CHAPTER PROBLEMS 2–16

1. 10^{3}
3. 10^{-2}
5. 10^{-2}
7. 10^{5}
9. 10^{3}

PRACTICE PROBLEMS 2–17

1. 3.97×10
2. 1.85×10^{3}
3. 1.38×10^{-3}
4. 2.21×10^{-7}
5. 4.53×10^{5}
6. 1.08×10^{-6}
7. 6.34×10
8. 7.68×10^{7}
9. 9.47×10^{-1}
10. 3.21×10^{-13}
11. 7.62×10^{-1}
12. 1.59
13. 2.50×10^{-1}
14. 4.96×10^{-5}
15. 1.21×10^{-1}
16. 3.00×10^{-1}
17. Area $= S^2 = (78.4\ \text{yd})^2 = 6.15 \times 10^3$ square yards
18. The area of the shower stall $= (36\ \text{in.})^2 = 1296\ \text{in}^2$. The area of one tile $= (4\ \text{in.})^2 = 16\ \text{in}^2$. $1296 \div 16 = 81$. 81 tiles will be needed.
19. $P = I^2R = (48.7 \times 10^{-4}\ \text{A})^2 \times 5.6 \times 10^3\ \Omega = 1.33 \times 10^{-1}\ \text{W}$

END OF CHAPTER PROBLEMS 2–17

1. 6.29×10^{3}
3. 3.17×10^{6}
5. 2.26×10^{-3}
7. 7.62×10^{-5}
9. 2.03×10^{9}
11. 9.00×10^{10}
13. 8.76×10^{-8}
15. 8.15×10^{-9}
17. 6.75
19. 2.17
21. 2.88×10^{3}
23. 144
25. 2.53×10 W

PRACTICE PROBLEMS 2–18

1. 6.04
2. 9.62×10^{-1}
3. 2.52
4. 1.37×10^{3}
5. 2.60×10^{-1}
6. 5.22×10
7. 2.34×10^{-2}
8. 1.32×10
9. 3.07×10^{-1}
10. 6.50×10^{3}
11. $I = \left(\dfrac{P}{R}\right)^{1/2} = \left(38.1 \times \dfrac{10^{-2}\ \text{W}}{91} \times 10^2\ \Omega\right)^{1/2} = 6.47 \times 10^{-3}\ \text{A}$

END OF CHAPTER PROBLEMS 2–18

1. 9.64
3. 1.54×10
5. 8.40×10^{-2}
7. 1.02×10^{-2}
9. 5.20×10^{3}
11. 6.71×10^{3}
13. 3.04×10^{-3}
15. 5.88
17. 1.07×10^{-2}
19. 7.96×10^{3}
21. 1.58×10^{4}
23. 1.05×10^{-2} A

SELF-TEST 2–15 THROUGH 2–18

1. 10^{15}
2. 10^{-8}
3. 10^{6}
4. 10^{-3}
5. 10^{2}
6. 10^{-1}
7. 6.13×10^{3}
8. 8.58×10^{-8}
9. 7.74×10^{13}
10. 5.39×10^{-1}
11. 1.83×10^{-3}
12. 1.50×10^{-1}
13. 1.65×10^{1}
14. 1.63×10^{-2}
15. 2.72×10^{-2}
16. 6.79×10
17. 9.21×10^{-3}
18. 8.12
19. 4.77×10^{4}
20. 5.04×10^{2}

CHAPTER 3

PRACTICE PROBLEMS 3–1

1. $27{,}000 = 27 \times 10^3 = 0.027 \times 10^6$
2. $330{,}000 = 330 \times 10^3 = 0.330 \times 10^6$
3. $5600 = 5.60 \times 10^3 = 0.0056 \times 10^6$
4. $390 \times 10^2 = 39 \times 10^3 = 0.039 \times 10^6$
5. $68 \times 10^4 = 680 \times 10^3 = 0.680 \times 10^6$
6. $1{,}200{,}000 = 1200 \times 10^3 = 1.20 \times 10^6$
7. $180 \times 10^4 = 1800 \times 10^3 = 1.8 \times 10^6$
8. $1500 \times 10^5 = 150{,}000 \times 10^3 = 150 \times 10^6$
9. $1800 \times 10^2 = 180 \times 10^3 = 0.180 \times 10^6$
10. $51 \times 10^5 = 5100 \times 10^3 = 5.1 \times 10^6$
11. $0.000423 = 0.423 \times 10^{-3} = 423 \times 10^{-6}$
12. $0.00716 = 7.16 \times 10^{-3} = 7160 \times 10^{-6}$

END OF CHAPTER PROBLEMS 3–1

1. $56 \times 10^3 = 0.056 \times 10^6$
3. $220 \times 10^3 = 0.220 \times 10^6$
5. $390 \times 10^3 = 0.39 \times 10^6$
7. $180 \times 10^3 = 0.18 \times 10^6$
9. $43 \times 10^3 = 0.043 \times 10^6$
11. $0.22 \times 10^{-3} = 220 \times 10^{-6}$

13. $2.13 \times 10^{-3} = 2130 \times 10^{-6}$
15. $0.0556 \times 10^{-3} = 55.6 \times 10^{-6}$
17. $0.122 \times 10^{-3} = 122 \times 10^{-6}$
19. $0.256 \times 10^{-3} = 256 \times 10^{-6}$
21. $667 \times 10^{-9} = 667{,}000 \times 10^{-12}$
23. $17.9 \times 10^{-9} = 17{,}900 \times 10^{-12}$
25. $6.74 \times 10^{-9} = 6740 \times 10^{-12}$
27. $0.0177 \times 10^{-9} = 17.7 \times 10^{-12}$
29. $70 \times 10^{-9} = 70{,}000 \times 10^{-12}$
31. $7.3 = 7300 \times 10^{-3} = 0.0073 \times 10^{3}$
33. $5.67 = 5670 \times 10^{-3} = 0.00567 \times 10^{3}$
35. $0.0178 = 17.8 \times 10^{-3} = 0.0000178 \times 10^{3}$
37. $78.3 = 78{,}300 \times 10^{-3} = 0.0783 \times 10^{3}$
39. $845 = 845{,}000 \times 10^{-3} = 0.845 \times 10^{3}$

PRACTICE PROBLEMS 3–2

1. 2700 Ω = 2.7 kΩ = 0.0027 MΩ
2. 12,000 Hz = 12 kHz = 0.012 MHz
3. 0.00076 A = 0.76 mA = 760 μA
4. 0.000023 A = 0.023 mA = 23 μA
5. 0.0000002 F = 0.2 μF = 200 nF
6. 0.000004 F = 4 μF = 4000 nF
7. 68,000 Ω = 68 kΩ = 0.068 MΩ
8. 120,000 Ω = 120 kΩ = 0.12 MΩ
9. 3,500,000 Hz = 3500 kHz = 3.5 MHz
10. 0.00037 S = 0.37 mS = 370 μS
11. 5×10^{-4} S = 0.5 mS = 500 μS
12. 5.6×10^{4} Ω = 56 kΩ = 0.056 MΩ
13. 21×10^{-2} A = 210 mA = 210,000 μA
14. 68×10^{-10} F = 6.8 nF = 6800 pF
15. 45×10^{-4} H = 4.5 mH = 4500 μH
16. 0.15×10^{-2} V = 1.5 mV = 1500 μV

END OF CHAPTER PROBLEMS 3–2

1. 0.26 mA = 260 μA
3. 0.632 mS = 632 μS
5. 7.63 kΩ = 0.00763 MΩ
7. 17.3 mA = 17,300 μA
9. 713 kΩ = 0.713 MΩ
11. 5630 kHz = 5.63 MHz
13. 2000 kΩ = 2 MΩ
15. 0.237 mS = 237 μS
17. 0.30 μF = 300 nF
19. 0.062 mA = 62 μA

PRACTICE PROBLEMS 3–3

1. 46.7 mA = 0.0467 A
2. 407 mA = 0.407 A
3. 68 kΩ = 68,000 Ω
4. 4.7 kΩ = 4700 Ω
5. 2.73 kHz = 2730 Hz
6. 300 kHz = 300,000 Hz
7. 670 μS = 0.000670 S
8. 37 mS = 0.037 S
9. 55 mH = 0.055 H
10. 465 μH = 0.000465 H
11. 120 kΩ = 120,000 Ω
12. 2 MΩ = 2,000,000 Ω
13. 150 mA = 0.150 A
14. 0.43 mA = 0.00043 A
15. 0.68 kΩ = 680 Ω

END OF CHAPTER PROBLEMS 3–3

1. 0.800 A
3. 0.0025 S
5. 33,000 Ω
7. 470 Ω
9. 12,500 Hz
11. 0.000100 F
13. 0.00025 A
15. 0.000900 S
17. 750,000 Ω
19. 0.30 A

PRACTICE PROBLEMS 3–4

1. 20 mA = 20,000 μA = 0.020 A
2. 0.01 μF = 10 nF = 10,000 pF
3. 2 H = 2000 mH = 2,000,000 μH
4. 2.5 kΩ = 2500 Ω = 0.0025 MΩ
5. 680 Ω = 0.680 kΩ = 0.00068 MΩ
6. 1.8 V = 1800 mV = 1,800,000 μV
7. 20 μA = 0.020 mA = 0.000020 A
8. 1.5 MΩ = 1500 kΩ = 1,500,000 Ω
9. 0.02 S = 20 mS = 20,000 μS
10. 50 nF = 0.050 μF = 50,000 pF
11. 4.7×10^{4} Ω = 47 kΩ = 0.047 MΩ
12. 16×10^{-7} F = 1.6 μF = 1600 nF
13. 2.4×10^{-2} A = 24 mA = 24,000 μA
14. 8.4×10^{2} Ω = 0.84 kΩ = 0.00084 MΩ
15. 56×10^{5} Ω = 5600 kΩ = 5.6 MΩ
16. 27.3 mS = 0.0273 S = 27,300 μS
17. 5600 μS = 0.00560 S = 5.60 mS
18. 170 mW = 0.170 W = 170,000 μW
19. 0.000642 V = 642 μV = 0.642 mV
20. 30 kHz = 0.030 MHz = 30,000 Hz

END OF CHAPTER PROBLEMS 3–4

1. 0.65 mA = 650 μA
3. 0.00805 mS = 8.05 μS
5. 56.2 mS = 56,200 μS
7. 0.613 mS = 613 μS
9. 7.50 kΩ = 0.00750 MΩ
11. 510 kΩ = 0.510 MΩ
13. 127 kHz = 0.127 MHz
15. 713 kΩ = 0.713 MΩ
17. 46,300 μA = 0.0463 A

19. 0.00005 μF = 50 pF
21. 3200 Ω = 0.0032 MΩ
23. 0.270 MΩ = 270,000 Ω
25. 0.000403 S = 0.403 mS
27. 1,030,000 Hz = 1030 kHz
29. 1430 pF = 0.00143 μF
31. 5.5 mS = 5500 μS
33. 0.00106 μF = 1060 pF
35. 463,000 Ω = 0.463 MΩ
37. 96.3 kHz = 0.0963 MHz
39. 7.8 mA = 7800 μA
41. 0.176 V = 176,000 μV
43. 1730 mW = 0.00173 kW
45. 25 mH = 25,000 μH
47. 0.173 S = 173,000 μS
49. 2.5 μF = 2500 nF

SELF-TEST 3–1 THROUGH 3–4

1. $4760 = 4.76 \times 10^3 = 0.00476 \times 10^6$
2. $32.4 \times 10^4 = 324 \times 10^3 = 0.324 \times 10^6$
3. $2.71 \times 10^{-1} = 271 \times 10^{-3} = 0.000271 \times 10^3$
4. $46.7 \times 10^{-2} = 467 \times 10^{-3} = 0.000467 \times 10^3$
5. 76.3 mA = 0.0763 A = 76,300 μA
6. 0.0055 μF = 5.5 nF = 5500 pF
7. 8340 μS = 8.34 mS = 0.00834 S
8. 20 kΩ = 20,000 Ω = 0.020 MΩ
9. 75,000 Hz = 0.075 MHz = 75 kHz
10. 146 mS = 0.146 S = 146,000 μS

PRACTICE PROBLEMS 3–5

1. 27 mA + 0.037 A = 0.064 A = 64 mA
2. 370 μS + 0.060 mS = 0.430 mS = 430 μS
3. $\dfrac{1}{68\text{ k}\Omega} + \dfrac{1}{47\text{ k}\Omega} = 0.036\text{ mS} = 36\ \mu\text{S}$
4. $\dfrac{1}{\dfrac{1}{4.7\text{ k}\Omega} + \dfrac{1}{1.8\text{ k}\Omega}} = 1300\ \Omega = 1.30\text{ k}\Omega$
5. $\dfrac{20\text{ V}}{27\text{ k}\Omega} = 0.000741\text{ A} = 0.741\text{ mA} = 741\ \mu\text{A}$
6. $\dfrac{25\text{ V}}{2.36\text{ mA}} = 10{,}600\ \Omega = 10.6\text{ k}\Omega = 0.0106\text{ M}\Omega$
7. $\dfrac{1}{6.28 \times 12\text{ kHz} \times 50\text{ nF}} = 265\ \Omega = 0.265\text{ k}\Omega$
8. $\dfrac{1}{2.7\text{ k}\Omega} + \dfrac{1}{3.3\text{ k}\Omega} + \dfrac{1}{4.7\text{ k}\Omega} = 0.886\text{ mS} = 886\ \mu\text{S} = 0.000886\text{ S}$
9. $\dfrac{1}{\dfrac{1}{22\text{ k}\Omega} + \dfrac{1}{33\text{ k}\Omega} + \dfrac{1}{56\text{ k}\Omega}} = 10{,}700\ \Omega = 10.7\text{ k}\Omega = 0.0107\text{ M}\Omega$
10. $\dfrac{1}{6340\ \mu\text{S}} = 158\ \Omega = 0.158\text{ k}\Omega$
11. 2.73 kΩ × 0.43 mA = 1.17 V = 1170 mV
12. 1.27 V + 48 mV + 5630 μV = 1.32 V = 1320 mV = 1,320,000 μV
13. 1000 kΩ = 1 MΩ
14. 1330 kΩ = 1.33 MΩ
15. 5450 Ω = 5.45 kΩ
16. 6.67 kΩ = 0.00667 MΩ
17. $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2 \times \pi \times 5 \times 10^3 \times 75 \times 10^{-9}} = 4.24 \times 10^2\ \Omega$

    $X_C = 424\ \Omega = 0.424\text{ k}\Omega = 0.000425\text{ M}\Omega$
18. $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = \dfrac{1}{\dfrac{1}{22\text{ k}\Omega} + \dfrac{1}{3.9 \times 10^4\ \Omega} + \dfrac{1}{180 \times 10^2\ \Omega}} = \dfrac{1}{1.27 \times 10^{-4}\text{ S}} = 7.90 \times 10^3\ \Omega$

    $R_T = 7900\ \Omega = 7.90\text{ k}\Omega$

END OF CHAPTER PROBLEMS 3–5

1. 2.04 mA = 2040 μA
3. 0.0957 mA = 95.7 μA
5. 0.167 mS = 167 μS
7. 0.0582 mS = 58.2 μS
9. 1.42 mS = 1420 μS
11. 964 Ω = 0.964 kΩ
13. 68,700 Ω = 68.7 kΩ = 0.0687 MΩ
15. 6850 Ω = 6.85 kΩ = 0.00685 MΩ
17. 115 Ω = 0.115 kΩ

19. 4690 Ω = 4.69 kΩ
21. 2.30 mS = 2300 μS
23. 0.273 mA = 273 μA
25. 70,000 Ω = 70.0 kΩ
27. 0.0417 mA = 41.7 μA
29. 9.52 V = 9520 mV
31. 4.48 V = 4480 mV
33. 84,900 Ω = 84.9 kΩ
35. 318 Ω = 0.318 kΩ
37. 0.00491 μF = 4.91 nF
39. 919 Hz = 0.919 kHz
41. 5.00×10^3 ohms = 5.00 kΩ
43. 9.99×10^3 ohms = 9.99 kΩ
45. 2.50×10^3 ohms = 2.50 kΩ
47. 571 ohms = 0.571 kΩ
49. 45.5 kΩ

PRACTICE PROBLEMS 3–6

1. 10.63 in
2. 133.9 in = 3.718 yd
3. 5470 yd = 3.107 mi
4. 68.58 cm
5. 402.3 m = 0.4023 km
6. 3.219 km = 3219 m
7. 1312 ft/s
8. 124.3 mi/h
9. 91.44 m/s
10. 104.6 km/h
11. 2.469 oz
12. 35.27 oz = 2.205 lb
13. 112.9 oz = 7.055 lb
14. 0.9072 kg
15. 69.46 g
16. 5 lb 4 oz = (5 lb × 16 oz/lb + 4 oz) × 2 = (80 oz + 4 oz) × 2 = 168 oz
    168 oz ÷ 16 oz/lb = 10 lb 8 oz = 4763 g
17. 5280 ft/mi × 1.25 mi = 6600 ft
    6600 ft ÷ 3 ft/yd = 2200 yds   1.25 mi × 1.609 km/mi = 2.011 km
18. 2 g × 1000 = 2000 g   2000 g ÷ 25 g/pkg = 80 packages
19. 1 gal = 3.785 L   3.785 L/gal × 55 gal = 208.2 L   55 gal × 8 pt/gal = 440 pt
20. 49 ft ÷ 3 ft/yd = 16.33 yd   16.33 yd × 0.9144 m/yd = 14.93 m
21. $2.12/gal ÷ 3.785 L/gal = $0.5600/L
22. 60 mi/hr × 1.609 km/mi = 96.54 km/hr
23. 20 boxes × 42 kg/box = 840.0 kg   840.0 kg × 2.205 lb/kg = 1852 lb
24. 24 cL = 240 mL   240 mL + 5 mL = 245 mL   245 mL ÷ 473.2 mL/pt = 0.5178 pt

END OF CHAPTER PROBLEMS 3–6

1. 39.37 in
3. 15.75 in
5. 2.3 m = 90.58 in = 2.516 yd
7. 10.94 yd
9. 10,940 yd = 6.216 mi
11. 25.40 cm
13. 90.42 cm
15. 804.7 m = 0.8047 km
17. 88.51 km/h
19. 160.9 km/h
21. 1640 ft/s
23. 328.1 ft/s
25. 2.116 oz
27. 5.291 oz
29. 28.22 oz = 1.764 lb
31. 176.4 oz = 11.02 lb
33. 0.6123 kg
35. 113.4 g
37. 7.796 g
39. 110 oz = 6.875 lb = $\frac{3}{118} \times 10^3$ gr
41. 1.254×10^4 ft = 4.180×10^3 yds = 3.821 km
43. 66.7
45. 208.21
47. 25 yd = 27.34 m
49. $0.927/1
51. 70 mi/hr = 113 m/hr
53. 525 kg = 1160 lb
55. 285 ml = 0.603 pts

CHAPTER 24

PRACTICE PROBLEMS 24–1

1. 5
2. 7
3. 9
4. 14
5. 22
6. 25
7. 37
8. 59
9. 84
10. 121
11. 100_2
12. 1000_2
13. 1010_2
14. 11011_2
15. 100111_2
16. 111001_2
17. 1001110_2
18. 1100100_2
19. 10010110_2
20. 11001000_2

END OF CHAPTER PROBLEMS 24–1

1. 7
3. 10
5. 13
7. 23
9. 38
11. 56
13. 101
15. 119
17. 240
19. 195
21. 101_2
23. 1100_2
25. 10101_2
27. 101101_2
29. 111011_2
31. 1000100_2
33. 1100000_2
35. 10000111_2
37. 11010010_2

PRACTICE PROBLEMS 24–2

1. (a) 38_{10}
   (b) 73_{10}
   (c) 132_{10}
   (d) 256_{10}
   (e) 511_{10}
2. (a) 33_8
   (b) 131_8
   (c) 144_8
   (d) 400_8
   (e) 620_8

END OF CHAPTER PROBLEMS 24–2

1. 12_{10}
3. 63_{10}
5. 190_{10}
7. 2174_{10}
9. 5068_{10}
11. 24_8
13. 120_8
15. 550_8
17. 2611_8
19. 7315_8

PRACTICE PROBLEMS 24–3

1. (a) 13_{10}
   (b) 63_{10}
   (c) 164_{10}
   (d) 461_{10}
   (e) 4286_{10}
2. (a) $1B_{16}$
   (b) 55_{16}
   (c) 64_{16}
   (d) 100_{16}
   (e) $5DC_{16}$

END OF CHAPTER PROBLEMS 24–3

1. 26_{10}
3. 76_{10}
5. 512_{10}
7. 4522_{10}
9. $41{,}003_{10}$
11. 16_{16}
13. 61_{16}
15. 200_{16}
17. $A8C_{16}$
19. $17BB_{16}$

PRACTICE PROBLEMS 24–4

1. (a) 1515_8
   (b) $34D_{16}$
2. (a) 1031_8
   (b) 219_{16}
3. (a) 1461_8
   (b) 331_{16}
4. (a) 7165_8
   (b) $E75_{16}$
5. (a) 75306_8
   (b) $7AC6_{16}$
6. 111011_2
7. 1010111_2
8. 110001111_2
9. 101000110_2
10. 1010000111_2
11. 10100110_2
12. 1001100_2
13. 110000001101_2
14. 101110100001_2
15. 111100100011_2

END OF CHAPTER PROBLEMS 24–4

1. $312_8 = CA_{16}$
3. $233_8 = 9B_{16}$
5. $201_8 = 81_{16}$
7. $6311_8 = CC9_{16}$
9. $5303_8 = AC3_{16}$
11. 1111_2
13. 100100_2
15. 10000000_2
17. 101110000_2
19. 1000000000_2
21. 1100_2
23. 100100_2
25. 1001111_2
27. 11100011_2
29. 110011010_2

PRACTICE PROBLEMS 24–5

1. $73_8 = 3B_{16} = 111011_2 = 59_{10}$
2. $273_8 = BB_{16} = 10111011_2 = 187_{10}$
3. $A3_{16} = 243_8 = 10100011_2 = 163_{10}$
4. $12E_{16} = 456_8 = 100101110_2 = 302_{10}$
5. $1101001_2 = 151_8 = 69_{16} = 105_{10}$
6. $10111100_2 = 274_8 = BC_{16} = 188_{10}$

END OF CHAPTER PROBLEMS 24–5

1. $75_8 = 111101_2 = 61_{10} = 3D_{16}$
3. $140_8 = 1100000_2 = 96_{10} = 60_{16}$
5. $2A_{16} = 101010_2 = 52_8 = 42_{10}$
7. $1A7_{16} = 110100111_2 = 647_8 = 423_{10}$
9. $100110_2 = 46_8 = 38_{10} = 26_{16}$
11. $111000101_2 = 705_8 = 453_{10} = 1C5_{16}$
13. $10_{10} = 1010_2 = 12_8 = A_{16}$
15. $290_{10} = 100100010_2 = 442_8 = 122_{16}$

SELF-TEST 24–1 THROUGH 24–5

1. 38
2. 190
3. 246
4. 3130
5. 91
6. 202
7. (a) 1010_2
   (b) 12_8
   (c) A_{16}
8. (a) 11100_2
   (b) 34_8
   (c) $1C_{16}$
9. (a) 1011101_2
   (b) 273_8
   (c) BB_{16}
10. (a) 1001110001_2
    (b) 1161_8
    (c) 271_{16}
11. (a) 1011101_2
    (b) 135_8
    (c) $5D_{16}$
12. (a) 111000011010_2
    (b) 7032_8
    (c) 3610_{10}
13. (a) 3226_8
    (b) 1686_{10}
    (c) 696_{16}
14. (a) 101111011_2
    (b) 379_{10}
    (c) $17B_{16}$

PRACTICE PROBLEMS 24–6

1. 522_8
2. 642_8
3. 1533_8
4. 1306_8
5. 1157_8

END OF CHAPTER PROBLEMS 24–6

1. 357_8
3. 775_8
5. 776_8
7. 1423_8
9. 1107_8

PRACTICE PROBLEMS 24–7

1. DF_{16}
2. $15B_{16}$
3. 1121_{16}
4. $E7D_{16}$
5. $111CF_{16}$

END OF CHAPTER PROBLEMS 24–7

1. ED_{16}
3. $CA5_{16}$
5. $D56_{16}$
7. $179D_{16}$
9. $1DA6C_{16}$

PRACTICE PROBLEMS 24–8

1. 10111_2
2. 11000_2
3. 101000_2
4. 110111_2
5. 1101010_2

END OF CHAPTER PROBLEMS 24–8

1. 10101_2
3. 110011_2
5. 101001_2
7. 1100010_2
9. 1101001_2

SELF-TEST 24–6 THROUGH 24–8

1. $E5_{16}$
2. $DC8_{16}$
3. $1F323_{16}$
4. 161_8
5. 124_8
6. 141_8
7. 1100_2
8. 11000_2
9. 110100_2

PRACTICE PROBLEMS 24–9

1. 15_8
2. 77_8
3. 475_8
4. 337_8
5. 2545_8

END OF CHAPTER PROBLEMS 24–9

1. 45_8
3. 42_8
5. 245_8
7. 337_8
9. 1515_8

PRACTICE PROBLEMS 24–10

1. $7AC_{16}$
2. 483_{16}
3. $4E60_{16}$
4. $3A8C_{16}$
5. $CE9F_{16}$

END OF CHAPTER PROBLEMS 24–10

1. $1A_{16}$
3. $3E1_{16}$
5. $5F1_{16}$
7. AFD_{16}
9. FDC_{16}

PRACTICE PROBLEMS 24–11

1. 101_2
2. 1001_2
3. 1011_2
4. 11010_2
5. 1101_2

END OF CHAPTER PROBLEMS 24–11

1. 101_2
3. 10_2
5. 10_2
7. 11_2
9. 10110_2

PRACTICE PROBLEMS 24–12

1. (a) 82 (b) 83
2. (a) 63 (b) 64
3. (a) 11 (b) 12
4. (a) 723 (b) 724
5. (a) 1023 (b) 1024
6. (a) 0010 (b) 0011
7. (a) 0110 (b) 0111
8. (a) 011001 (b) 011010
9. (a) 001110 (b) 001111
10. (a) 00111100 (b) 00111101

11. (a) 1

$$\begin{array}{rl} 11001 & \\ \underline{10010} & \text{(one's complement)} \\ 1\ \ 01011 & \\ \llcorner\!\!\longrightarrow 1 & \\ \hline 01100 & \end{array}$$

(b) 1

$$\begin{array}{rl} 11001 & \\ \underline{10011} & \text{(two's complement)} \\ \cancel{1}\ \ 01100 & \end{array}$$

12. (a) 1

$$\begin{array}{rl} 101101 & \\ \underline{110100} & \text{(one's complement)} \\ 1\ \ 100001 & \\ \llcorner\!\!\longrightarrow 1 & \\ \hline 100010 & \end{array}$$

(b) 1

$$\begin{array}{rl} 101101 & \\ \underline{110101} & \text{(two's complement)} \\ \cancel{1}\ \ 100010 & \end{array}$$

13. (a) 1

$$\begin{array}{rl} 11 & \\ 1100011 & \\ \underline{1100110} & \text{(one's complement)} \\ 1\ \ 1001001 & \\ \llcorner\!\!\longrightarrow 1 & \\ \hline 1001010 & \end{array}$$

(b) 1

$$\begin{array}{rl} 111 & \\ 1100011 & \\ \underline{1100111} & \text{(two's complement)} \\ \cancel{1}\ \ 1001010 & \end{array}$$

14. (a) 1

$$\begin{array}{rl} 10110010 & \\ \underline{11100010} & \text{(one's complement)} \\ 1\ \ 10010100 & \\ \llcorner\!\!\longrightarrow 1 & \\ \hline 10010101 & \end{array}$$

(b) 1

$$\begin{array}{rl} 11\ \ 1 & \\ 10110010 & \\ \underline{11100011} & \text{(two's complement)} \\ \cancel{1}\ \ 10010101 & \end{array}$$

15. (a) 1

$$\begin{array}{rl} 11\ \ 1 & \\ 11100100 & \\ \underline{11100100} & \text{(one's complement)} \\ 1\ \ 11001000 & \\ \llcorner\!\!\longrightarrow 1 & \\ \hline 11001001 & \end{array}$$

(b) 1

$$\begin{array}{rl} 11\ \ 1 & \\ 11100100 & \\ \underline{11100101} & \text{(two's complement)} \\ \cancel{1}\ \ 11001001 & \end{array}$$

END OF CHAPTER PROBLEMS 24–12

1. 0110_2
3. 010011_2
5. 01101_2
7. 100011_2
9. 01011_2
11. 1000011_2

PRACTICE PROBLEMS 24–13

1. (a) 1 (b) 2
2. (a) 65 (b) 66
3. (a) 21 (b) 22
4. (a) 604 (b) 605

5. (a) 051 (b) 052
6. (a) 5 (b) 6
7. (a) D3 (b) D4
8. (a) 78 (b) 79
9. (a) B5C (b) B5D
10. (a) 0DF6 (b) 0DF7

11. (a)
$$\begin{array}{rl} & 11010011 \\ & \underline{11000011}\ \text{(two's complement)} \\ \cancel{1} & 10010110_2 \end{array}$$
(b)
$$\begin{array}{llcl} 1101 & 0011 = & & \text{D3} \\ 1100 & 0011 = & & \underline{\text{C3}}\ \text{(sixteen's complement)} \\ & & \cancel{1} & 96_{16} \end{array}$$
(c)
$$\begin{array}{lllcl} 011 & 010 & 011 = & & 323 \\ 111 & 000 & 011 = & & \underline{703}\ \text{(eight's complement)} \\ & & & \cancel{1} & 226_8 \end{array}$$

The subtrahend is 00111101. The two's complement is 11000011. Since we have already found the two's complement of 00111101 to be 11000011, we separate the two's complement into two 4-bit groups. Then 1100 0011 = C3, which is the sixteen's complement. To find the eight's complement, we must separate the subtrahend into three 3-bit groups. Since we have only 8 bits, we add a zero to the subtrahend in the original problem to get 000 111 101. The two's complement is 111 000 011. The eight's complement is 703.

12. (a)
$$\begin{array}{rl} & 10011101 \\ & \underline{11001101}\ \text{(two's complement)} \\ \cancel{1} & 01101010_2 \end{array}$$
(b)
$$\begin{array}{llcl} 1001 & 1101 = & & \text{9D} \\ 1100 & 1101 = & & \underline{\text{CD}}\ \text{(sixteen's complement)} \\ & & \cancel{1} & \text{6A}_{16} \end{array}$$
(c)
$$\begin{array}{lllcl} 010 & 011 & 101 = & & 235 \\ 111 & 001 & 101 = & & \underline{715}\ \text{(eight's complement)} \\ & & & \cancel{1} & 152_8 \end{array}$$

13. (a)
$$\begin{array}{rl} & 10111100 \\ & \underline{10110010}\ \text{(two's complement)} \\ \cancel{1} & 01101110_2 \end{array}$$
(b)
$$\begin{array}{llcl} 1011 & 1100 = & & \text{BC} \\ 1011 & 0010 = & & \underline{\text{B2}}\ \text{(sixteen's complement)} \\ & & \cancel{1} & \text{6E}_{16} \end{array}$$
(c)
$$\begin{array}{lllcl} 010 & 111 & 100 = & & 274\ \text{(eight's complement)} \\ 110 & 110 & 010 = & & \underline{662} \\ & & & \cancel{1} & 156_8 \end{array}$$

14. (a)
$$\begin{array}{rl} & 11110000 \\ & \underline{11000100}\ \text{(two's complement)} \\ \cancel{1} & 10110100_2 \end{array}$$
(b)
$$\begin{array}{llcl} 1111 & 0000 = & & \text{F0} \\ 1100 & 0100 = & & \underline{\text{C4}}\ \text{(sixteen's complement)} \\ & & \cancel{1} & \text{B4}_{16}\ \ \text{(eight's complement)} \end{array}$$
(c)
$$\begin{array}{lllcl} 011 & 110 & 000 = & & 360 \\ 111 & 000 & 100 = & & \underline{704} \\ & & & \cancel{1} & 264_8 \end{array}$$

15. (a) 11001111

$\underline{01111101}$ (two's complement)

$\cancel{1}$ 01001100_2

(b) 1100 1111 = CF

0111 1101 = $\underline{7D}$ (sixteen's complement)

$\cancel{1}$ $4C_{16}$

(c) 011 001 111 = 317

101 111 101 = $\underline{575}$ (eight's complement)

$\cancel{1}$ 114_8

END OF CHAPTER PROBLEMS 24–13

1. 14_8
3. 143_8
5. $5D_{16}$
7. $3E2_{16}$
9. 36_8
11. 227_8
13. 16_{16}
15. $1A3_{16}$

SELF-TEST 24–9 THROUGH 24–13

1. 10011110_2
2. 101011_2
3. 25_8
4. 217_8
5. BC_{16}
6. 255_{16}

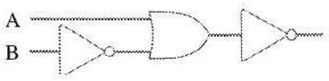
CHAPTER 25

PRACTICE PROBLEMS 25–1

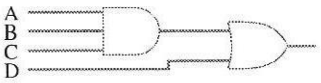
6. $F = \overline{\overline{A} + B}$
7. $F = \overline{A\overline{B}}$
8. $F = A + \overline{B} + C$
9. $F = (A + B)(B + C)$
10. $A = XY + XZ$

END OF CHAPTER PROBLEMS 25–7

1.



3.



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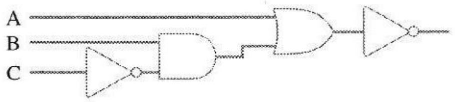

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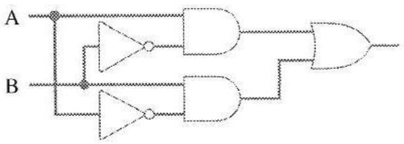

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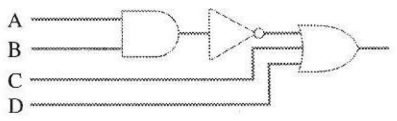
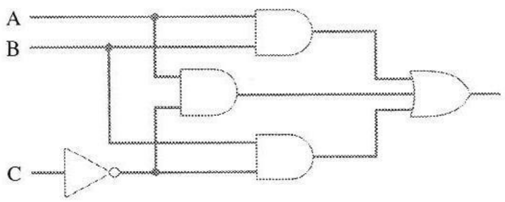
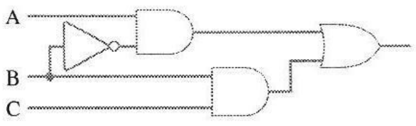


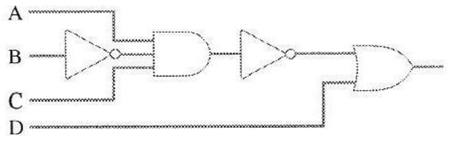
11.



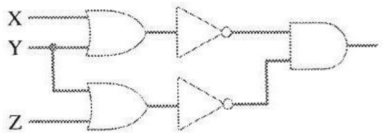
13.



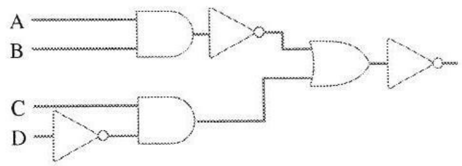
15.



17.



19.



21. $A\overline{B} + \overline{B}C$

23. $\overline{\overline{X}YZ}$

25. $\overline{\overline{A}\,\overline{B}} + C$

27. $(A + B)\overline{(A + C)}D$

29. $Z(X + Y)$

31. $\overline{\overline{AB}\,(A + B)}$

33. $\overline{X + \overline{Y}} \cdot \overline{\overline{Y} + Z}$

35. $\overline{\overline{A + B}(B + C)(C + D)}$

37. $\overline{XY + \overline{YZ}}$

39. $\overline{A\overline{B} + \overline{B}C}$

SELF-TEST 25–1

1. $F = \overline{A}\,\overline{B} + C$
2. $F = \overline{C(\overline{A + B})}$
3. $X = (\overline{A + B})(B + C)$
4. $A = (\overline{X}Y)(Y + Z)$
5. $F = \overline{\overline{A}(\overline{B + C})}$

See Figure C–55.

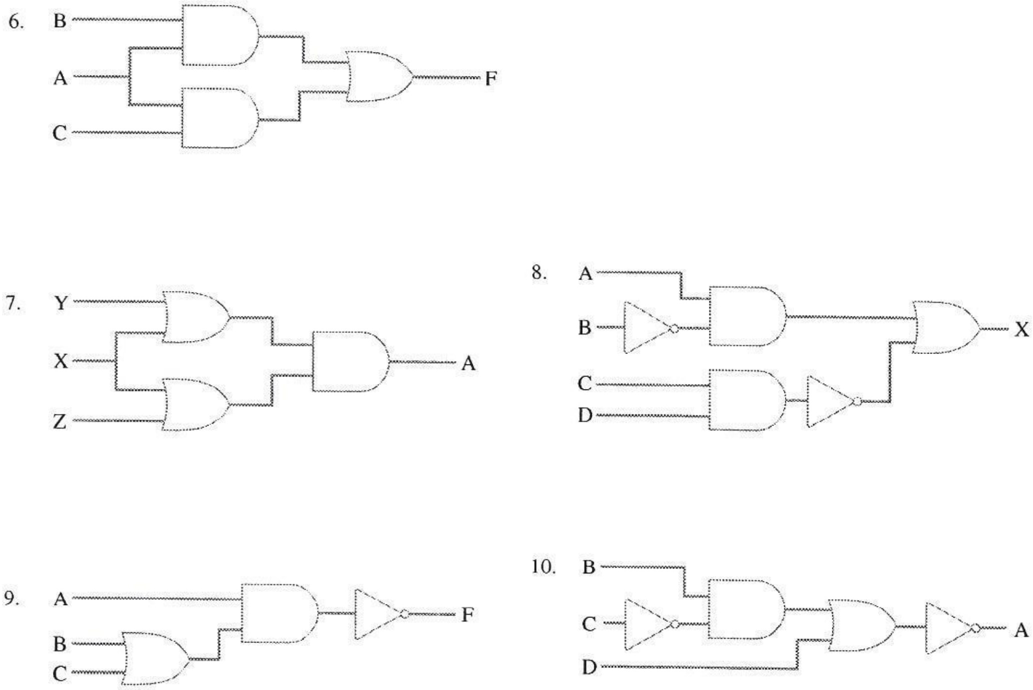


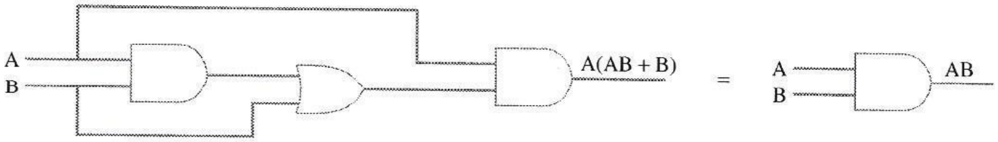
FIGURE C–55 Answers to questions 6 through 10 for Self-test 25–1.

PRACTICE PROBLEMS 25–2

1. See Figure C–56.
2. See Figure C–57.
3. See Figure C–58.
4. See Figure C–59.

FIGURE C–56 Answer to problem 1, Practice Problems 25–2.

A(AB + B)	Original expression
AAB + AB	8a
AB + AB	11a
AB	11b



A	B	AB	AB + B	A(AB + B)
0	0	0	0	0
0	1	0	1	0
1	0	0	0	0
1	1	1	1	1

Equal

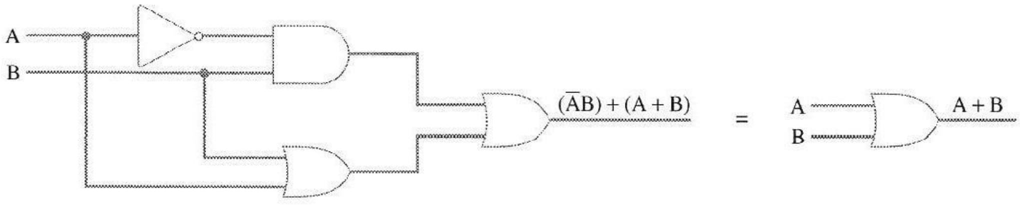


FIGURE C-57 Answer to problem 2, Practice Problems 25–2.

$(\overline{A}B) + (A + B)$	Original expression
$\overline{A}B + A + B$	7b
$\overline{A}B + B + A$	6b
$B(\overline{A} + 1) + A$	8a
$B(1) + A$	10b
$B + A$	10a

A	B	$\overline{A}$	$\overline{A}B$	A + B	$\overline{A}B + (A + B)$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

Equal

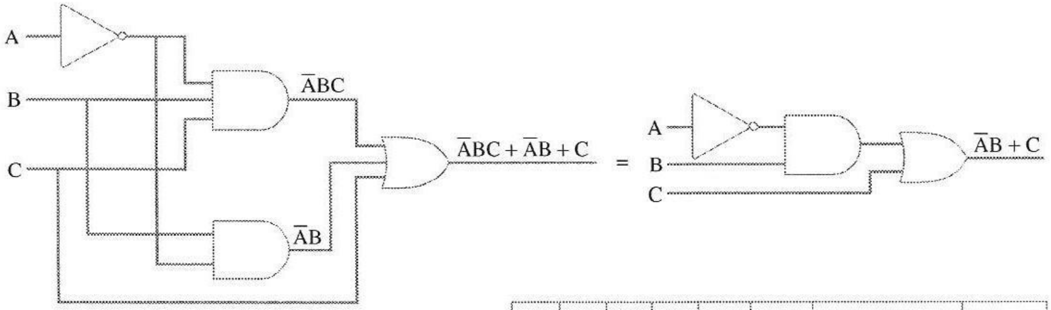


FIGURE C-58 Answer to problem 3, Practice Problems 25–2.

$\overline{A}BC + \overline{A}B + C$	Original expression
$\overline{A}B\,(C + 1) + C$	8a
$\overline{A}B\,(1) + C$	10b
$\overline{A}B + C$	10a

A	B	C	$\overline{A}$	$\overline{A}B$	$\overline{A}BC$	$\overline{A}BC + \overline{A}B + C$	$\overline{A}B + C$
0	0	0	1	0	0	0	0
0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	1
1	1	0	0	0	0	0	0
1	1	1	0	0	0	1	1

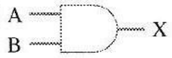
Equal

END OF CHAPTER PROBLEMS 25–2

1. (a) $A((A + B)B)$

(b) $AB(A + B) = AAB + ABB = AB + AB = AB$

(c)



(d)

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

A + ABC	Original expression
A(1 + BC)	8a
A(1)	10b
A	10a

A	B	C	AB	ABC	A + ABC
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	1	1	1

Equal

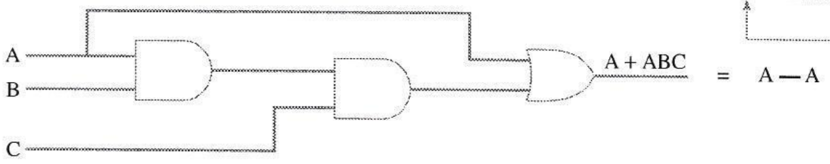
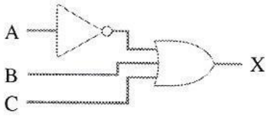


FIGURE C–59 Answer to problem 4, Practice Problems 25–2.

3. (a) $(\overline{A} + B + C) + (\overline{A} + B) + C$

(b) $\overline{A} + B + C + \overline{A} + B + C = \overline{A} + B + C$

(c)



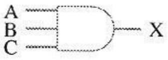
(d)

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

5. (a) $(AB)(\overline{B} + C)C$

(b) $ABC(\overline{B} + C) = AB\overline{B}C + ABCC = 0 + ABC = ABC$

(c)



(d)

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

PRACTICE PROBLEMS 25–3

1. See Figure C–60.
2. See Figure C–61.
3. See Figure C–62.
4. See Figure C–63.
5. See Figure C–64.

$\overline{\overline{A}+B}$ Original expression
$\overline{\overline{A}}\cdot\overline{B}$ 14a
$A\overline{B}$ 13a

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}+B$	$\overline{\overline{A}+B}$	$A\overline{B}$
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	1	0	0

Equal

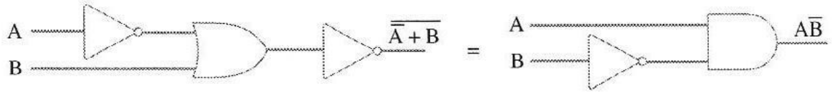


FIGURE C–60 Answer to problem 1, Practice Problems 25–3.

$\overline{A\overline{B}}$ Original expression
$\overline{A}+\overline{\overline{B}}$ 14b
$\overline{A}+B$ 13b

A	B	$\overline{A}$	$\overline{B}$	$A\overline{B}$	$\overline{A\overline{B}}$	$\overline{A}+B$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	1	0	0
1	1	0	0	0	1	1

Equal

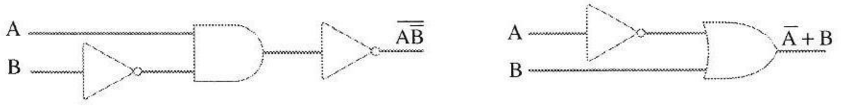


FIGURE C–61 Answer to problem 2, Practice Problems 25–3.

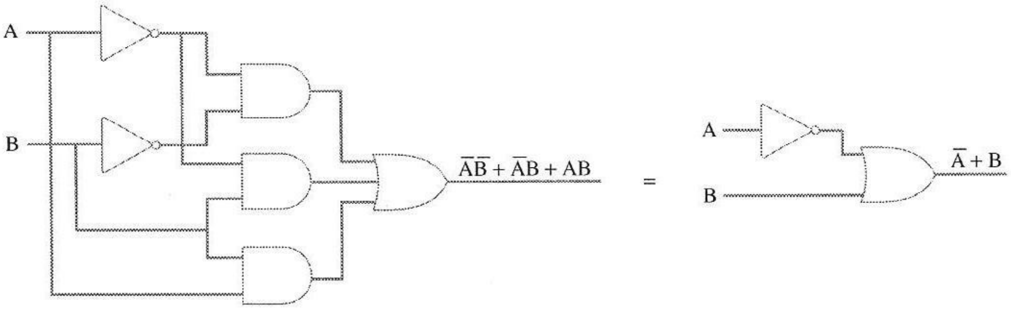
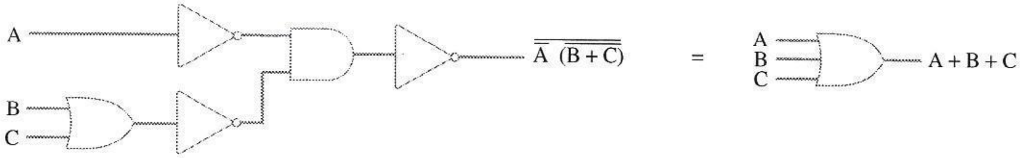


FIGURE C–62 Answer to problem 3, Practice Problems 25–3.

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$	AB	$\overline{A}\,\overline{B}+\overline{A}B+AB$	$\overline{A}+B$
0	0	1	1	1	0	0	1	1
0	1	1	0	0	1	0	1	1
1	0	0	1	0	0	0	0	0
1	1	0	0	0	0	1	1	1

Equal

$\overline{\overline{A}\,\overline{(B+C)}}$ Original expression
$\overline{\overline{A}} + \overline{\overline{(B+C)}}$ 14b
$A + (B+C)$ 13a
$A + B + C$ 7b

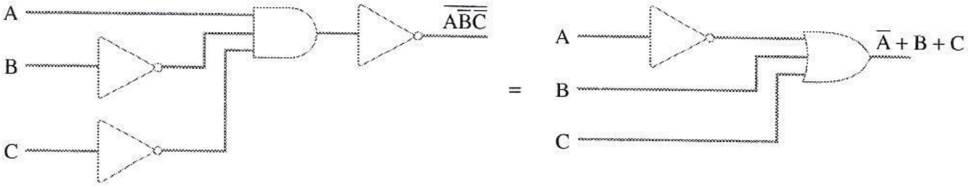


A	B	C	$\overline{A}$	B + C	$\overline{B+C}$	$\overline{A}(\overline{B+C})$	$\overline{\overline{A}(\overline{B+C})}$	A + B + C
0	0	0	1	0	1	1	0	0
0	0	1	1	1	0	0	1	1
0	1	0	1	1	0	0	1	1
0	1	1	1	1	0	0	1	1
1	0	0	0	0	1	0	1	1
1	0	1	0	1	0	0	1	1
1	1	0	0	1	0	0	1	1
1	1	1	0	1	0	0	1	1

Equal

FIGURE C–63 Answer to problem 4, Practice Problems 25–3.

$\overline{A\overline{B}\,\overline{C}}$ Original expression
$\overline{A} + B + C$ 14b



A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$A\overline{B}\,\overline{C}$	$\overline{A\overline{B}\,\overline{C}}$	$\overline{A} + B + C$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	1	1
0	1	0	1	0	1	0	1	1
0	1	1	1	0	0	0	1	1
1	0	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	0	1	1
1	1	1	0	0	0	0	1	1

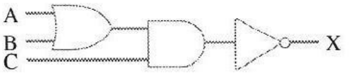
Equal

FIGURE C–64 Answer to problem 5, Practice Problems 25–3.

END OF CHAPTER PROBLEMS 25–3

1. (a) $\overline{(A + B)} + \overline{C} = \overline{(A + B)C}$

(b)

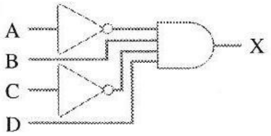


(c)

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

3. (a) $(A + B)\overline{(A + C)}D = ((A + B)(\overline{A}\,\overline{C}))D = A\overline{A}\,\overline{C}D = \overline{A}B\overline{C}D = \overline{A}B\overline{C}D$

(b)

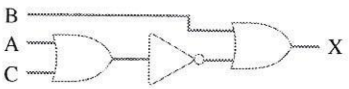


(c)

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

5. (a) $\overline{A\overline{B} + \overline{B}C} = \overline{\overline{B}(A + C)} = B + \overline{(A + C)}$

(b)

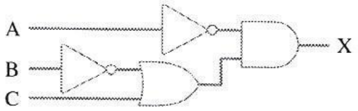


(c)

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

7. (a) $\overline{A + B\overline{C}} = \overline{A}(\overline{B\overline{C}}) = \overline{A}(\overline{B} + C)$

(b)

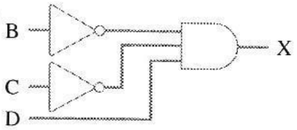


(c)

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

9. (a) $(\overline{B + C})(C + D) = (\overline{BC})\,(C + D) = \overline{B}\,\overline{C}C + \overline{B}\,\overline{C}D = \overline{B}\,\overline{C}D$

(b)

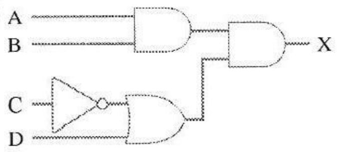


(c)

B	C	D	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

11. (a) $\overline{\overline{AB} + C\overline{D}} = \overline{\overline{(AB)}}\;\overline{(C\overline{D})} = (AB)\,(\overline{C} + D)$

(b)



(c)

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

PRACTICE PROBLEMS 25–4

1. (ON) + ((AUTO) · (IR)) = (Lights on)
2. (TV pressed) · (ON pressed) = (Turn TV on)
   (VCR pressed) · (ON pressed) = (Turn VCR on)
3. (SHIFT) · (SIN) = (calculate inverse sine)
   ($\overline{\text{SHIFT}}$) · (SIN) = (calculate sine)
4. (Pointer on print icon) · (left mouse button) = (print document)
   (Pointer on page icon) · (left mouse button) = (display page)

END OF CHAPTER PROBLEMS 25–4

1. (PARK) · (key) = (start)
3. (Power) · (ON) · (OS) = (start)
5. (Down arrow) + (page down) + ((scroll bar) · (left mouse button)) = (scroll down)

SELF-TEST 25–2

1. See Figure C–65.
2. See Figure C–66.
3. See Figure C–67.

$\overline{A\overline{B}C}$	Original expression
$\overline{A} + B + \overline{C}$	14b

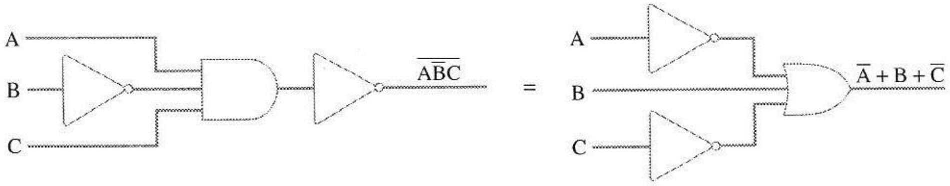

A
B
C
$\overline{A\overline{B}C}$
=
A
B
C
$\overline{A} + B + \overline{C}$

FIGURE C–65 Logic diagrams and truth table for $\overline{A\overline{B}C} = \overline{A} + B + \overline{C}$. Answer to problem 1, Self-test 25–2.

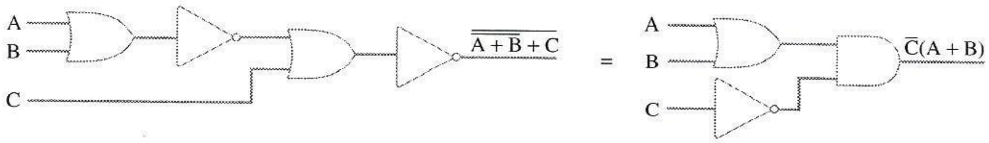
FIGURE C-65 *(cont.)*

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$A\overline{B}C$	$\overline{A\overline{B}C}$	$\overline{A}+B+\overline{C}$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	1	1
0	1	0	1	0	1	0	1	1
0	1	1	1	0	0	0	1	1
1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	1	0	0
1	1	0	0	0	1	0	1	1
1	1	1	0	0	0	0	1	1

Equal

FIGURE C-66 Logic diagram and truth table for $\overline{\overline{A+B}+C} = \overline{C}(A+B)$. Answer to problem 2, Self-test 25–2.

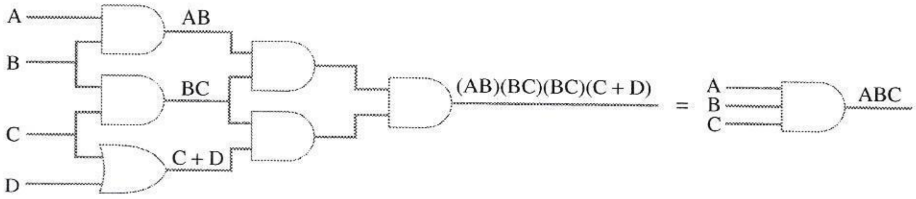
$\overline{\overline{A+B}+C}$	Original expression
$\overline{\overline{A+B}}\cdot\overline{C}$	14a
$(A+B)\overline{C}$	13a
$\overline{C}(A+B)$	6a



A	B	C	$\overline{C}$	$A+B$	$\overline{A+B}$	$\overline{A+B}+C$	$\overline{\overline{A+B}+C}$	$\overline{C}(A+B)$
0	0	0	1	0	1	1	0	0
0	0	1	0	0	1	1	0	0
0	1	0	1	1	0	0	1	1
0	1	1	0	1	0	1	0	0
1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	0
1	1	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	0

Equal

FIGURE C-67 Answer to problem 3, Self-test 25–2.



$(AB)(BC)(BC)(C+D)$	Original expression
$(AB)(BC)(C+D)$	11b
$ABC\,(C+D)$	11a
$ABCC+ABCD$	8a
$ABC+ABCD$	11a
$ABC\,(1+D)$	8a
ABC	10b

4. See Figure C–68.
5. See Figure C–69.
6. See Figure C–70.
7. See Figure C–71.
8. See Figure C–72.
9. See Figure C–73 (p. 696).
10. See Figure C–74 (p. 696).

A	B	C	D	AB	BC	C + D	(AB)(BC)	(BC)(C + D)	(AB) (BC) (BC) (C + D)	ABC
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0
0	0	1	1	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	1	0	0	0	0
0	1	1	0	0	1	1	0	1	0	0
0	1	1	1	0	1	1	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0
1	0	1	1	0	0	1	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0
1	1	0	1	1	0	1	0	0	0	0
1	1	1	0	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1

Equal

FIGURE C–67 *(cont.)*

FIGURE C–68 Answer to problem 4, Self-test 25–2.

$\overline{\overline{A}B + \overline{AC}}$	Original expression
$\overline{\overline{A}B} \cdot AC$	14a
$(A + \overline{B})\,AC$	14b
$AAC + A\overline{B}C$	8a
$AC + A\overline{B}C$	11a
$AC\,(1 + \overline{B})$	8a
$AC\,(1)$	10b
AC	10a

A	B	C	$\overline{A}$	$\overline{A}B$	AC	$\overline{AC}$	$\overline{A}B + \overline{AC}$	$\overline{\overline{A}B + \overline{AC}}$
0	0	0	1	0	0	1	1	0
0	0	1	1	0	0	1	1	0
0	1	0	1	1	0	1	1	0
0	1	1	1	1	0	1	1	0
1	0	0	0	0	0	1	1	0
1	0	1	0	0	1	0	0	1
1	1	0	0	0	0	1	1	0
1	1	1	0	0	1	0	0	1

Equal

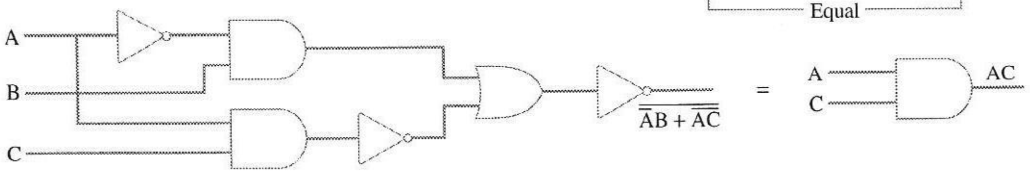
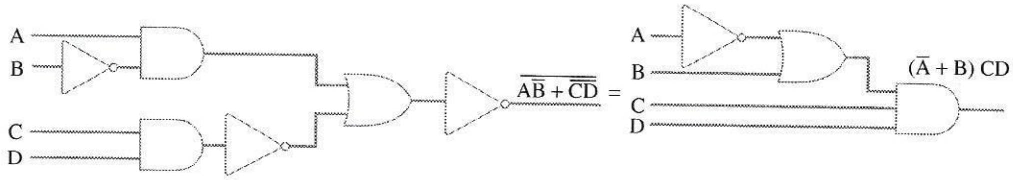


FIGURE C–69 Answer to problem 5, Self-test 25–2.

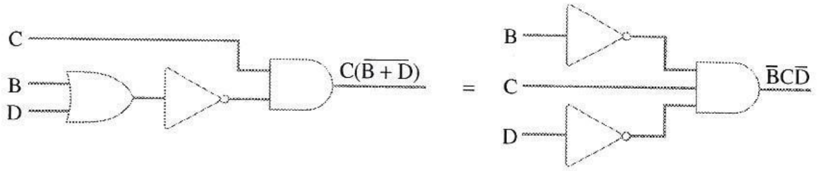
$\overline{A\overline{B}+\overline{CD}}$	Original expression
$\overline{A\overline{B}}\cdot\overline{\overline{CD}}$	14a
$\overline{A\overline{B}}\cdot CD$	13a
$(\overline{A}+B)\,CD$	14b
$CD\,(\overline{A}+B)$	7a



A	B	C	D	$\overline{A}$	$\overline{B}$	CD	$\overline{CD}$	$A\overline{B}$	$A\overline{B}+\overline{CD}$	$\overline{A\overline{B}+\overline{CD}}$	$\overline{A}+B$	$CD\,(\overline{A}+B)$
0	0	0	0	1	1	0	1	0	1	0	1	0
0	0	0	1	1	1	0	1	0	1	0	1	0
0	0	1	0	1	1	0	1	0	1	0	1	0
0	0	1	1	1	1	1	0	0	0	1	1	1
0	1	0	0	1	0	0	1	0	1	0	1	0
0	1	0	1	1	0	0	1	0	1	0	1	0
0	1	1	0	1	0	0	1	0	1	0	1	0
0	1	1	1	1	0	1	0	0	0	1	1	1
1	0	0	0	0	1	0	1	1	1	0	0	0
1	0	0	1	0	1	0	1	1	1	0	0	0
1	0	1	0	0	1	0	1	1	1	0	0	0
1	0	1	1	0	1	1	0	1	1	0	0	0
1	1	0	0	0	0	0	1	0	1	0	1	0
1	1	0	1	0	0	0	1	0	1	0	1	0
1	1	1	0	0	0	0	1	0	1	0	1	0
1	1	1	1	0	0	1	0	0	0	1	1	1

Equal

FIGURE C–70 Answer to problem 6, Self-test 25–2.



$C(\overline{B+D})$	Original expression
$C(\overline{B}\overline{D})$	14a
$C\overline{B}\overline{D}$	7a
$\overline{B}C\overline{D}$	6a

B	C	D	$\overline{B}$	$\overline{D}$	B + D	$\overline{B+D}$	$C(\overline{B+D})$	$\overline{B}C\overline{D}$
0	0	0	1	1	0	1	0	0
0	0	1	1	0	1	0	0	0
0	1	0	1	1	0	1	1	1
0	1	1	1	0	1	0	0	0
1	0	0	0	1	1	0	0	0
1	0	1	0	0	1	0	0	0
1	1	0	0	1	1	0	0	0
1	1	1	0	0	1	0	0	0

Equal

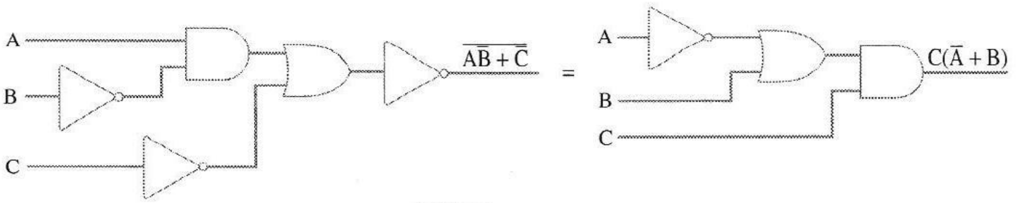


FIGURE C–71 Answer to problem 7, Self-test 25–2.

$\overline{A\overline{B}+\overline{C}}$	Original expression
$\overline{A\overline{B}}\cdot\overline{\overline{C}}$	14a
$\overline{A\overline{B}}\cdot C$	13a
$(\overline{A}+\overline{\overline{B}})C$	14b
$(\overline{A}+B)C$	13a
$C(\overline{A}+B)$	7a

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$A\overline{B}$	$A\overline{B}+\overline{C}$	$\overline{A\overline{B}+\overline{C}}$	$\overline{A}+B$	$C(\overline{A}+B)$
0	0	0	1	1	1	0	1	0	1	0
0	0	1	1	1	0	0	0	1	1	1
0	1	0	1	0	1	0	1	0	1	0
0	1	1	1	0	0	0	0	1	1	1
1	0	0	0	1	1	1	1	0	0	0
1	0	1	0	1	0	1	1	0	0	0
1	1	0	0	0	1	0	1	0	1	0
1	1	1	0	0	0	0	0	1	1	1

Equal

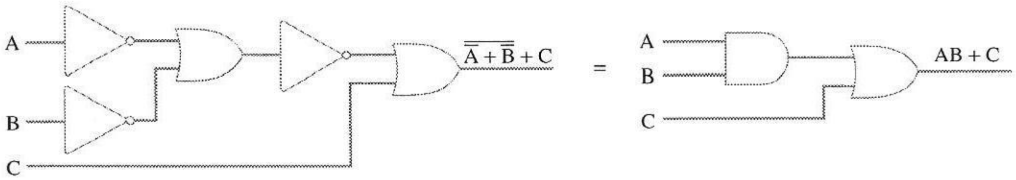


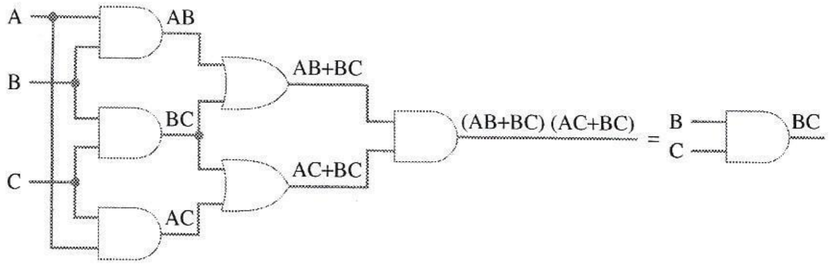
FIGURE C–72 Answer to problem 8, Self-test 25–2.

$\overline{\overline{A}+\overline{B}}+C$	Original expression
$\overline{\overline{A}}\cdot\overline{\overline{B}}+C$	14a
$AB+C$	13a

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$	$\overline{\overline{A}+\overline{B}}$	$\overline{\overline{A}+\overline{B}}+C$	AB	AB + C
0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	1	0	1	0	1
0	1	0	1	0	1	0	0	0	0
0	1	1	1	0	1	0	1	0	1
1	0	0	0	1	1	0	0	0	0
1	0	1	0	1	1	0	1	0	1
1	1	0	0	0	0	1	1	1	1
1	1	1	0	0	0	1	1	1	1

Equal

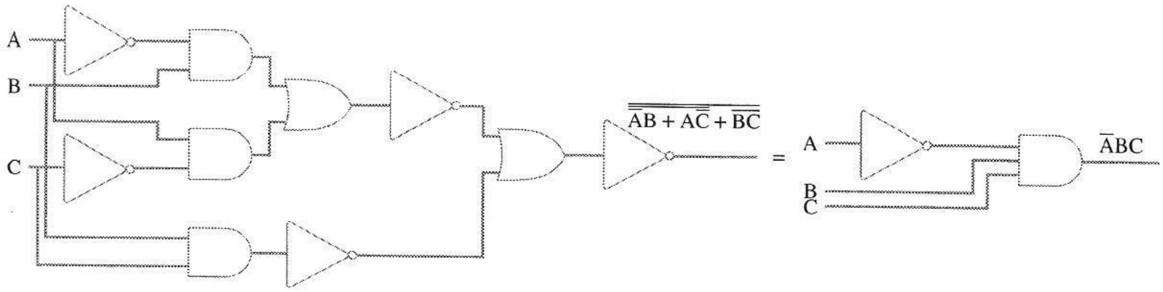
FIGURE C–73 Answer to problem 9, Self-test 25–2.



(AB+BC) (AC+BC)	Original expression
(BC+AB) (BC+AC)	6b
BC + (AB) (AC)	8b
BC + AABC	6a
BC + ABC	11a
BC (1+A)	8a
BC(1)	11b
BC	10a

A	B	C	AB	BC	AC	AB+BC	AC+BC	(AB+BC)(AC+BC)
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	1	0	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	1	0	1	0
1	1	0	1	0	0	1	0	0
1	1	1	1	1	1	1	1	1

Equal


A
B
C
$\overline{\overline{\overline{A}B + A\overline{C}} + \overline{BC}}$
=
A
B
C
$\overline{A}BC$

$\overline{\overline{\overline{A}B + A\overline{C}} + \overline{BC}}$	Original expression
$(\overline{A}B + A\overline{C})BC$	14a
$\overline{A}BBC + ABC\overline{C}$	8a
$\overline{A}BC + 0$	11a, 12a
$\overline{A}BC$	9b

FIGURE C–74 Answer to problem 10, Self-test 25–2.

A	B	C	$\overline{A}$	$\overline{C}$	$\overline{A}B$	$A\overline{C}$	$\overline{A}B + A\overline{C}$	$\overline{\overline{A}B + A\overline{C}}$	BC	$\overline{BC}$	$\overline{\overline{A}B + A\overline{C}} + \overline{BC}$	$\overline{\overline{\overline{A}B + A\overline{C}} + \overline{BC}}$	$\overline{A}BC$
0	0	0	1	1	0	0	0	1	0	1	1	0	0
0	0	1	1	0	0	0	0	1	0	1	1	0	0
0	1	0	1	1	1	0	1	0	0	1	1	0	0
0	1	1	1	0	1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	1	1	0	0	1	1	0	0
1	0	1	0	0	0	0	0	1	0	1	1	0	0
1	1	0	0	1	0	1	1	0	0	1	1	0	0
1	1	1	0	0	0	0	0	1	1	0	1	0	0

Equal

FIGURE C–74 (cont.)

CHAPTER 26

PRACTICE PROBLEMS 26–1

1. The frequency distribution table for problem 1 appears in Figure C–75. The histogram appears in Figure C–76.
2. The frequency distribution for problem 2 is shown in Figure C–77. The histogram for problem 2 appears in Figure C–78. Notice in our histogram that the midpoint of each interval is the data value and each interval is 10 Ω wide.

Scores	***60***	***65***	***70***	***75***	***80***	***85***	***90***	***95***
Tally	|	|||	||	𝍸||	𝍸	|||	|||	|
	1	3	2	7	5	3	3	1

FIGURE C–75 Frequency distribution table for Practice Problems 26–1 #1.

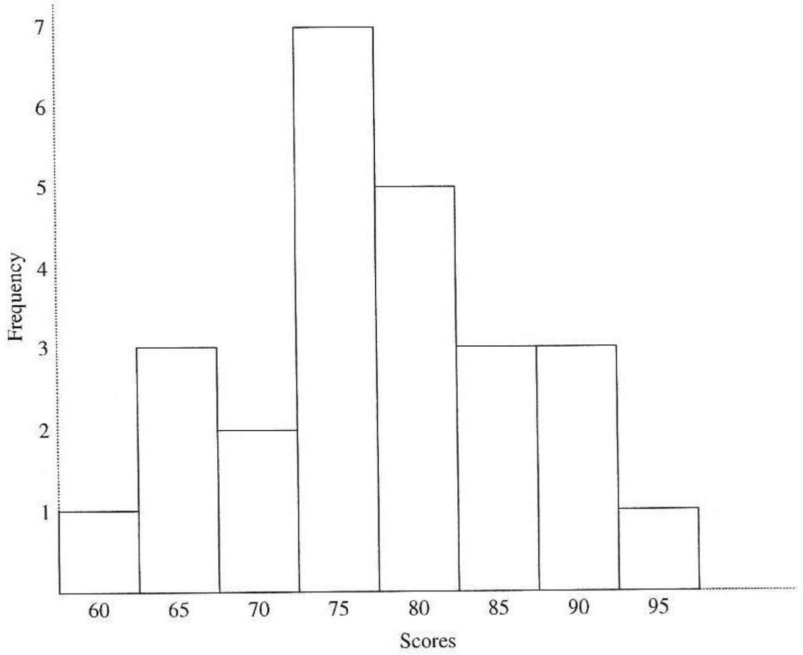


FIGURE C–76 Histogram for Practice Problems 26–1 #1.

3. The frequency distribution table for Problem 3 is shown in Figure C–79. The histogram is shown in Figure C–80.

4. An examination of the scores shows that the highest score was 91 and the lowest score was 62.

$$\text{range} = 91 - 62 = 29$$

With a range of 29 we could conveniently choose six groups with an interval of 5. $29 \div 6 = 4.85 \approx 5$. Construct the frequency distribution table. Tally the number of data in each interval. The frequency distribution for problem 4 is shown in Figure C–81. The midpoints of the intervals are 64, 69, 74, 79, 84, and 89. The histogram is in Figure C–82.

5. Let's use eight groups. $54 \div 8 = 6.75 \approx 7$. Each interval width is 7. Your groups should look like Figure C–83. The histogram for problem 5 appears in Figure C–84 (p. 700).

6. Using 6 intervals, your frequency distribution table should look like the one in Figure C–85 (p. 700). The histogram is shown in Figure C–86 (p. 700).

FIGURE C–77 Frequency distribution table for Practice Problems 26–1 #2.

Resistor values in ohms	***980***	***990***	***1000***	***1010***	***1020***	***1030***	***1040***
Tally	||	𝍸	𝍸	𝍸||	||||	|	|
	2	5	5	7	4	1	1

FIGURE C–78 Histogram for Practice Problems 26–1 #2.

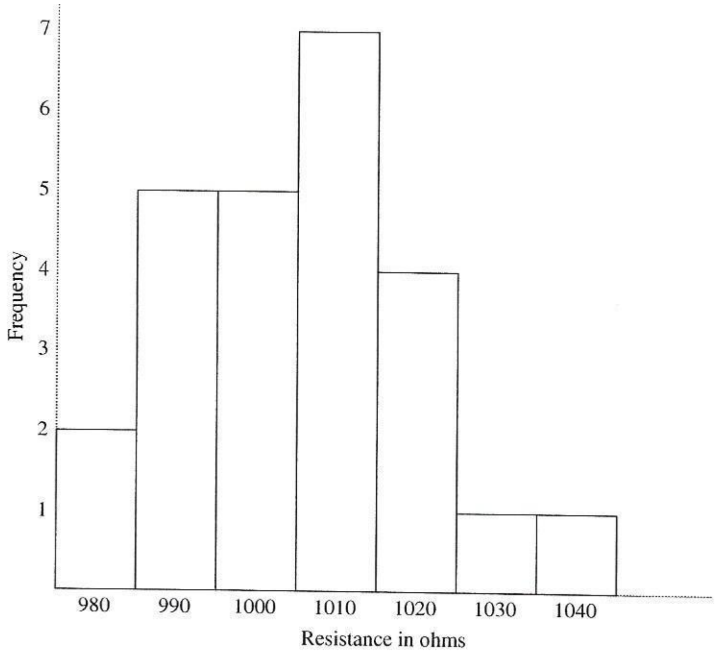


FIGURE C–79 Frequency distribution table for Practice Problems 26–1 #3.

Number of defective parts/100	***1***	***2***	***3***	***4***	***5***	***6***
Tally	||	|||	||||	𝍸||	|||	|
	2	3	4	7	3	1

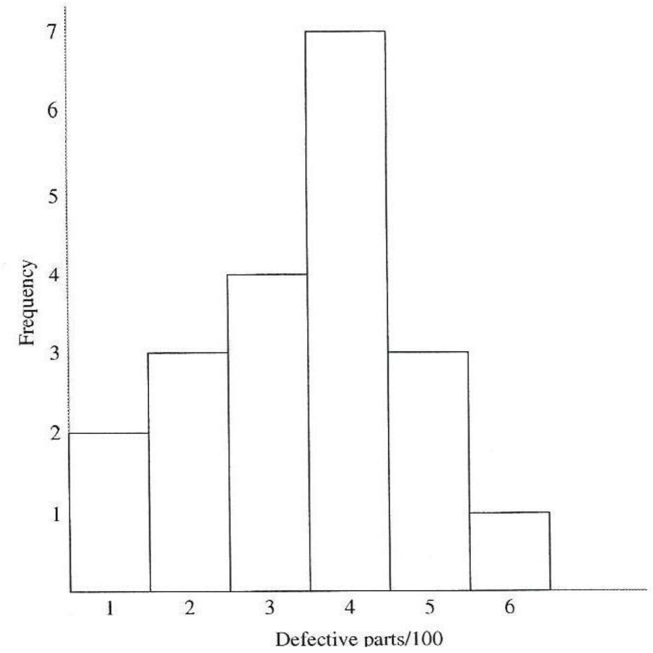


FIGURE C–80 Histogram for Practice Problems 26–1 #3.

Score intervals	***62–66***	***67–71***	***72–76***	***77–81***	***82–86***	***87–91***
Tally	||	|||	~~||||~~ ||	~~||||~~	~~||||~~	|||
	2	3	7	5	5	3

FIGURE C–81 Frequency distribution table for Practice Problems 26–1 #4.

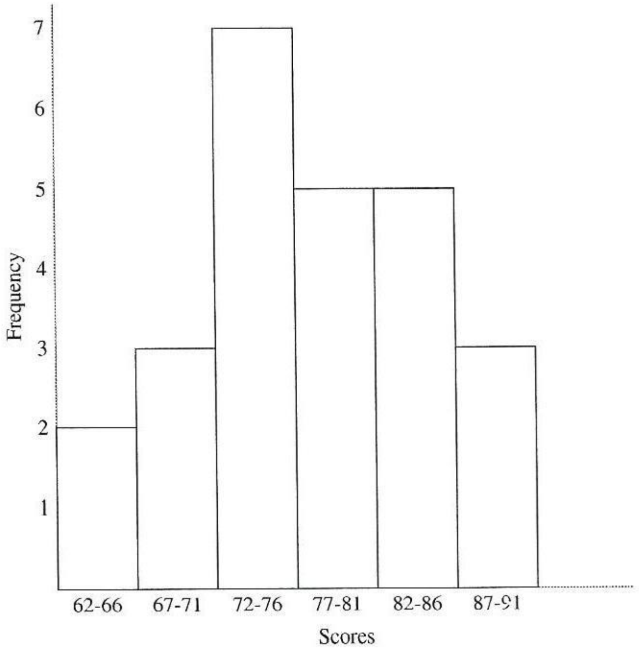


FIGURE C–82 Histogram for Practice Problems 26–1 #4.

Score intervals	***25–31***	***32–38***	***39–45***	***46–52***	***53–59***	***60–66***	***67–73***	***74–80***
Tally	|||	~~||||~~	~~||||~~ |||	~~||||~~ ||	||	|||	|	||
	3	5	8	7	2	3	1	2

FIGURE C–83 Frequency distribution table for Practice Problems 26–1 #5.

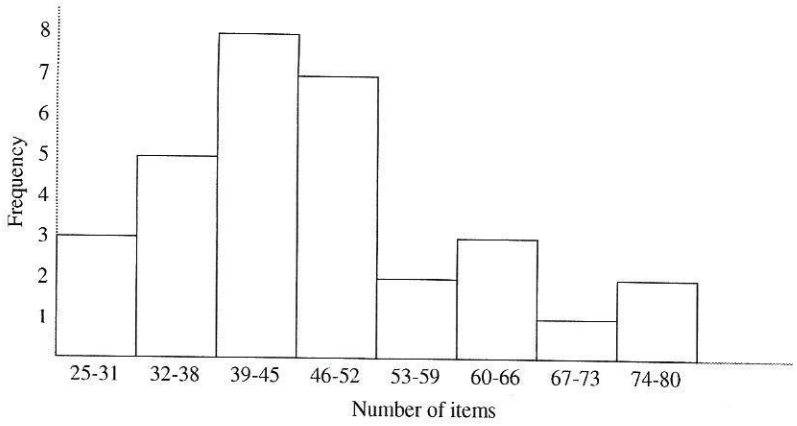


FIGURE C–84 Histogram for Practice Problems 26–1 #5.

FIGURE C–85 Frequency distribution table for Practice Problems 26–1 #6.

Gain in dB	***184–188***	***189–193***	***194–198***	***199–203***	***204–208***	***209–213***
Tally	|	|||	卌	卌 卌	||||	||
	1	3	5	10	4	2

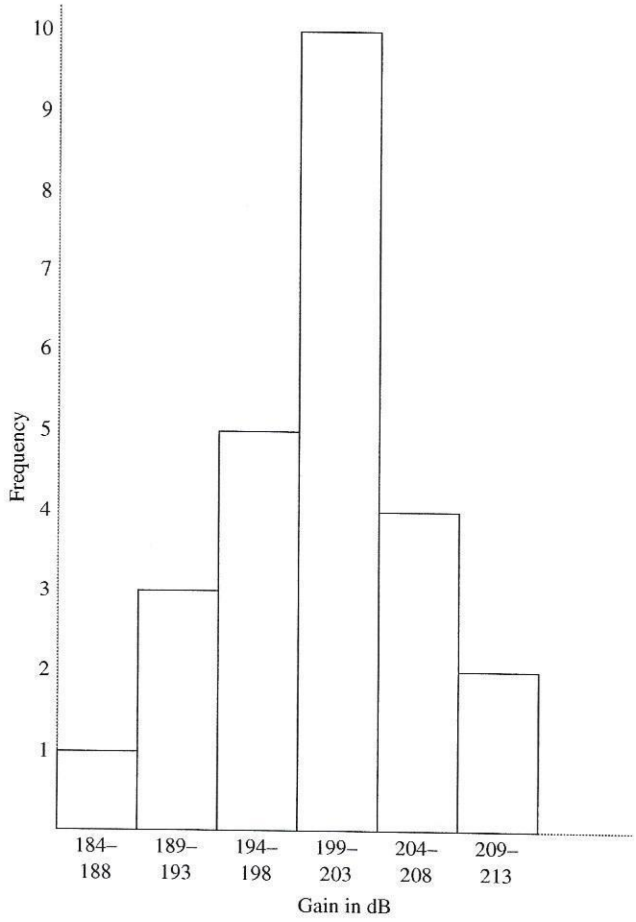


FIGURE C–86 Histogram for Practice Problems 26–1 #6.

END OF CHAPTER PROBLEMS 26–1

1. Figure C–87 is the frequency distribution table. Figure C–88 is the histogram.

3. Figure C–89 is the frequency distribution table. Figure C–90 is the histogram. Range = 98 − 60 = 38. Eight groups of five starting at 60 and extending to 99 are used. Interval midpoints are 62, 67, 72, 77, 82, 87, 92, and 97.

Scores	***60***	***65***	***70***	***75***	***80***	***85***	***90***
Tally	𝍷	𝍷𝍷	𝍷𝍷	𝍸	𝍷𝍷	𝍷𝍷	𝍷
	1	2	2	5	2	2	1

FIGURE C–87 Frequency distribution table for problem 1 of End of Chapter Problems 26–1.

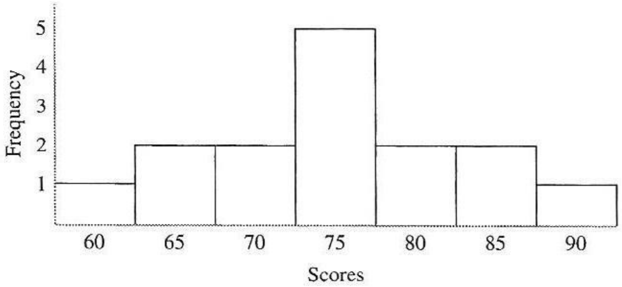


FIGURE C–88 Histogram for problem 1 of End of Chapter Problems 26–1.

Scores	***60-64***	***65-69***	***70-74***	***75-79***	***80-84***	***85-89***	***90-94***	***95-99***
Tally	𝍷𝍷	𝍷𝍷𝍷	𝍷𝍷𝍷𝍷	𝍸𝍷𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷	𝍷𝍷	𝍷
	2	3	4	7	4	2	2	1

FIGURE C–89 Frequency distribution table for problem 3 of End of Chapter Problems 26–1.

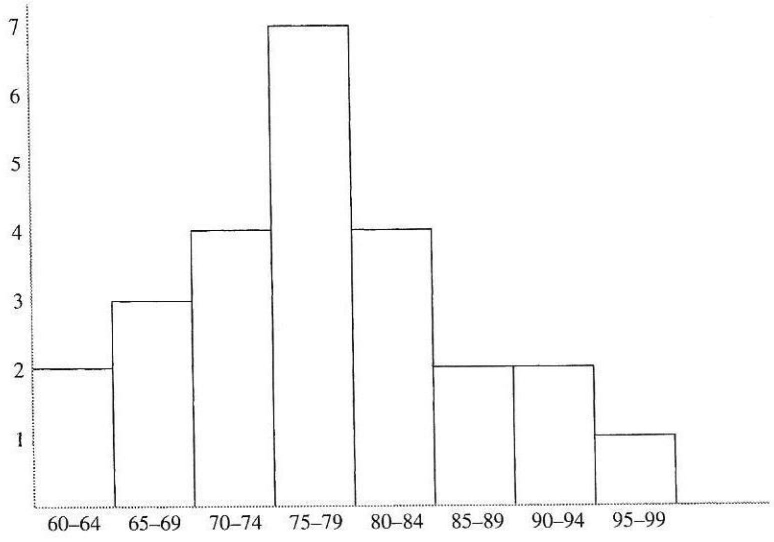


FIGURE C–90 Histogram for problem 3 of End of Chapter Problems 26–1.

5. Range = 432 dB − 370 dB = 62. Seven groups of 9-dB intervals from 370 dB to 433 dB are used. Interval midpoints (in dB) are 374, 383, 392, 401, 410, 419, and 428. Figure C–91 is the frequency distribution table and Figure C–92 is the histogram.

7. Range = 340 − 260 = 80 items. Nine groups of 9 items ranging from 260 to 341 are used. Interval midpoints are 264, 273, 282, 291, 300, 309, 318, 327, and 336. Figure C–93 is the frequency distribution table. Figure C–94 is the histogram.

Gain	***370–378***	***379–387***	***388–396***	***397–405***	***406–414***	***415–423***	***424–432***
Tally	𝍷	𝍷𝍷	𝍷𝍷𝍷	𝍸𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷	𝍷𝍷
	1	2	3	6	4	2	2

FIGURE C–91 Frequency distribution table for problem 5 of End of Chapter Problems 26–1.

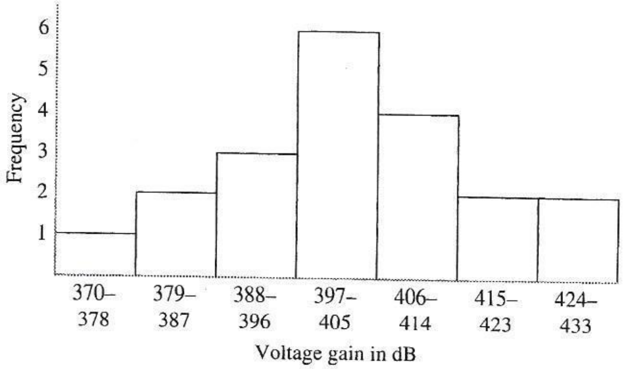


FIGURE C–92 Histogram for problem 5 of End of Chapter Problems 26–1.

Items produced	*260–268*	*269–277*	*278–286*	*287–295*	*296–304*	*305–313*	*314–322*	*323–331*	*332–340*
Tally	|	|||	||	𝍸 |	||||	|||	|||	||	|
	1	3	2	6	4	3	3	2	1

FIGURE C–93 Frequency distribution table for problem 7 of End of Chapter Problems 26–1.

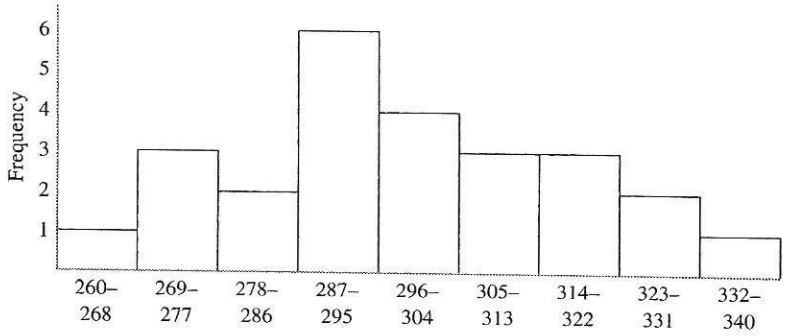


FIGURE C–94 Histogram for problem 7 of End of Chapter Problems 26–1.

9. Range = 2090 Ω − 1900 Ω = 190 Ω. Eight groups of 25 Ω ranging from 1900 Ω to 2099 Ω are used. Interval midpoints (in ohms) are 1912, 1937, 1962, 1987, 2012, 2037, 2062, and 2087. Figure C–95 is the frequency distribution table and Figure C–96 is the histogram.

11. Figure C–97 is the frequency distribution table and Figure C–98 is the histogram.

Resistor values	*1900–1924*	*1925–1949*	*1950–1974*	*1975–1999*	*2000–2024*	*2025–2049*	*2050–2074*	*2075–2099*
Tally	|	|	||	||||	𝍸 ||	𝍸	|||	||
	1	1	2	4	7	5	3	2

FIGURE C–95 Frequency distribution table for problem 9 of End of Chapter Problems 26–1.

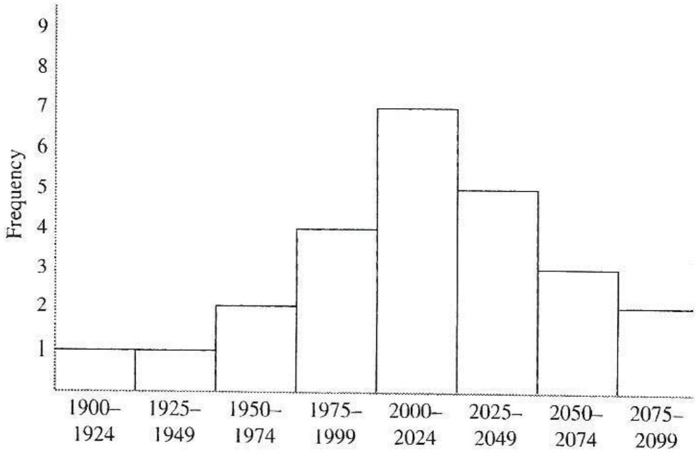


FIGURE C–96 Histogram for problem 9 of End of Chapter Problems 26–1.

Defective parts/100	*1*	*2*	*3*	*4*	*5*	*6*
Tally	||	|||	𝍸||	||||	|||	|
	2	3	7	4	3	1

FIGURE C–97 Frequency distribution table for problem 11 of End of Chapter Problems 26–1.

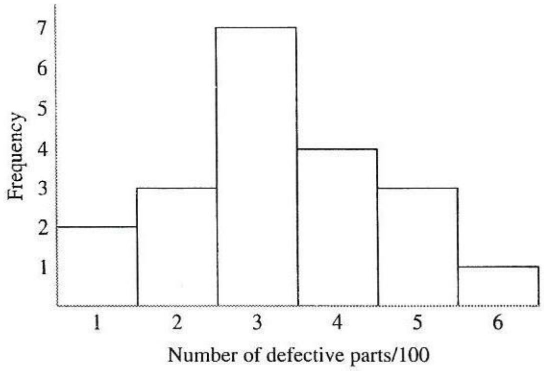


FIGURE C–98 Histogram for problem 11 of End of Chapter Problems 26–1.

PRACTICE PROBLEMS 26–2

1. $\overline{x} = \dfrac{1940}{25} = 77.6$

The median is the thirteenth value, which is 75. The mode is 75.

mean = 77.6   median = 75   mode = 75

2. $\overline{x} = \dfrac{1935}{25} = 77.4$

The mean is found by using the midpoints of each interval, which are 64, 69, 74, 79, 84, and 89. The median is the thirteenth value, which is in the interval 77 through 81. The midpoint of this interval, 79, is the median. The mode is the midpoint of the interval 72 through 76, or 74.

mean = 77.4   median = 79   mode = 74

3. mean = $\overline{x} = \dfrac{\Sigma x}{25} = 1005$

mean = 1005   median = 1010   mode = 1010

4. mean = $\overline{x}$ = 47.2
mean = 47.2   median = 42   mode = 42

5. mean = $\overline{x}$ = 3.45
mean = 3.45   median = 4   mode = 4

6. mean = $\overline{x}$ = 200
mean = 200   median = 201   mode = 201

END OF CHAPTER PROBLEMS 26–2

1. Mean = 75.0; Median = 75; Mode = 75
3. Mean = 77.4; Median = 77; Mode = 77
5. Mean = 40.3; Median = 40.1; Mode = 40.1
7. Mean = 299; Median = 300; Mode = 291
9. Mean = 2014; Median = 2012; Mode = 2012
11. Mean = 3.30; Median = 3; Mode = 3

ANSWERS TO SELF-TEST 26–1 THROUGH 26–2

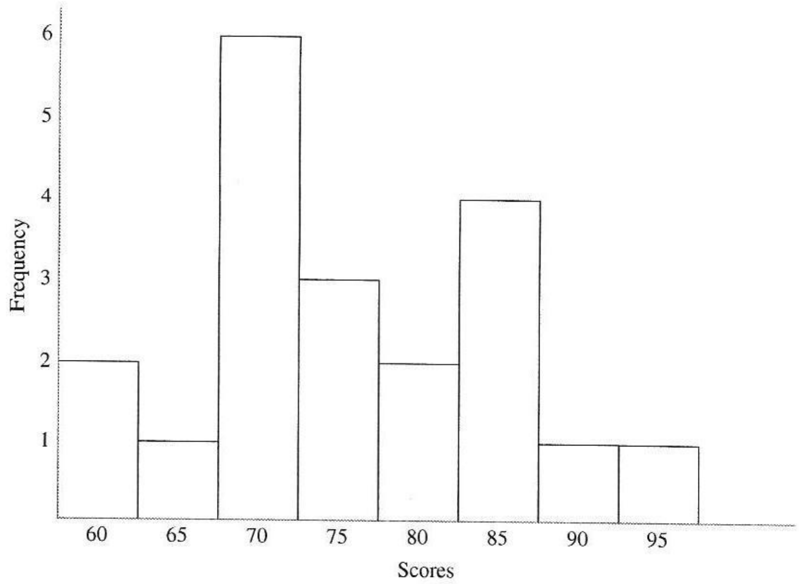
1. Figure C–99 is the frequency distribution.
Figure C–100 is the histogram.
mean = 75.8   median = 75   mode = 70

2. Figure C–101 is the frequency distribution.
Figure C–102 is the histogram. The midpoints of the intervals are 118, 127, 136, 145, 154, 163, 172, 181.
mean = 149   median = 145   mode = 154

Scores	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*
Tally	||	|	𝍸|	|||	||	||||	|	|
	2	1	6	3	2	4	1	1

FIGURE C–99

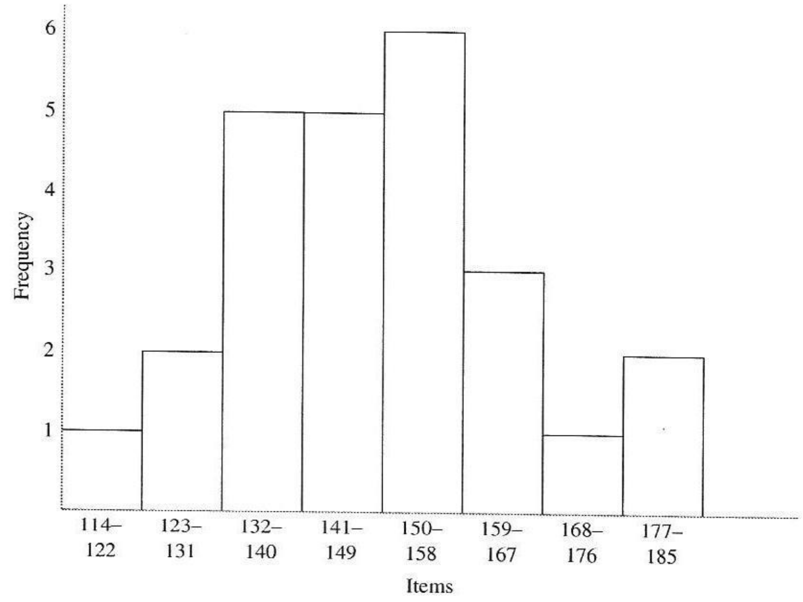
FIGURE C–100



Item intervals	*114–122*	*123–131*	*132–140*	*141–149*	*150–158*	*159–167*	*168–176*	*177–185*
Tally	|	||	卌	卌	卌 |	|||	|	||
	1	2	5	5	6	3	1	2

FIGURE C–101

FIGURE C–102



PRACTICE PROBLEMS 26–3

1. $\bar{x} = 78.1$ median = 80
   mode = 80 range = 35
   $\sigma = 9.20$
3. Midpoints are 61, 66, 71, 76, 81, 86, and 91.
   $\bar{x} = 72.8$ median = 71
   mode = 71 range = 34
   $\sigma = 7.59$
2. $\bar{x} = 632$ median = 630
   mode = 630 range = 60
   $\sigma = 14.2$
4. Midpoints are 62, 67, 72, 77, 82, 87, 92, and 97.
   $\bar{x} = 77.3$ median = 77
   mode = 72 range = 39
   $\sigma = 8.44$

5. Midpoints are 208, 225, 242, 259, 276, 293, 310, and 327.
$\overline{x} = 272$ median = 276
mode = 259 range = 135
$\sigma = 29.3$

END OF CHAPTER PROBLEMS 26–3

1. Mean = 77.4; Median = 75; Mode = 75; $\sigma = 9.41$. Range = 40
3. Mean = 77.4; Median = 80; Mode = 80; $\sigma = 10.2$. Range = 40
5. Mean = 73.4; Median = 74; Mode = 74; $\sigma = 11.2$. Range = 48
7. Mean = 71.9; Median = 72; Mode = 82; $\sigma = 11.2$. Range = 44
9. Mean = 473; Median = 484; Mode = 484; $\sigma = 67.4$. Range = 327
11. Mean = 561; Median = 552; Mode = 552; $\sigma = 32.3$. Range = 146

PRACTICE PROBLEMS 26–4

1. If $\sigma = 2$, then:

$$25\text{ dB} - 1\,\sigma = 23\text{ dB}$$
$$25\text{ dB} + 1\,\sigma = 27\text{ dB}$$

68% of all the amplifiers have an output between 23 dB and 27 dB. 600 × 0.34 = 204. 34% or 204 amplifiers have an output between the mean (25 dB) and 23 dB. 34% or 204 amplifiers have an output between the mean and 27 dB. 95% of all amplifiers have an output between $\pm 2\,\sigma$ from the mean.

$$25\text{ dB} - 2\,\sigma = 25\text{ dB} - 4\text{ dB} = 21\text{ dB}$$
$$25\text{ dB} + 2\,\sigma = 25\text{ dB} + 4\text{ dB} = 29\text{ dB}$$
$$600 \times 0.475 = 285$$

47.5% of 600 amplifiers or 285 amplifiers will have outputs between 21 dB and 25 dB ($-2\,\sigma$).

47.5% of 600 amplifiers or 285 amplifiers will have outputs between 25 dB and 29 dB ($+2\,\sigma$). 99.7% of all amplifiers will have outputs between $\pm 3\,\sigma$ from the mean.

$$25\text{ dB} - 3\,\sigma = 25\text{ dB} - 6\text{ dB} = 19\text{ dB}$$
$$25\text{ dB} + 3\,\sigma = 25\text{ dB} + 6\text{ dB} = 31\text{ dB}$$
$$600 \times 0.4985 = 299$$

49.85% of the amplifiers or 299 amplifiers will have outputs between 19 dB and 25 dB.

49.85% of the amplifiers or 299 amplifiers will have outputs between 25 dB and 31 dB.

299 + 299 = 598. Two amplifiers will have outputs beyond $\pm 3\,\sigma$.

2. 1 σ above 70 = 70 + 4 = 74.1 σ below 70 = 70 − 4 = 66.

$$1500 \times 34\% = 510$$

510 students scored between 66 and 70.
510 students scored between 70 and 74.
2 σ above 70 = 70 + 8 = 78.2 σ below 70 = 70 − 8 = 62.

$$1500 \times 47.5\% = 713$$

713 students scored between 62 and 70.
713 students scored between 70 and 78.
3 σ above 70 = 70 + 12 = 82.3 σ below 70 = 70 − 12 = 58.

$$1500 \times 49.85\% = 748$$

748 students scored between 58 and 70.
748 students scored between 70 and 82.
748 + 748 = 1496
Two students had scores greater than 82.
Two students had scores less than 58.

END OF CHAPTER PROBLEMS 26–4

1. 340 (68%) are between 39 mm and 41 mm long.
475 (95%) are between 38 mm and 42 mm long.
499 (99.7%) are between 37 mm and 43 mm long.
1 will be greater than 3 σ from the mean.

3. During 20 days (68%), between 590 and 610 parts will be produced per day.
During 28 days (95% rounded down to a full day) between 580 and 620 parts will be produced each day.
Between 570 and 630 parts will be produced each day.

5. 3400 of the scores will fall between 67.4 and 78.6.
4750 of the scores will fall between 61.8 and 84.2.
4985 of the scores will fall between 56.2 and 90.8.
15 of the scores will be less than 56.2 or greater than 90.8.

ANSWERS TO SELF TEST 26–3 THROUGH 26–4

1. $\overline{x} = 77.8$ $\sigma = 9.26$ median = 75 mode = 75 range = 40
2. Midpoints are 620, 661, 702, 743, 784, 825, 866.
   $\overline{x} = 737$ $\sigma = 50.3$ median = 743 mode = 743 range = 286
3. If $\sigma = 6$, then:
   $70 - 1\sigma = 70 - 6 = 64.70 + 1\sigma = 70 + 6 = 76.$
   68% of all the grades fall between 64 and 76.34% (half of 68%) fall between 64 and 70 and 34% fall between 70 and 76.
   $3000 \times 0.34 = 1020$
   1020 of the students' scores were between 64 and 70.
   1020 of the students' scores were between 70 and 76.
   95% of all the scores fall $\pm 2\sigma$ from the mean.
   $70 - 2\sigma = 70 - 12 = 58.\ 70 + 2\sigma = 70 + 12 = 82.$
   47.5% of the scores (half of 95%) fall between 58 and 70.
   47.5% are in the range from 70 to 82.
   $3000 \times 0.475 = 1425$
   1425 of the students' scores were between 58 and 70.
   1425 of the students' scores were between 70 and 82. $1425 + 1425 = 2850$
   The normal curve shows that almost all the scores (99.7%) are $\pm 3\sigma$ or less from the mean.
   $3000 \times 0.4985 = 1496$ (rounded up from 1495.5).
   $70 - 3\sigma = 70 - 18 = 52.\ 70 + 3\sigma = 70 + 18 = 88.$
   1496 of the students' scores were between 52 and 70.
   1496 of the students' scores were between 70 and 88.
   Some scores were *greater than* $\pm 3\sigma$ from the mean. In this case, 9 (0.3% of 3000).

Power gain in Decibels (dB):

$$A_p = 10 \times \log \frac{P_{out}}{P_{in}}$$

Table 23-1 Power Gains and Losses using Decibels (dB).

POWER GAIN	dB VALUE*	LINEAR MULTIPLIER EQUIVALENT
Gain	90	10^9
Gain	80	10^8
Gain	70	10^7
Gain	60	10^6
Gain	50	10^5
Gain	40	10^4
Gain	30	10^3
Gain	27	512
Gain	24	256
Gain	21	128
Gain	20	100
Gain	18	64
Gain	15	32
Gain	12	16
Gain	10	10
Gain	9	8
Gain	6	4
Gain	3	2
No change	0	1
Loss	−3	1/2
Loss	−6	1/4
Loss	−9	1/8
Loss	−10	1/10
Loss	−12	1/16
Loss	−15	1/32
Loss	−18	1/64
Loss	−20	1/100
Loss	−21	1/128
Loss	−24	1/256
Loss	−27	1/512
Loss	−30	10^{-3}
Loss	−40	10^{-4}
Loss	−50	10^{-5}
Loss	−60	10^{-6}
Loss	−70	10^{-7}
Loss	−80	10^{-8}
Loss	−90	10^{-9}

*dB values for non-decade multipliers are approximate.

Power measurement in dBm is:

$$P_{dBm} = 10 \times \log \frac{P_{mW}}{1mW}$$

Table 23-2 Power Levels using dBm.

dBm VALUE*	MILLIWATTS
90	10^9
80	10^8
70	10^7
60	10^6
50	10^5
40	10^4
30	10^3
27	512
24	256
21	128
20	100
18	64
15	32
12	16
10	10
9	8
6	4
3	2
0	1
−3	1/2
−6	1/4
−9	1/8
−10	1/10
−12	1/16
−15	1/32
−18	1/64
−20	1/100
−21	1/128
−24	1/256
−27	1/512
−30	10^{-3}
−40	10^{-4}
−50	10^{-5}
−60	10^{-6}
−70	10^{-7}
−80	10^{-8}
−90	10^{-9}

*dBm values for non-decade multipliers are approximate.